

Suppose that we are given a coin with an unknown probability p of landing heads. By tossing this coin repeatedly, a classical trick due to von Neumann (1952) shows we can simulate an unbiased coin. Iterating von Neumann's trick leads to an efficient unbiasing procedure matching the entropy rate (Peres, Ann. Stat., 1992) and is connected to an interesting functional equation satisfied by the entropy function. Different problems arise when we try to simulate a coin with probability $f(p)$ of landing heads. In particular, only rational functions can be simulated using finite automata, and only real-analytic functions can be simulated if the simulation must have exponential tail. The talk will survey more recent works with E. Mossel, S. Nacu and O. Holtz on this topic.