On Joint Decoding and Random CDMA Demodulation

Christian Schlegel, Dmitri Truhachev, and Łukasz Krzymień

Department of Electrical and Computer Engineering
University of Alberta
Edmonton, AB, CANADA
email:{schlegel,dmitrytr,lukaszk}@ece.ualberta.ca

Abstract

We consider a CDMA system on an additive white Gaussian noise (AWGN) channel with K concurrent users. Each user transmits equal-rate data and employs an LDPC error control code (ECC). Each user's bits are modulated with a random spreading waveform. These spreading waveforms are furthermore partitioned in M sections which are in turn interleaved and spread in time. This creates a rate $1/M$ repetition code which interfaces the LDPC-ECC with the CDMA channel. This method, called *partitioned spreading*, significantly improves convergence properties and maximum system load in conjunction with iterative (turbo) detection. Different decoding iteration schedules, viz. a low-complexity two-stage schedule which separates CDMA detection and LDPC decoding into two successive processes, and a full iteration schedule (full decoding) which invokes all message exchanges in a parallel decoding approach are presented and analyzed. We show that, under certain power and rate conditions, two-stage decoding achieves virtually identical performance as full decoding.

I. Introduction

Joint decoding of CDMA has the potential to increase the spectral efficiency of CDMA by allowing more users to be active without overwhelming the receiver. Furthermore, joint decoding, or multiuser detection can mitigate or even completely overcome the detrimental effects of unequal received powers. However, optimal joint decoding, so-called multiuser detection, is known to be very complex [9]. For reasons of acceptable complexity, much work has concentrated on linear multiuser detection method, such as decorrelation, and, in particular, minimum-mean square error (MMSE) filtering. The MMSE filter is a linear matrix receiver [5] that separates the channel into parallel channels, one for each user. These parallel channels have typically a significantly increased signal-to-noise ratio, and single user decoding principles can be applied to each channel's signal. The methodology whereby a front-end processor is used to separate the joint channel into parallel sub-channels is referred to as signal layering, a term first used in the treatment of multiple-antenna channels.

Often in CDMA the concept of randomly spread signals is used whereby the data signals of each user are spread by a (pseudo)-random spreading waveform. Typically, this spreading waveform is a direct-sequence signature waveform which is assumed known at the receiver for all participating users. The information theoretic capacity of such random CDMA channels is well understood [6], as is the information theoretic capacity of a number of popular layered random CDMA channels, such as those using decorrelation or MMSE receivers [4]. While the layered channels are more feasible from an eventual implementation point of view, the full channel can, under certain circumstances, deliver significantly higher capacity. However, since optimal decoding of the full CDMA channel is feasible only for very small numbers of users, a garden variety of limited search algorithms and approximations to the ideal receiver have been conceived and are discussed in the literature.

Joint iterative decoding-detection of coded CDMA [4], in particular, has received much attention following the invention of turbo coding and iterative decoding methods, summarized as message passing decoding. However, a coherent theory on what is possible in terms of performance as a function of decoding complexity is still lacking. All that is known is that these systems typically achieve a performance somewhere between that of linear layered receivers and optimal decoding. It is worth mentioning that even linear layering typically has a complexity which grows with the cubic power of the number of participating users, albeit there exist very efficient iterative inversion approximations [4] to the required matrix inversions.

This work was supported in part by textitiCore Alberta, and the Alberta Ingenuity Fund.

In this paper we present a design as well as an analysis of signaling method for (random) CDMA channels which combines the iterative decoding of error control codes with an iterative demodulation processing which can be viewed as a non-linear layering function. The resulting decoder is full joint decoder in the sense discussed above, which, under certain operating parameters behaves like a linearly layered CDMA receiver. The methodology derived from an observation that direct application of error control codes designed for the (single-user) Gaussian channel often did not lead to very good joint detectors, on the contrary, the convergence properties of such ad-hoc combinations can be shown to be typically very poor, resulting in low spectral efficiencies.

This methodology is based on an observation that breaking the spreading sequences into smaller partitions, so-called partitioned spreading [2], proves a very effective choice in conjunction with joint iterative decoding. In partitioned spreading the spreading sequences are segmented, interleaved, and spread in time. This can be viewed as placing a repetition code between the ECC and the CDMA channel. At the receiver, each user bit receives a number of different, albeit very noisy but (largely) uncorrelated estimates. These are refined in an iterative manner by a message passing decoder/demodulator. The combination of error control coding and partitioned spreading can be viewed as serially concatenated coding or as distributed modulation. We will simply resort to a factor graph model of the overall system, which integrates most logically with the iterative message passing decoding we apply.

As the error control codes of choice we choose low-density parity-check (LDPC), both due to their superior error control capabilities as well as for analytical tractability. In principle, our analysis can be extended to virtually any ECC using a graphical code representation and graph-based message passing decoding. We furthermore concentrate on regular LDPC codes. While irregular LDPC codes can appreciably improve performance for lower rates $R < 0.5$ on Gaussian channels, they offer little improvement for higher rates, nor do they offer much improvement in the application discussed here. In order to gauge possible improvements over regular LDPC codes, we will also compute results using the BPSK capacity limits, rather than the signal-to-noise thresholds of the LDPC codes. From these any maximum possible gain of more powerful codes versus regular LDPC codes can can be estimated.

Partitioned spreading interfaces the poor match of complex ECC and equal-power CDMA and achieves significant gains over applying ECC directly to the CDMA channel, even if ECC is optimized. We present results which quantify these gains and investigate the performance of the two most natural iterative decoding schedules. The first schedule completes iterative demodulation of the partitioned spreading before invoking LDPC decoding. This is a low-complexity, two-stage schedule with no feedback communication between the ECC and the iterative CDMA channel demodulator. For this schedule we show that the best user power distribution given a fixed average received power is bimodal. The second schedule, termed full iteration schedule (full decoding), allows for feedback communication between the LDPC decoder and the CDMA demodulator. It is shown that the best performance is achieved by performing a large number of iterations ($\rightarrow \infty$) in each of the subprocesses, i.e., LDPC decoding and CDMA demodulation, before exchanging messages between the processes. In this sense, full decoding is a complex extension of two-stage decoding which essentially repeats the two-stage process an arbitrary number of times.

Not surprisingly, full decoding outperforms two-stage decoding. However, we show that if high-rate LDPC codes are used, the gain of full decoding becomes marginal. That is, full decoding achieves the same performance as layered two-stage decoding. Furthermore, for low code rates, this system has a performance which is identical to that of an MMSE filter, and thus allows direct comparisons of complexity between the two methods. However, if the code rate is large than a certain threshold, both the two-stage iterative decoding process as well as full decoding achieve significantly improved spectral efficiencies.

Due to size, this paper is a summary and outlook of results. It is organized as follows. Section II contains a description of coded CDMA using partitioned spreading and the two iteration schedules discussed in this paper. Section III is devoted to iterative decoding and the presentation of our current results. In Section **??** we present our results for the achievable spectral efficiencies with either schedule and contrast it to that achievable without partitioned spreading.

II. LDPC Coded Partitioned-Spreading CDMA

A. System Description

Consider a system where K users generate independent information bit streams, which are encoded by K parallel LDPC encoders and interleaved. We consider regular $(d_\mathrm{v}, d_\mathrm{c})$ LDPC codes with block length L, thus the encoded user data is broken into frames of length L.

Consider a particular user $k \in \{1, 2, \ldots, K\}$ and a transmitted frame of encoded bits $\boldsymbol{v}_k = (v_{k,1}, v_{k,2}, \ldots, v_{k,L})$. The lth bit $v_{k,l}$ of user k is spread by the direct spreading sequence $s_{k,l,i}$, $i = 1, 2, \ldots, N$ consisting of N randomly and independently chosen chips from $\{-1/\sqrt{N}, 1/\sqrt{N}\}$, and N is the spreading gain. Each spread symbol is then partitioned into M equal-length partitions, where $M|N$. The partitions of all the symbols are then permuted and spread over one or several frame lengths. Denoting the power of user k by P_k, the transmitted signal of user k is given by

$$x_k(t) = \sum_{l=0}^{L-1} \sqrt{P_k} v_{k,l} \sum_{m=0}^{M-1} c_{k,l,m} \left(t - lT - \tau_k - \pi_k(m) \frac{T}{M} \right) \tag{1}$$

where

$$c_{k,l,m}(t) = \sum_{n=0}^{N/M-1} s_{k,l,n+m\frac{N}{M}} p(t - nT_c) \tag{2}$$

is the m-th section of the spreading waveform for bit $v_{k,l}$, T the duration of a symbol, and $p(t)$ is the unit-energy chip pulse. Furthermore, $T_c = T/N$, and we allow the system to be asynchronous with $\tau_k < T$. $\pi_k(m)$ is the permuter of the position of the mth spreading partition for the symbols of user k, and is assumed to be random. The combined signal from all users is transmitted over an additive white Gaussian channel (AWGN) with power spectral density N_0, giving the received signal

$$y(t) = \sum_{k=1}^{K} x_k(t) + n(t) \tag{3}$$

where $n(t)$ is the noise process.

B. *Two-Stage Decoding*

While iterative decoding can be organized in many feasible schedules, a natural and low-complexity schedule is the following two-stage schedule. In a first stage iterative demodulation is applied only to the different partitions from partitioned spreading, using message passing principles and utilizing the repetition property of the partitioned symbols. This first stage layers the channel into K subchannels, each with reduced interference. In a second stage each user's LDPC decoder operates on the improved channel and completes decoding. Convergence of the LDPC decoder depends only on the output signal-to-noise ratio of the second stage.

More precisely, the received signal is filtered by filters matched to the spreading partitions. The received signal of the mth partition of bit $v_{k,l}$ is therefore

$$z_{m,k,l} = \sqrt{\frac{P_k}{M}} v_{k,l} + I_{m,k,l} + \eta_{m,k,l} \tag{4}$$

and the log-likelihood ratio from the observations (4) is given by

$$\ln \left(\frac{P(v_{k,l} = 1 | z_{1,k,l}, z_{2,k,l}, \ldots, z_{M,k,l})}{P(v_{k,l} = -1 | z_{1,k,l}, z_{2,k,l}, \ldots, z_{M,k,l})} \right) = \frac{2}{\sigma_0^2} \sum_{m'}^{M} z_{m',k,l} \tag{5}$$

where σ_0^2 is the variance of $I_{m,k,l} + \eta_{m,k,l}$.

For iterative demodulation, consider the top part of the factor graph of the combined CDMA channel – ECC system, shown in Figure 1. This figure illustrates the partial factor graph for user k. The LDPC code graph, at the bottom, is connected to the channel/partition spreading portion of the graph via the equality nodes, since they represent the same coded LDPC bit. These equality node acts as interface between the channel and the LDPC code. We now iteratively exchange messages only between the CDMA channel nodes (top circles) and the interleaved partitions (equality nodes). We call such an iteration a demodulation iteration. At each iteration $i : 0, \cdots, I$, the equality node of bit $v_{k,l}$ returns a soft bit estimate of $v_{k,l}$ to the channel partition m, which is calculated as

$$\tilde{v}_{k,l,m}(i) = \tanh\left(\frac{1}{\sigma^2_{i-1}} \sum_{m \neq m'}^{M} z_{k,l,m'}(i-1)\right) \tag{6}$$

i.e., from all the incoming received partitioned channel signals, expect the one on the outgoing edge. From these soft bits, the channel nodes compute interference canceled signals for each user k as

$$y_k^{(i)}(t) = x_k(t) + \sum_{k \neq k'}^{K} \sum_{l=0}^{L-1} \sqrt{P_{k'}} \sum_{m=1}^{M} (v_{k',l} - \tilde{v}_{k,l,m}) c_{k',l,m}\left(t - lT - \tau_{k'} - \pi_{k'}(m)\frac{T}{M}\right) + n(t) \tag{7}$$

from which new received signal $z_{k,l,m'}(i)$ are generated by matched filtering again. It is straight forward to see that the combined noise and interference variance of $I_{m,k,l} + \eta_{m,k,l}$ at iteration i, and for sufficiently large K and N, is given by

$$\sigma_i^2 = \frac{1}{N}\sum_{k=1}^{K} P_k \sigma^2_{d,i,k} + \sigma^2, \quad \text{where } \sigma^2_{d,i,k} = \mathrm{E}\left[(v_k - \tilde{v}_{k,i})^2\right] \tag{8}$$

Demodulation proceeds for I iteration, where it can be shown that σ_i^2 is monotonically decreasing with $i$and therefore $\sigma^2_\infty \geq \sigma^2$ is the minimum value possible. This completes the first stage.

LDPC decoding at the second stage in turn proceeds as customary. It is successful if and only if

$$\frac{P_k}{\sigma_I^2} \geq 2R\gamma_{\text{code}} \tag{9}$$

where γ_{code} is the code's convergence threshold signal-to-noise ratio (in E_b/N_0), and R is the code's rate. Alternatively, if we use an "ideal" code, the second stage will decode successfully if and only if

$$R < C_B\left(\frac{P_k}{\sigma_I^2}\right) \tag{10}$$

where $C_B(\mathrm{snr})$ is the binary capacity of a channel with signal-to-noise ratio snr.

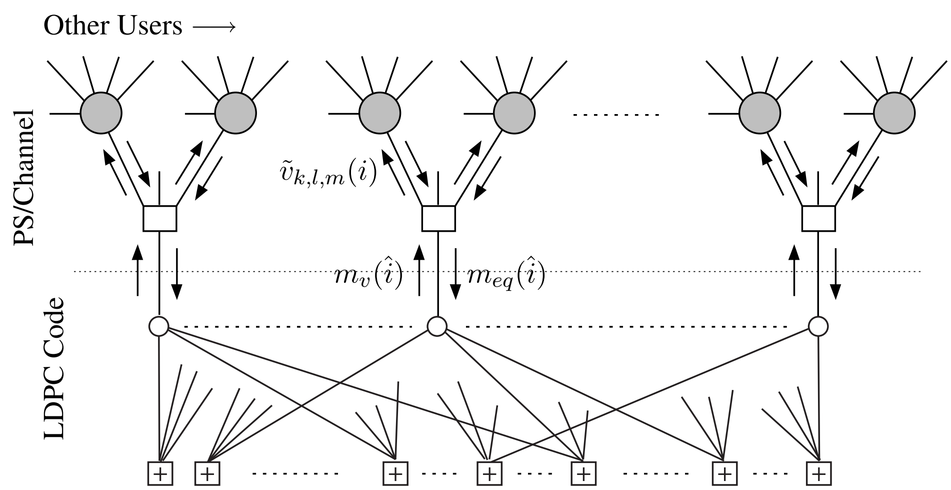

Fig. 1. Factor graph for a coded CDMA system with partitioned spreading using a two-stage iterative decoding approach.

C. Full Iteration Schedule

In order to analyze the information exchange in the joint detection process we utilize again the factor graph Figure 1. Contrary to the two-stage schedule, in full decoding we allow messages to be sent back to the channel from the LDPC code. As we will see, this does improve performance, and for low-rate codes this improvement is significant. However, the additional computational cost is substantial.

Full decoding consists essentially of repeated two-stage decoding. Assume that at each overall iteration I_1 detection iterations through the partitioned spreading and channel nodes are executed, and I_2 LDPC decoding iterations. For $\hat{i} = 1, \ldots, I$: carry out I_1 detector iterations, $i = 1, \ldots, I_1$ (for message notation, see Figure 1). Compute $\tilde{v}_{k,l,m}(\hat{i}, i)$ in (6) where the argument in $\tanh(\cdot)$ is given by

$$m_c(\hat{i}, i) = \frac{2}{\sigma^2(\hat{i}, i)} \sum_{m' \neq m} z_{k,l,m'}(i-1) + m_v(\hat{i}-1) \qquad k = 1, \ldots, K \tag{11}$$

and $\sigma^2(\hat{i}, i) = \sigma_i^2$ from (8). This iteration process is then followed by I_2 LDPC decoding iterations with messages

$$m_{eq}(\hat{i}) = \frac{2}{\sigma^2(\hat{i}, i)} \sum_{m} z_{k,l,m'}(i-1) \tag{12}$$

The message passing inside the LDPC code sub-graph is executed according to the well-established flooding schedule with parallel updating [3]. The optimal performance schedule for full decoding is given by

Lemma 1: Full decoding achieves its best performance if $I_1, I_2 \to \infty$, in fact, performance is monotonically non-decreasing with I_1 and I_2.

Proof: The monotonicity of the performance of the LDPC decoding algorithm with iterations is well known. The monotonicity of the demodulation iterations has been shown in [1].

Figure 3 shows achievable systems loads $\alpha = K/N$ for equal received power levels for three cases: two-stage and full decoding, as well as linear MMSE filtering for comparison. It can be seen that with increasing code rate R and at moderate and high values of E_b/N_0, two-stage and full decoding substantially outperform linear MMSE filtering[1] Two-stage decoding approaches the far more complex full decoding performance rapidly with increasing code rate R.

The 2-stage and MMSE capacities are obtained by using the BPSK capacity formula instead of the regular LDPC threshold values. The larger gap for low rates results from the fact that low-rate regular LDPC codes have a relatively large gap to capacity, while the thresholds of high-rate LDPC codes are come very close.

Furthermore, at low code rates R, the performance of two-stage decoding is equivalent to linear MMSE filtering. This is seen better in Figure 4 which shows the supportable system loads as a function of the code rate for regular LDPC codes for two values of the signal-to-noise ratio E_b/N_0, viz. for 5dB and 10dB. Note in particular that for the higher signal-to-noise ratio, the performance of the 2-stage decoder quickly approaches that of full decoding. It can also be seen that partitioned spreading significantly outperforms MMSE filtering as soon as the code rate $R > 0.4$.

For a quantitative analysis, recall that the interference and noise power experienced by a given user are given by (8), where we abbreviate

$$\sigma_{d,i,k}^2 \leq \mathrm{E}\left[(v_k - \tilde{v}_{k,i})^2\right] = g\left(\frac{P_k(M-1)}{M\sigma_{i-1}^2}\right) \tag{13}$$

and the function $g(x)$ can be bounded tightly by [1]

$$g(x) \leq \begin{cases} \frac{1}{1+x}, & x < 1 \\ \pi Q(\sqrt{x}), & x \geq 1 \end{cases} \tag{14}$$

As long as $P_k(M-1)/(M\sigma_{i-1}^2) < 1$ the first inequality is biting, and

$$\sigma_\infty^2 = \sigma_{\mathrm{MMSE}} = \frac{\alpha P_k}{1 + \frac{P_k(M-1)}{M\sigma_\infty^2}} + \sigma^2 \tag{15}$$

[1]This point has already been proven in [1].

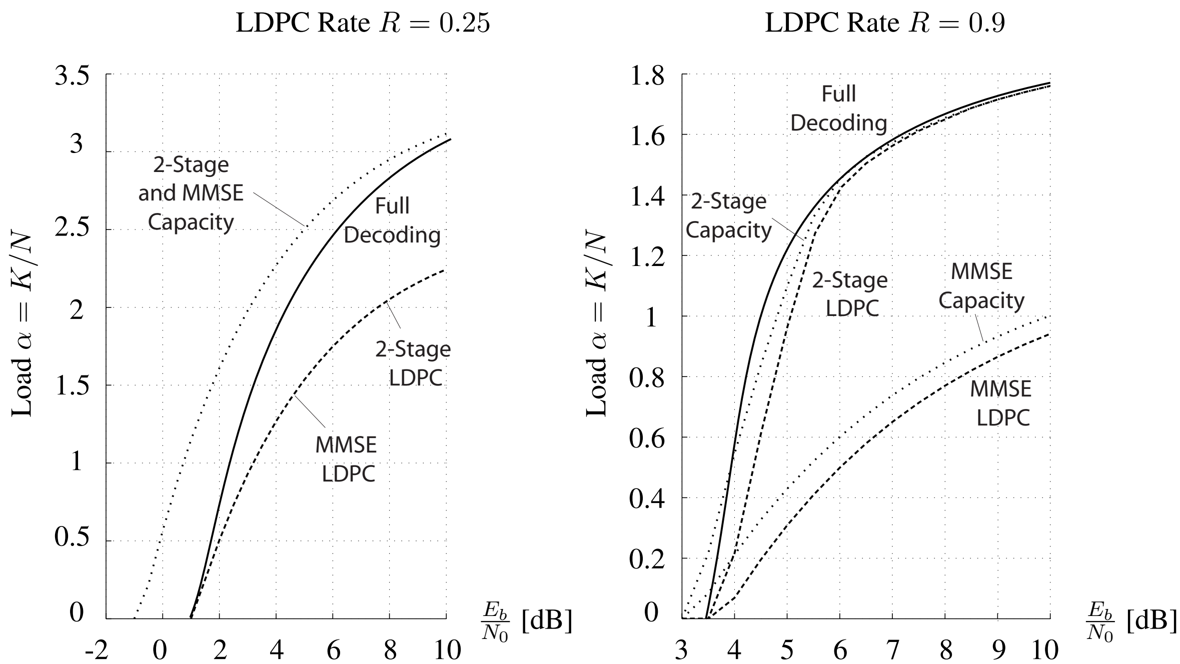

Fig. 2. Supportable system loads with partitioned spreading, plotted as a function of E_b/N_0 for regular LDPC ECC for two representative rates $R = 0.25$ and $R = 0.9$.

which equals the variance of a linear MMSE filter [5]. Therefore, for rates

$$R\gamma_{\text{code}} < \frac{M}{2(M-1)} \tag{16}$$

the iterative two-stage demodulation of partitioned spreading has a performance equal to that of the linear MMSE filter, where E_b/N_0 is the symbol energy to noise power ratio required by the code. For regular LDPC codes, and large values of M, (16) evaluates to $R < 0.397$, and for ideal binary coding to $R < 0.488$, see Figure 3.

A more complex analysis shows that there exists a region of the signal-to-noise ratio and code rate where the performance of a (layered) two-stage decoding algorithm achieves the performance of full decoding, still substantially outperforming linear MMSE detection. This is achieved for higher values of E_b/N_0, see e.g. Figures 3 and 4.

The remarkable fact is that there exist threshold signal-to-noise ratios for given code rates and system loads, below which the decoder converges with only a minimal dependency of the SNR on the system load. Above these thresholds a rapid asymptotic increase in required SNR signals the hard limits of both two-stage and full decoding. These results are illustrated in Figure 5. We omit the theoretical description, which is not yet complete.

III. Optimal Power Profiles

The results in the previous section were derived for equal received power systems. In practice this implies some sort of power control, which may be highly undesirable for packet networks. The extension of analogous results to the case of unequal received powers is not complete yet, but we do have preliminary results.

It is well-known that successive cancellation using an MMSE filter at each stage can achieve the capacity of the joint multiple access channel [7], [8]. However, for equal rate systems, this imposes an exponential received power profile. Such a power profile would need to be carefully controlled, and therefore negates the benefits achievable. For any other received power profile, the rates will have to follow a careful distribution which needs to be communicated to the users. Furthermore, code families which allow transmission at all these required exact code rates may be difficult to construct. We conclude preliminarily that it is more important to guarantee that a receiver is robust to (large) changes in the received power rather than requiring a special prescribed power distribution for optimal operation.

Nonetheless, here we assume that controlling the received power levels is somehow possible, and ask the question which distribution of received power levels optimizes spectral efficiency.

Insight into the optimal received power distribution can then be gleaned from the following theorem:

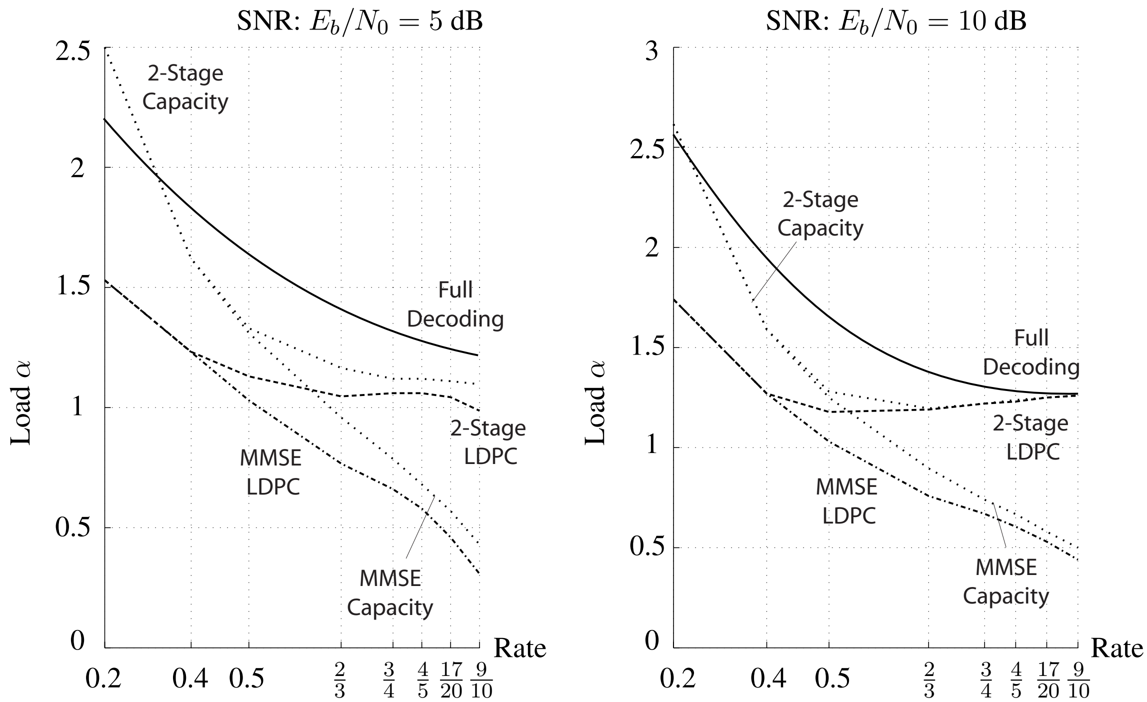

Fig. 3. Supportable system loads with partitioned spreading, plotted as a function rate for regular LDPC ECC for two representative signal-to-noise ratio values of $E_b/N_0 = 5$dB and 10dB .

Theorem 1: Assume σ_{i-1}^2, the average power P_{av}, and the minimal power threshold P_{thr} are fixed. Then

$$\sigma_i^2 = \frac{1}{N}\sum_{k=1}^{K} P_k g\left(\frac{P_k(M-1)}{M\sigma_{i-1}^2}\right) + \sigma^2 \tag{17}$$

is minimized by the bimodel distribution $P_1 = P_2 = \ldots = P_k = P_{\text{thr}}$ and $P_{k+1} = P_{k+2} = \ldots = P_K$ for some $k \in (1; K)$.

Proof: Omitted.

From this result we conjecture that a bimodal distribution is the optimal power distribution for the two-stage decoding schedule. We will present achievable loads and spectral efficiencies and compare them equal power performance.

References

[1] M.V. Burnashev, C. Schlegel, W.A. Krzymien, and Z. Shi, "Characteristics analysis of successive interference cancellation methods", *Problemy Peredachi Informatsii*, November 2005.

[2] R. Kempter and C. Schlegel, "Packet Random Access in CDMA Radio Networks", *Allerton Conference*, September 2005.

[3] C. Schlegel and L. Perez, *Trellis and Turbo Coding*, IEEE/Wiley, Piscataway, NJ, 2004.

[4] C. Schlegel and A. Grant, "Coordinated Multiple User Communications", *Springer Publishers*, 2005.

[5] D. Tse and S. Hanly, "Linear Multiuser Receivers: Effective Interference, Effective Bandwidth and User Capacity", *IEEE Trans. Inform. Theory*, pp. 641–657, March 1999.

[6] A.M. Tulino and S. Verdu, *Random Matrix Theory And Wireless Communications*, in Foundations and Trends in Communications and Information, now Publisers, 2004.

[7] M. K. Varanasi and T. Guess, "Achieving Vertices of the Capacity Region of the Synchronous Gaussian Correlated-Waveform Multiple-Access Channel with Decision-Feedback Receivers", *International symposium on Information Theory*, Ulm, Germany, June 1997.

[8] M. K. Varanasi and T. Guess, "An Information-Theoretic Framework for Deriving Canonical Decision-Feedback Receivers in Gaussian Channels", *IEEE Transactions on Information Theory*, Vol. 51, No 1, January 2005.

[9] S. Verdu, *Multiuser Detection*, Cambridge Press, 1998.

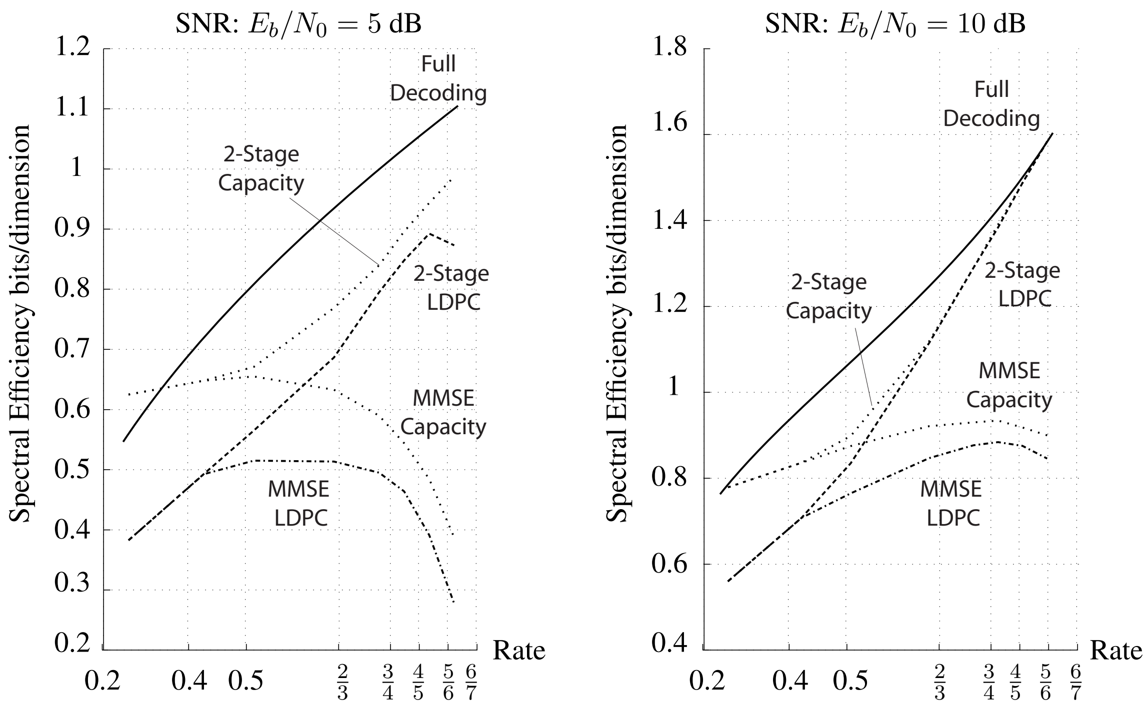

Fig. 4. Achievable spectral efficiencies in bits/dimension for two representative signal-to-noise ratio values of $E_b/N_0 = 5$dB and 10dB.

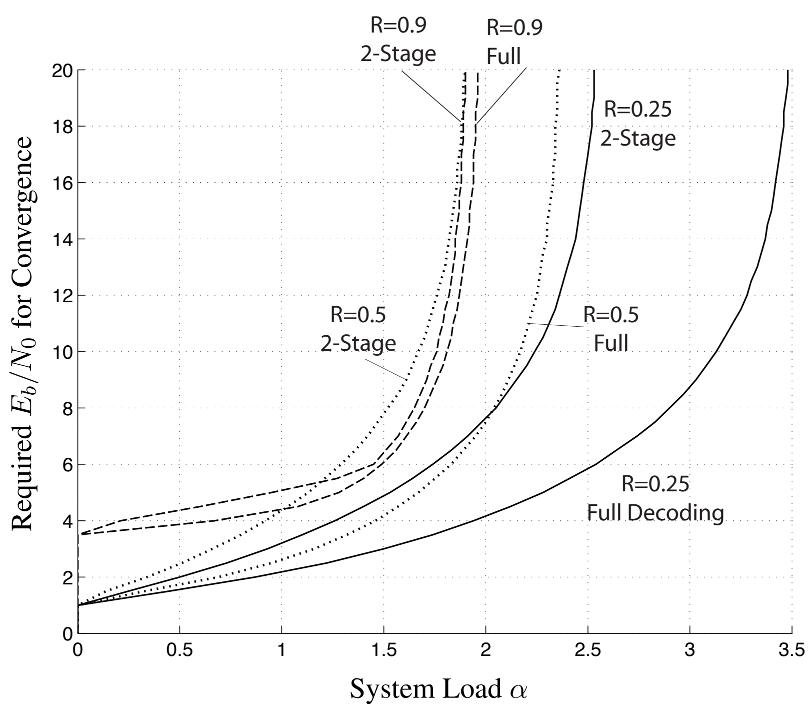

Fig. 5. Achievable spectral efficiencies in bits/dimension for two representative signal-to-noise ratio values of $E_b/N_0 = 5$dB and 10dB.