Shared Sensing and Communications in Sensor Networks : The Multihop Case

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***Abstract*— A joint sensing/communication problem is considered for sensor networks. Herein, the channel(s) between a source and destination are the parameters to be sensed and communicated over the network. Lower bounds on the end-to-end distortion are developed for a multihop, linear network. Inter-node communication is assumed to be done via an encode-and-forward approach. For a many-to-one topology with two hops, orthogonal communication schemes are compared to other possible schemes and found to be optimal in the sense of minimizing the sum distortion of all the channels in the network.**

I. Introduction

In this paper, we study a special class of sensor networks where the unknown time-varying (communication) channel between the nodes is the sensor data of interest. Thus, communication and sensing share both the bandwidth and transmit power at each node in the network, in contrast to most sensor network formulations in the work. The bulk of the prior literature on communication and sensing in sensor networks focuses on estimating a parameter or process extrinsic to the network and then using the sensor network for communicating an estimate or a pre-estimate to a fusion center or base-station type node (*e.g.* [1–3]). While the work in [4] also considers extrinsic parameter estimation; there is a tradeoff to be made between sensing (estimation) and communication. In [4], each node in the network observes a single phenomenon and thus each node has correlated observations. The tradeoff therein considers the rate to be assigned to each node for transmitting the innovation at each node.

In [5], we introduced our joint sensing and communication problem, where end-to-end distortion was considered for simple two-hop networks. In the current work, we extend our results to multiple hops and make rigorous a conjecture made in [5] for many-to-one network topologies. Herein, we shall focus on the *encode-and-forward* protocol for shared modality sensor networks. This protocol is inspired by well-known protocols for data forwarding in relay channels [6, 7]. A lower bound on the distortion for channel estimation for linear networks (nodes arranged in a line as depicted in Figure 1) is developed and analyzed for asymptotically high SNR, thus generalizing results in [5].

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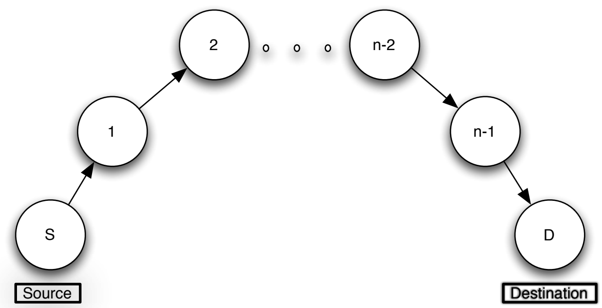

Fig. 1. Multi-hop linear network.

To consider problems seen more often in the real world, we next examine a more complex network topology. This is a two level tree-network where the first level has M nodes connected to a single relay node. The relay node then communicates (and senses) with the destination node as shown in Figure 2. The encode-and-forward results are generalized for this topology. We also prove that time orthogonal communication is optimal for the first hop of the network.

The joint sensing and communication leads to some interesting differences compared to traditional sensor networks. The single hop results of [8] suggest an intuitive scheme for state estimation when the transmitter has perfect knowledge of the state. However, this is not the case in most practical systems. Moreover, as the number of hops increases, the problem becomes more complicated because the number of observations is fixed. A few general observations from the problem illustrate its specific characteristics. First, more transmit power not only improves communication performance (throughput and/or error rates), but also simultaneously improves sensing accuracy. Thus, for sensor nodes with limited energy resources, the sensing and communication tasks do not "contend" for the same source of power. In contrast, in most sensor networks, the battery power is shared between sensing and communication subsystems such that more power for one task does not necessarily improve the performance of the other task. In fact, this is the major reason why we can study sensing performance asymptotics as a function of received

SNR for small networks, while most asymptotic analysis in sensor networks requires a large network [9, 10]. However, note that for the shared modality networks, the resource of bandwidth or time is one for which there is contention.

Second, the shared sensing and communication networks have fewer unknowns to estimate than most other sensor networks. Consider two sensor networks: one in which the channel is the sensor data of interest (the network analyzed in this paper) and one in which sensor data is independent of the channel (traditionally analyzed sensor networks). In both cases, the channel is unknown and hence the communication task has to account for the time-varying unknown channel. However, in the first network, there is no other unknown while in the second network, the additional unknown is the output of the sensors (the sensor data). Thus, the number of unknowns in shared modality networks is smaller. We make no direct comparisons between these two classes of networks as they cannot be interchanged in practice.

In this paper, we develop lower bounds on the end-to-end distortion for encode-and-forward based estimation/communication protocols. For the encode-and-forward scheme, as presented in this paper, it is interesting to note that the bottleneck link (the one that transmits for the least time) affects the estimates of all links before it. This lends itself to heurestic schemes for improving network performance in an intuitive minimax way. It will be interesting to see if these results carry over to other network topologies and/or other communication schemes like the amplify-and-forward protocol.

The rest of the paper is organized as follows. Section II introduces the signal model and the formulation of the problem as a minimization problem and defines the different communication schemes. In Section III we consider encode-and-forward for a linear network. These results are extended to a variation of the linear network in Section IV. Section V presents and discusses a few numerical results and finally, we summarize and discuss avenues for further research in Section VI.

II. Preliminaries

A. Channel Model

Consider the N-node network in Figure 1. At each node,n, Y_n is the received message and X_n is the corresponding sent message. The channel between node $i-1$ and i is denoted by h_i; thus the channel between the source (node 0) and node 1 is h_1. All channels, h_i, are assumed to be narrow-band and flat fading. The channel gains are standard Gaussian random variables, $h_i \sim \mathcal{N}(0,1)$ and the additive channel noises are Gaussian as well, $Z_i \sim \mathcal{N}(0,\sigma_i^2)$. The channel coefficients and noises are also assumed to be mutually independent. Finally the channels are assumed to have a common coherence interval of T seconds, such that all channels change to a new realization every T seconds. We make the following simplifying assumption – communication in the network is time orthogonal, *i.e* when one node is transmitting data, every other node is silent.

B. Problem Formulation

The observed distortion in the channels is of interest at the destination. The distortion between the estimate and the actual channel is given by their mean-squared error,

$$D_i = \mathbb{E}|h_i - h_{i,d}|^2$$

where $h_{i,d}$ is the reconstruction at the destination. The feasible distortion region $\mathcal{D}$ is then described as all those $(D_1, D_2, \ldots, D_n)$ n-tuples which can be simultaneously achieved. Of interest is investigating the achievable *diversity* of the joint communication and sensing problem. We shall define diversity as the exponent on the decay rate of the distortion as a function of the signal-to-noise ratio. That is, we achieve diversity r if

$$\lim_{\forall j \;\; \mathrm{SNR}_j \to \infty} D_i \quad = \quad O\left(\mathrm{SNR}_i^{-r}\right)$$

where $\mathrm{SNR}_j \doteq \frac{P_j}{\sigma_j^2}$.

As previously noted, we assume encode-and-forward based inter-node communication. That is, each relay node, j, forms an estimate of the channel for all preceding nodes. , node j estimates h_1^j and optimally encodes this vector for communication over channel h_j, to the next node in the network $j+1$.

III. Linear Network

The following equations describe the signal model for the n-hop linear network,

$$\begin{aligned}
X_S = X_0 &= \sqrt{P_0}, t \in I_0 \\
Y_1 &= \sqrt{P_0} h_1 + Z_1, t \in I_0 \\
X_1 &= \sqrt{P_1}\beta_1 f_1(\hat{h}_1), t \in I_1 \\
Y_2 &= \sqrt{P_1}\beta_1 f_1(\hat{h}_1) h_2 + Z_2, t \in I_1 \\
&\vdots \\
Y_D = Y_n &= \sqrt{P_{n-1}}\beta_{n-1} f_{n-1}(\hat{h}_{1,\ldots,n-1}) h_n + Z_n, t \in I_{n-1}
\end{aligned}$$

where I_j is the interval $[\sum_{i=0}^{i=j-1} T_i, \sum_{i=0}^{i=j} T_i]$ ($I_0 = [0, T_0]$ and $\sum_{i=0}^{n} T_i = T$). Given our focus on mean-squared error, the optimal estimator is the minimum mean-squared error estimate (MMSE). We denote the MMSE for channel i at node j as, $\hat{h}_i^j = \mathbb{E}[h_i|Y_j]$. In the encode-and-forward scheme, preliminary estimates of all channels preceding are made at a node, thus we define $\hat{h}_{i,\ldots,j} = [\hat{h}_i \hat{h}_{i+1} \ldots \hat{h}_j]$ as the MMSE of $h_{i,\ldots,j}$. The minimum distortion for the estimate of h_i at the destination node is denoted D_i. Finally, we define D_i^j to be the contribution to the minimum distortion in h_i at the jth node. Note that $D_i = \sum_{j=i}^{n} D_j^n$. This decomposition holds due to the orthogonality of the MMSE detector(see also [11]).

Lemma 1: D_i^j is lower bounded by:

$$\begin{aligned}
D_i^j &\geq \mathrm{var}(\hat{h}_i^{j-1})(1+\mathrm{SNR}_j)^{-\frac{T_j}{T}} && (1) \\
\text{where } \hat{h}_i^j &= \mathbb{E}[h_i|Y_j] && (2)
\end{aligned}$$

Proof: We use the expression for the rate distortion function for Gaussian channels, use the coherent capacity as a bound for the non-coherent capacity and then use Jensen's inequality to bound the coherent capacity conditional on the channel realization. (See [5] for an extended, more methodical derivation of this lemma which exploits the results of [11]). ■

Lemma 2: Given Y_i, we can lower bound D_i as,

$$D_i \geq \frac{1}{\text{SNR}_i T_i + 1} \tag{3}$$

Proof: If we assume that we know $h_{1,\ldots,i+1}$ in addition to Y_i, this bound follows from the computation of the estimation error for the MMSE. ■

Theorem 1 (An n-hop Encode and Forward Bound): Given the signal model described above, we can form the following lower bounds:

$$D_n \geq \frac{1}{\text{SNR}_n T_n + 1} \tag{4}$$

$$D_i \geq L_i = \sum_{j=i}^{n} L_i^j, \; i < n \tag{5}$$

where, L_i is the bound on D_i and the bound L_i^j on D_i^j is formed as:

$$L_i^i = \frac{1}{\text{SNR}_i T_i + 1} \tag{6}$$

$$L_i^j = (1 - L_i^{j-1})(1 + SNR_j)^{-\frac{T_j}{T}} \, j > i \tag{7}$$

Proof: Recall that $\hat{h}_i^j$ is the MMSE of h_i at node j. Then, by definition, $\text{var}(\hat{h}_i^j) = \mathbb{E}\left[\hat{h}_i^j\right]^2$. By the orthogonality property of MMSE's, we can show that:

$$D_i^{j-1} = 1 - \text{var}(\hat{h}_i^{j-1}) \tag{8}$$

We also have:

$$\text{var}(\hat{h}_i^j) \geq \text{var}(\hat{h}_i^{j-1}) \tag{9}$$

since the variance of the estimator can only increase as we get further away from hop i.

Summing Equation (8) and Equation (9), we have:

$$\text{var}(\hat{h}_i^j) \geq 1 - D_i^{j-1} \tag{10}$$

Using Equation (10) and Lemma 1, yields

$$D_i^j \geq (1 - D_i^{j-1})(1 + \text{SNR}_j)^{-\frac{T_j}{T}} \tag{11}$$

Initializing with L_i^i from Lemma 2 and applying Equation (11) recursively leads to the bound in the theorem statement. ■

Corollary 1 (Diversity Factor for n-hops): For encode-and-forward, assuming that $\text{SNR}_j = \text{SNR} \;\; \forall j$, the diversity factor for estimating h_i is bounded as,

$$r_i \quad \leq \quad \min_{j>i} \frac{T_j}{T} \tag{12}$$

Proof:

We define $A_j = 1 + \text{SNR}_j$. Using Equations (6) and (7), the expression for L_i^j is:

$$\begin{aligned} L_i^j &= (1 - L_i^{j-1}) A_j^{-\frac{T_j}{T}} \\ &= A_j^{-\frac{T_j}{T}} - (1 - L_i^{j-2}) A_{j-1}^{-\frac{T_{j-1}}{T}} A_j^{-\frac{T_j}{T}} \\ &= A_j^{-\frac{T_j}{T}} - A_{j-1}^{-\frac{T_{j-1}}{T}} A_j^{-\frac{T_j}{T}} + \ldots \\ &= \sum_{k=i+1}^{j} (-1)^{j-k} \prod_{m=k}^{j} A_m^{-\frac{T_m}{T}} + (-1)^{j-i} \prod_{k=i+1}^{j} A_k^{-\frac{T_k}{T}} L_i^i \end{aligned}$$

Now, let $\text{SNR}_j = \text{SNR} \; \forall \; j$, then $A_j = A \; \forall \; j$ and the above summation simplifies to:

$$L_i^j = \sum_{k=i+1}^{j} (-1)^{j-k} A^{-\frac{\sum_{m=k}^{j} T_m}{T}} + (-1)^{j-i} A^{-\frac{\sum_{k=i+1}^{j} T_k}{T}} L_i^i$$

As $\text{SNR} \to \infty$, the term in the sum above corresponding to $k = j$ dominates the expression (since it has the largest exponent). To form the lower bound, the L_i^j's are summed. As such, the diversity factor for estimating h_i is upperbounded by $\min_{j>i} \frac{T_j}{T}$. ■

We note that subsequent channel estimates will achieve lower end-to-end distortion than "earlier" channels. We next formalize this notion for the three-hop case,which is easily extended to n hops. First, we normalize the observation interval T to 1 over which communication and estimation is conducted. The bounds from Theorem 1 can be shown to have the following relationships:

$$\begin{aligned} L_3 &= \frac{1}{\text{SNR}_3 T_3 + 1} && (13) \\ &< (1 + \text{SNR}_3)^{-T_3} && (14) \\ &< L_2 && (15) \end{aligned}$$

Consider $f(x) = (1 + x)^y - 1 - xy$, then $f(0) = 0$ and $f'(x) < 0$ whenever $x > 0$ and $y < 1$. This proves Equation (14) while Equation (15) follows since D_2 can be written as a convex sum of 1 and the quantity in Equation (14) which is upper bounded by 1. Similarly, we have;

$$\begin{aligned} L_2 < L_1 \iff & (1 - \alpha) + \alpha(1 + \text{SNR}_3)^{-T_3} \\ & > (1 - \beta) + \beta(1 + \text{SNR}_3)^{-T_3} \end{aligned} \tag{16}$$

where $\alpha = 1 - (1 + \text{SNR}_2)^{-T_2}$ and $\frac{1}{1-\beta} = \text{SNR}_2 T_2 + 1$ (this statement follows by simple algebra after setting $\text{SNR}_1 \to \infty$). Again we appeal to the properties of the function f defined above and those of convex combinations to verify that Equation (16) is valid.

IV. Many To One Relay

Our ultimate goal is to investigate the joint sensing and communication problem for arbitrary network topologies. To this end, an important generalization of the linear network is to consider a two-level network where multiple nodes communicate to a relay and then the relay communicates over a single channel to the destination. We denote this topology as

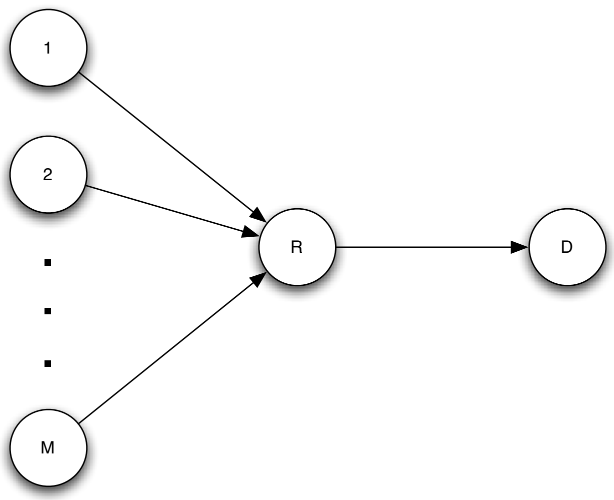

Fig. 2. Node Topology for the Many to One Relay Problem

the *many-to-one* network. Our goal in this section is determine a good communication protocol for the first level of the many-to-one network described in Figure 2. The quality of a communication scheme is gauged by the sum distortion of all the channels at the destination. In this scenario, the relay receives training signals from M sources for the M channels. The relay then jointly codes the estimated channels and transmits them over the final link to the destination. The destination must then decode/estimate all $M+1$ channels of interest. We observe that several protocols are possible for this problem and we prove that any orthogonal communication scheme is optimal for this network. In particular, time orthogonal communication is one optimal scheme and is employed for its ease of use.

Theorem 2 (Orthogonal Communication is optimal): Any orthogonal communication scheme is optimal, in the sense of minimizing the sum distortion of the channels, for the many-to-one network shown in Figure 2.

Proof: The signal received at the relay can be written in vector form as:

$$\mathbf{y} = A\mathbf{h} + \mathbf{n} \tag{17}$$

where A is a matrix that depends on the signalling scheme used, and $\mathbf{n}$ is the noise vector. Note that the dimensions of $\mathbf{y}$, $\mathbf{h}$ and $\mathbf{n}$ are $M \times 1$ and A is $M \times M$. The ith column of the matrix A is formed at the relay node by collecting all elements of the received signal which have contributions from h_i. The only constraint in the problem is $\sum_{i=1}^{M} \lambda_i^2 \leq P$ where λ_i is the i-th eigenvalue of the matrix A and P is the total power transmitted by the source nodes. Since we assume that the channel is independent for each source and that the channel is normalized, mean-zero and Gaussian, we have $\mathbf{h} \sim \mathcal{N}(0, I)$. Finally, since we assume that the channel noises are mutually independent, we have $\mathbf{n} \sim \mathcal{N}(0, \text{diag}(\sigma_1^2, \ldots, \sigma_M^2))$. For convenience, we define $S = \text{diag}(\sigma_1^2, \ldots, \sigma_M^2)$.

Given the signal model in Equation (17), since the relay node knows $\mathbf{y}$ and A, we can form an MMSE of $\mathbf{h}$, say $\hat{\mathbf{h}}$. The covariance matrix of the error $\mathbf{e} = \hat{\mathbf{h}} - \mathbf{h}$ is then given by:

$$K = I - A^T[AA^T + S]^{-1}A$$

and so

$$D = \text{trace}(K) = M - \sum_{i=1}^{M} \frac{\lambda_i^2}{\lambda_i^2 + \sigma_i^2} = \sum_{i=1}^{M} \frac{\sigma_i^2}{\lambda_i^2 + \sigma_i^2} \tag{18}$$

where D is the total sum distortion observed at the relay node. To minimize D, we then need to minimize the sum on the right hand side of Equation (18):

$$\begin{aligned} \text{minimize } & \sum_{i=1}^{M} \frac{\sigma_i^2}{\lambda_i^2 + \sigma_i^2} \\ \text{subject to } & \sum_{i=1}^{M} \lambda_i^2 \leq P \end{aligned}$$

This equation can be solved by Lagrange multipliers and the Karush-Kuhn-Tucker (since λ_i^2 must be positive) conditions to to yield the solution:

$$\lambda_i^2 = \sigma_i(\nu - \sigma_i)^+$$

where ν is chosen such that

$$\sum_{i=1}^{M} \sigma_i(\nu - \sigma_i)^+ = P$$

Here, $(x)^+$ denotes the positive part of x, *i.e.*

$$(x)^+ = \begin{cases} x & \text{if } x > 0, \\ 0 & \text{if } x \leq 0. \end{cases}$$

In particular, we can create a diagonal matrix for A, the eigenvalues of which satisfy the conditions outlined above thus proving the theorem. ■

Note that Theorem 2 only claims the optimality of orthogonal communication schemes for the problem of minimizing the sum distortion of all channels at the relay and that these claims need not hold when the criterion for optimality is changed, for example, to minimizing the sum of the probability of error for all the channels. Once time orthogonality is accepted as optimal, the end to end distortion faced by individual sources can be bounded by an application of Lemmas 1 and 2:

$$\begin{aligned} D_i &\geq \alpha_i + (1 - \alpha_i)(1 + \text{SNR}_{M+1})^{-\frac{T_{M+1}}{T}} \;\; i \leq M \\ \text{where, } \;\; \alpha_i &= \frac{1}{\text{SNR}_i T_i + 1} \\ D_{M+1} &\geq \frac{1}{SNR_{M+1}T_{M+1} + 1} \end{aligned}$$

where we assume we are given the source powers $\{P_i\}_{i=1}^{M+1}$, the channel noises variances $\{\sigma\}_{i=1}^{M+1}$ and the optimal transmission times $\{T_i\}_{i=0}^{M+1}$ for all the nodes in the network.

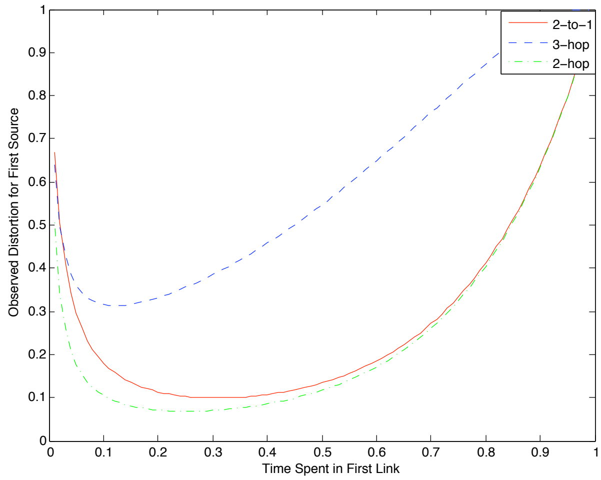

Fig. 3. Comparison of the bounds on D_1 for linear and many-to-one networks

V. Results

A comparison of the lower bound derived in Theorems 1 and 2 for the case when the network comprises of two and three links is presented in Figure 3. We consider sources with an SNR of 20dB and total time T set to 1 for normalization. Note that in the three hop case, we optimize for D_1 over the time spent in the second and third channels and that in the 2-to-1 network, the time spent in the first link is divided evenly between the two sources (which is intuitive given the equal powers for the two sources).

Observe that the bounds are very steep at low T_1. This is to be expected for the conditions assumed ($\text{SNR}_1 = \text{SNR}_2 = \text{SNR}_3 =$20dB), since at low T_1, any increase provides a large improvement in distortion. Also note that the bound for the two-hop network is less sensitive to changes in T_1 for a large interval of time while the three-hop bound is "sharper". As the number of nodes in the network grow, the bounds become more sensitive to T_1 due to the larger number of variables that influence the bound and need to be optimized over. Further, the slope of the curves changes as $T_1 \to 1$. ($D_1 = 1$ trivially when T_1 is 1). This is because when T_1 is close to 1, while relay node 1 has a good estimate of h_1, since this information now has to travel over more hops (in the same time) to reach the destination, the distortion faced is greater which translates to a lower slope for more nodes. As the number of nodes tends to ∞, it is to be expected that the curve approaches the point (1,1) at a zero slope.

It is intuitive to observe that the distortion in the 2-hop topology and the 2-to-1 topology are similar. Further, the 2-to-1 case is lower bounded by the 2-hop case since the time spent in the first link is now used to estimate two different sources thus leading to a higher distortion in both. Also note that while the 2-to-1 topology has the same number of channels of interest as the 3-hop network, the hierarchically layered nature of the 2-to-1 network leads to a reduced distortion at the destination. This feature is expected to become more prominent as the number of nodes increases while the time available for communication remains fixed.

VI. Conclusions

We have derived lower bounds on the distortion in channel estimates for a simple linear network as in Figure. 1 with encode-and-forward based communication between the relays. For a symmetric case, where each link has the same SNR, the distortion for the estimation of channel i is limited by the smallest transmission duration between node $i-1$ and the destination. This suggests that for more general cases there is an effective SNR and transmission duration measure that should be equalized over all links in order to ensure equal distortion at the destination. In addition, we have shown that under very mild conditions, orthogonal communication yields lower distortions than any other communication scheme for the many-to-one network in Figure. 2. Ongoing work is completing the derivation of distortion bounds for amplify-and-forward based communication for multihop networks and applying the current results to more complex networks such as those with tree topologies.

References

[1] R. Nowak, U. Mitra, and R. Willett, "Estimating inhomogeneous fields using wireless sensor networks," *IEEE Journal on Selected Areas of Communications*, vol. 22, no. 6, pp. 999–1006, August 2004.

[2] S. Cui, J.-J. Xiao, A. J. Goldsmith, Z.-Q. Luo, and H. V. Poor, "Energy-efficient joint estimation in sensor networks: analog versus digital," in *Proc. of ICASSP*, March 2005, pp. 745–748.

[3] G. Thatte and U. Mitra, "Sensor selection and power allocation in sensor networks," *Information Processing in Sensor Networks*, 2006, submitted.

[4] S. Draper and G. Wornell, "Side information aware coding strategies for sensor networks," *IEEE J. Selected Areas Commun.*, vol. 22, 2004.

[5] S. Vedantam, U. Mitra, and A. Sabharwal, "Sensing the channel: Sensor networks with shared sensing and communications," *Information Processing in Sensor Networks*, 2006, accepted.

[6] T. M. Cover and A. E. Gamal, "Capacity theorems for the relay channel," *IEEE Info. Theory*, vol. 25, no. 5, pp. 572–584, September 1979.

[7] J. N. Laneman, D. N. C. Tse, and G. W. Wornell, "Cooperative diversity in wireless networks: Efficient protocols and outage behavior," *IEEE Transactions on Information Theory*, vol. 50, no. 12, pp. 3062 – 3080, December 2004.

[8] A. Sutivong, M. Chiang, T. M. Cover, and Y.-H. Kim, "Channel capacity and state estimation for state-dependent gaussian channels," *IEEE Transactions on Information Theory*, vol. 51, no. 4, pp. 1486–1496, April 2005.

[9] M. Gastpar and M. Vetterli, "Power-bandwidth-distortion scaling laws for sensor networks," in *Proc. of Information Processing in Sensor Networks*, April 2004, pp. 320–329.

[10] ——, "Power, spatio-temporal bandwidth and distortion in large sensor networks," *IEEE Journal on Selected Areas in Communications*, vol. 23, no. 4, pp. 745–754, April 2005.

[11] J. K. Wolf and J. Ziv, "Transmission of noisy information to a noisy receiver with minimum distortion," *IEEE Transactions on Information Theory*, vol. 16, no. 4, pp. 406–411, July 1970.