Achieving Queue-Length Stability Through Maximal Scheduling in Wireless Networks

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***Abstract*— We address the question of attaining stability guarantees through distributed scheduling in wireless networks. We consider a simple, local information based, distributed scheduling strategy, *maximal scheduling*, and prove that it attains a guaranteed fraction of the maximum stability region. By considering the notion of queue-length stability, we strengthen existing rate stability results for maximal scheduling. The queue-length stability guarantees provided by maximal scheduling can differ across sessions, and depends on the "interference degree" in the two-hop neighborhood of the session.**

I. Introduction

Until recently, the question of attaining throughput guarantees through distributed scheduling had remained largely unexplored. Maximizing the network throughput, or equivalently, attaining the maximum stability region of the network, through appropriately scheduling is a key design goal in wireless networks. Tassiulas *et al.* characterize the maximum attainable stability region in an arbitrary wireless network, and provide a scheduling strategy that attains this region [11]. The policy, however, is centralized and can have exponential complexity depending on the network topology considered. Tassiulas [10] and Shah *et al.* [9] provide linear complexity randomized scheduling schemes that attain the maximum stability region; however, these scheduling strategies also require centralized control. Although [5], [6] consider distributed scheduling policies, these do not provide any analytical performance characterization of the policies.

In this paper, we study the throughput performance of a class of distributed scheduling policies called *maximal scheduling*. Maximal scheduling only ensures that if a transmitter u has a packet to transmit to a receiver v, either (u, v) or a transmitter-receiver pair that can not simultaneously transmit with (u, v) is scheduled for transmission; the scheduling is otherwise arbitrary. It is worth noting that a maximal scheduling policy can be implemented using only local topology information. Several recent works have obtained performance guarantees for maximal scheduling under various interference models. Lin *et. al.* [4] have shown that for the node-exclusive spectrum sharing model (in which case the maximal scheduling policy reduces to the maximal matching policy), maximal scheduling attains at least half of the maximum throughput region. Dai *et. al.* [2] have also obtained a similar guarantee for the maximal matching policy in input-queued switches where the scheduling constraints are similar to that in the node-exclusive spectrum sharing model. Chaporkar *et. al.* [1], [8] and Wu *et. al.* [12] have studied the performance of maximal scheduling under generalized as well as certain specialized interference models.

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In this paper, we extend the results in [1], [8] by proving stronger performance guarantees for the maximal scheduling policy under arbitrary interference models. More specifically, whereas [1], [8] obtain stability results in terms of rates, our stability results are in terms of the queue-lengths. Rate stability results, as shown in [1], [8] only imply that the arrival and departure rates are the same. Queue-length stability results, as provided in this paper, however imply that the queue-lengths remain bounded at all times. Therefore, although queue-length stability implies rate stability, the converse is not necessarily true.

Although our stability results are similar in nature to the queue-length stability results provided in [12], the performance bounds that we provide are significantly tighter than those provided in the latter. More specifically, we provide session-specific stability guarantees which depend only on the "interference degree" in the two-hop neighborhood of every session. The "interference degree" of any session is defined as the maximum number of sessions that interfere with the given session *and do not interfere with each other*. The stability guarantees in [12] however depend on the maximum number of sessions that interfere with any session in the network, which could be significantly larger than the session-specific interference degree, as defined above.

The paper is structured as follows. In Section II, we describe the system model and provide the necessary definitions used later in the paper. In Section III, we state and prove our result on the queue-length stability guarantee provided by maximal scheduling. We conclude in Section IV.

II. System Model

We consider scheduling at the medium access control (MAC) layer in a wireless network. We assume that time is slotted. The topology in a wireless network can be modeled as a directed graph $G = (V, E)$, where V and E respectively denote the sets of nodes and links. A link exists from a node

u to another node v if and only if v can receive u's signals. The link set E depends on the transmission power levels of nodes and the propagation conditions in different directions.

Next we introduce terminologies that we use throughout the paper. These are also defined in [1], [8], some of these being well-known in graph theory. We mention these for completeness.

Definition 1: A node i is a *neighbor* of a node j, if there exists a link from i to j, i.e., $(i, j) \in E$.

At the MAC layer, each session traverses only one link. If a session i traverses link (u, v) then u and v are i's transmitter and receiver respectively, and the session is completely specified by the $3-$tuple, (i, u, v). Multiple sessions may traverse the same link. Without loss of generality, we assume that every node in V is either the transmitter or the receiver of at least one session. If this assumption does not hold, we can consider G to be a subgraph obtained from the original topology by removing the nodes that are not the end points of sessions.

Definition 2: A session i *interferes* with session j if j can not successfully transmit a packet when i is transmitting.

A wireless network $\mathcal{N}$ can be described by the topology $G = (V, E)$, the $3-$tuple specifications of the sessions and the pair-wise interference relations between the sessions. We consider a network with N sessions.

Definition 3: The *interference set* of a session i, S_i, is the set of sessions j such that either i interferes with j or j interferes with i.
Note that if $j \in S_i$, then $i \in S_j$.

We elucidate these definitions through examples in Fig. 1. Note that the interference sets of the sessions will depend on the communication and interference models; [1] describes the broad classes of communication and interference models, and how pairwise interference relations can be obtained for these classes.

We now describe the arrival process. We assume that at most $\alpha_{\max}$ packets arrive for any session in any slot. Let $\alpha_j(t)$ and $\tilde{D}_j(t)$ denote the number of arrivals and departures, respectively, for session j in slot t. We assume that the arrival process $(\alpha_1(.), \ldots, \alpha_N(.))$ constitutes an irreducible, aperiodic markov chain with a finite number of states. We refer to this assumption as the *jointly markovian* assumption. Note that such an arrival process satisfies a strong law of large numbers (SLLN). In other words, if $A_i(n)$ denotes the number of packets that session i generates in interval $(0, n]$, $i = 1, \ldots, N$, then there exist non-negative real numbers $\lambda_i, i = 1, \ldots, N$, such that with probability 1,

$$\lim_{n\to\infty} A_i(n)/n = \lambda_i, \; i = 1, \ldots, N. \tag{1}$$

Definition 4: The *arrival rate* of session i is λ_i, $i = 1, \ldots, N$. The *arrival rate vector* $\vec{\lambda}$ is an $N-$dimensional vector whose components are the arrival rates.

Definition 5: A *scheduling policy* is an algorithm that decides in each slot the subset of sessions that would transmit packets in the slot.
Clearly, a subset S of sessions can transmit packets in any slot if no two sessions in S interfere with each other and every session in S has a packet to transmit. Every packet has length 1 slot. Thus if a session is scheduled in a slot, it transmits a packet in the slot.

We now describe the "maximal scheduling" policy we consider. This policy schedules a subset S of sessions such that (i) every session in S has a packet to transmit, (ii) no session in S interferes with any other session in S, (iii) if a session i has a packet to transmit, then either i or a session in S_i, is included in S. Clearly, many subsets of sessions satisfy the above criteria in each slot, e.g., in Fig. 1(b), $\{S1, S7\}, \{S2, S3, S6\}$ satisfy the above criteria in any slot in which all sessions have packets to transmit. Maximal scheduling can select any such subset. If each session knows its interference set, maximal scheduling can be implemented in distributed manner using standard algorithms [7]. In most cases of practical interest, sessions can determine their interference sets using local message exchange.

Now we define our notion of stability, *queue-length stability*, which guarantees that the expected queue-lengths of sessions are finite in stable systems. Let $Q_i(n)$ be the number of packets waiting for transmission at the source of session i at the beginning of slot n.

Definition 6: The network is said to be *queue-length stable* if there exists non-negative real numbers q_i, $i = 1, \ldots, N$, such that with probability 1,

$$\lim_{n\to\infty} Q_i(n)/n = q_i, \; i = 1, \ldots, N. \tag{2}$$

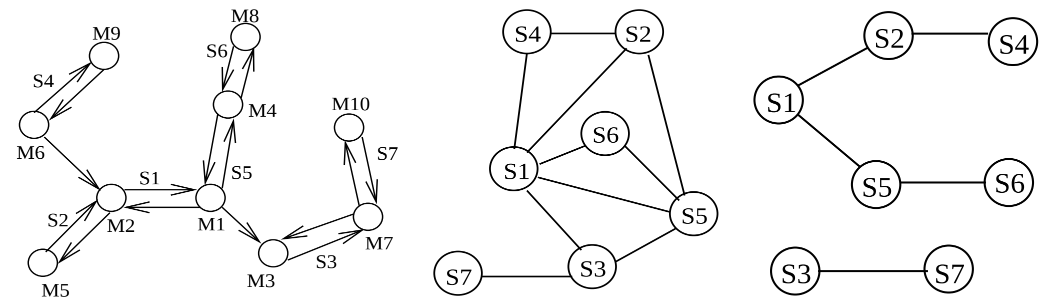
The *queue-length stability region* of a scheduling policy is the set of arrival rate vectors $\vec{\lambda}$ such that the network is queue-length stable under the policy, for any arrival process that satisfies the jointly markovian assumption and has arrival rate vector $\vec{\lambda}$. The *maximum queue-length-stability region* Λ_{Q} is the union of the queue-length-stability regions of all scheduling policies. Let $\Lambda_{\mathrm{Q}}^{\mathrm{MS}}$ denote the queue-length stability region attained by maximal scheduling.

III. Queue-Length Stability Guarantees with Maximal Scheduling

In this section, we state and prove our main result on queue-length stability. More specifically, we relate the queue-length stability region attained by maximal scheduling to the maximum queue-length stability region by providing neighborhood-specific throughput guarantees for the individual sessions.

We consider the notion of "interference degree" of a session, as introduced in [1]. The *interference degree* of a session i in network $\mathcal{N}$, $K_i(\mathcal{N})$ is (i) the maximum number of sessions in its interference set S_i that can simultaneously transmit, if S_i is non-empty, and (ii) 1, if S_i is empty. The *two-hop interference degree* of session-link i, is defined as $\beta_i(\mathcal{N}) = \max_{j \in S_i \cup \{i\}} K_i(\mathcal{N})$.

In the following result, we show that the performance of each session i under maximal scheduling can be characterized by the its two-hop interference degree, $\beta_i(\mathcal{N})$. More specifically, the result shows that maximal scheduling ensures queue-length stability as long as the arrival rates of every session i is within a factor of $1/\beta_i(\mathcal{N})$ of the stable arrival rates of



(a) Network $\mathcal{N}'$ (b) (c)

Fig. 1. Panel (a) shows a directed graph with $V = \{M1, \ldots, M10\}$. The arrows between the nodes indicate the directed links. There are 7 sessions: $S1, \ldots, S7$. Nodes $M2, M5, M3, M6, M1, M8$ and $M10$ are the transmitters of sessions $S1, S2, S3, S4, S5, S6$ and $S7$, respectively. Node $M2$ has 3 neighbors: $M1$, $M5$, $M6$. Sessions $S5$ and $S6$ interfere with each other, as $M4$ has a single transceiver.
Panels (b) and (c) show the interference graphs for the network shown in (a) under bidirectional and unidirectional communication models, respectively. As panels (b) and (c) show, the interference sets of $S6$ are $\{S1, S5\}$ and $\{S5\}$ under the bidirectional and unidirectional communication models, respectively.

the sessions. Thus, due to the use of local information based scheduling, the performance of each session i decreases by a factor of $\beta_i(\mathcal{N})$; the penalty for each session therefore depends only on its two-hop neighborhood.

Theorem 1: Consider an arrival rate vector $(\lambda_1^{'}, \ldots, \lambda_N^{'})$ *such that* $\lambda_1^{'} < \lambda_1/\beta_1(\mathcal{N}), \ldots, \lambda_N^{'} < \lambda_N/\beta_N(\mathcal{N})$*, where* $(\lambda_1, \ldots, \lambda_N) \in \Lambda_{\text{Q}}$*. Then,* $(\lambda_1^{'}, \ldots, \lambda_N^{'}) \in \Lambda_{\text{Q}}^{\text{MS}}$.

Proof: Let $\vec{\lambda} \in \Lambda_{\text{Q}}$. Then, under $\vec{\lambda}$, for some scheduling policy π, there exists a non-negative real vector $(q_1, \ldots, q_N)$ such that for all i, $\lim_{n\to\infty} \sum_n Q_i(n)/n = q_i$ w.p. 1. Now, since $Q_i(n) = Q_i(0) + A_i(n-1) - D_i(n-1)$, $\sum_n Q_i(n)/n = Q_i(0) + \sum_n \frac{A_i(n-1)-D_i(n-1)}{n}$. Thus, for all i, $\lim_{n\to\infty} \frac{A_i(n-1)-D_i(n-1)}{n} = 0$ w.p. 1. Since for all i, $\lim_{n\to\infty} A_i(n-1)/n = \lim_{n\to\infty} A_i(n)/n = \lambda_i$ w.p. 1, for all i, $\lim_{n\to\infty} D_i(n)/n = \lim_{n\to\infty} D_i(n-1)/n = \lambda_i$ w.p. 1. Thus, the arrival and departure rates are the same, and the system is rate stable. Therefore, for all i, $\sum_{j\in S_i\cup\{i\}} \lambda_j/\beta_j(\mathcal{N}) \le 1$ (Lemma 5, [8]). Hence

$$\sum_{j\in S_i\cup\{i\}} \lambda_j' < 1 \;\forall\; i. \tag{3}$$

Let the arrival rate vector be $(\lambda_1', \ldots, \lambda_N')$. Consider a maximal scheduling policy. Clearly, $\vec{Q}(.)$ constitutes an irreducible aperiodic markov chain.

Consider the lyapunov function $f(t)$, where

$$f(t) = \sum_i \sum_{j\in S_i\cup\{i\}} Q_i(t)Q_j(t).$$

Clearly, $f(t) > 0$ if $Q_i(t) > 0$ for some i.

$$\begin{aligned}
&\mathbb{E}[f(n+1) - f(n)|\vec{Q}(n)]\\
=\;& \sum_i \sum_{j\in S_i\cup\{i\}} \mathbb{E}[Q_i(n+1)Q_j(n+1) - Q_i(n)Q_j(n)|\vec{Q}(t)]\\
=\;& \sum_i \sum_{j\in S_i\cup\{i\}} \mathbb{E}\left[\left(Q_i(n) + \alpha_i(n) - \tilde{D}_i(n)\right)(Q_j(n)\right.\\
&\left. +\alpha_j(n) - \tilde{D}_j(n)\right) - Q_i(n)Q_j(n)|\vec{Q}(n)\Big]\\
\le\;& \sum_i \sum_{j\in S_i\cup\{i\}} \mathbb{E}[Q_i(n)\alpha_j(n) - Q_i(n)\tilde{D}_j(n)\\
&+Q_j(n)\alpha_i(n) - Q_j(n)\tilde{D}_i(n)|\vec{Q}(n)]\\
&+(N+1)N(\alpha_{\max}^2 + 1).
\end{aligned} \tag{4}$$

Now,

$$\sum_i \sum_{j\in S_i\cup\{i\}} Q_i(n)\alpha_j(n) = \sum_i \sum_{j\in S_i\cup\{i\}} Q_j(n)\alpha_i(n),$$

and
$$\sum_i \sum_{j\in S_i\cup\{i\}} Q_i(n)\tilde{D}_j(n) = \sum_i \sum_{j\in S_i\cup\{i\}} Q_j(n)\tilde{D}_i(n).$$

Thus,

$$\begin{aligned}
&\mathbb{E}[f(n+1) - f(n)|\vec{Q}(n)]\\
\le\;& 2\sum_i Q_i(n) \sum_{j\in S_i\cup\{i\}} \mathbb{E}[\alpha_j(n) - \tilde{D}_j(n)|\vec{Q}(n)]\\
&+(N+1)N(\alpha_{\max}^2 + 1)\\
=\;& 2\sum_i Q_i(n)\left[\sum_{j\in S_i\cup\{i\}} \lambda_j' - \mathbb{E}[\sum_{j\in S_i\cup\{i\}} \tilde{D}_j(n)|\vec{Q}(n)\right]\\
&+(N+1)N(\alpha_{\max}^2 + 1).
\end{aligned}$$

Let $\delta = 1 - \max_i \sum_{j\in S_i\cup\{i\}} \lambda_j'$. From (3), $\delta > 0$. Next, under maximal scheduling, if $Q_i(n) > 0$, $\sum_{j\in S_i\cup\{i\}} \tilde{D}_j(n) = 1$. Thus, for all $\vec{Q}(n)$,

$$\mathbb{E}[f(n+1) - f(n)|\vec{Q}(n)] \le -2\delta \sum_i Q_i(n) + (N+1)N(\alpha_{\max}^2 + 1).$$

Hence, by Foster's theorem (Theorem 2.2.3 in [3]), $\vec{Q}(.)$ is a positive recurrent markov chain. Thus, under maximal scheduling, there exists a non-negative real vector $(q_1, \ldots, q_N)$ such that for all i, $\lim_{n\to\infty} \sum_n Q_i(n)/n = q_i$ w.p. 1. Thus, $(\lambda_1', \ldots, \lambda_N') \in \Lambda_{\text{Q}}^{\text{MS}}$. This concludes the proof. ■

IV. CONCLUDING REMARKS

In this paper, we have investigated the question of attaining stability guarantees using a simple distributed scheduling policy, maximal scheduling, in an arbitrary wireless network. We consider the notion of queue-length stability, and show that guarantees provided by maximal scheduling can be characterized in terms of the local interference degree of every session.

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