Outage-Optimal Cooperative Relaying

A. Salman Avestimehr
Dept of EECS
U.C. Berkeley
Berkeley, CA 94720, USA
Email: avestime@eecs.berkeley.edu

David N.C. Tse
Dept of EECS
U.C. Berkeley
Berkeley, CA 94720, USA
Email: dtse@eecs.berkeley.edu

***Abstract*— In this paper we look at the outage capacity of the fading relay channel with half-duplex constraint in the low SNR regime. When the channel state information (CSI) is available only at the receiver it was shown that a Bursty Amplify-Forward (BAF) protocol is optimal and achieves the max-flow min-cut upper bound on the outage capacity of this network [11]. But as the channel estimation is quite challenging in the low SNR regime in this paper we focus on the scenario that neither the transmitter nor the receiver know the channel state (non coherent). We show that the outage capacity in this scenario is the same as before, hence the receiver does not need to estimate the channel to get the same rate as before. We also investigate another extreme that the channel state information is available at both the transmitter and the receiver (full CSI). We show that this additional information will just slightly increase the outage capacity while the communication protocol gets quite complicated.**

I. Introduction

Cooperative diversity has been shown to be an effective way of creating diversity in wireless fading networks [1], [2], [3]. In the slow fading scenario, once a channel is weak due to deep fade coding no longer helps the transmission. In this situation cooperative transmission can dramatically improve the performance by creating diversity using the antennas available at the other nodes of the network. This observation leads to recent interest in the design and analysis of efficient cooperative transmission protocols. In particular in [4] authors looked at different cooperative strategies for relay networks applied two several wireless channels with different geometries and fading conditions.

In this paper, the cooperative diversity scenario is modelled by a slow Rayleigh fading relay channel. We also impose a practical constraint on the relay, which is the relay operates on a half-duplex mode and transmits and receives on different frequency bands (so called frequency-division (FD) relay channel). There are two regimes of interest that one can look at for this channel : high SNR and low SNR. The design and analysis of cooperative protocols at high SNR have been studied in [3] and [5]. In the high SNR regime, the main performance measure is the *diversity-multiplexing tradeoff* [9], which can be viewed as a high SNR approximation of the outage probability curve. In [3] the authors looked at the scenario that the channel state information (CSI) is only available at the receiver and they introduced several simple transmission protocols and analyzed the diversity-multiplexing tradeoff achieved by these schemes. While these schemes extract the maximal available diversity in the channel, they are sub-optimal in terms of achieving the diversity-multiplexing tradeoff. Then in [5] more efficient cooperative transmission protocols were introduced. In particular, they proposed a dynamic decode and forward scheme that achieves the optimal diversity-multiplexing tradeoff in a range of low multiplexing gain.

While at high SNR regime the main challenge is to use the degrees of freedom efficiently, the energy efficiency becomes the important measure in the low SNR regime. Therefore in the low SNR regime we should look for the cooperative schemes that are efficient in the transfer of energy into the network. Moreover based on this intuition the behavior of all protocols can be summarized in how they transfer energy in the network.

In this paper we focus on the outage performance at the low SNR regime. There are two reasons to study the low SNR regime. First, the impact of diversity on capacity is much more significant in low SNR than high SNR. Second, in energy-limited scenarios, the key performance measure is the maximum number of bits per unit energy that one can communicate for a given ϵ outage probability. So analogous to [6] one can define the ϵ-outage capacity per unit energy or $\mathbf{C}_\epsilon$. It is easy to show that this capacity is achieved in the low SNR limit and so our results on low SNR outage capacity directly translates to results on the outage capacity per unit cost.

The scenario that the channel state information is available only at the receiver was considered in [11]. In order to find the outage capacity of that network first a max-flow min-cut bound was stated to find an upper bound on the ϵ*-outage capacity* of the frequency-division (FD) relay channel. Then the outage performance of two classes of cooperative protocols was investigated: Amplify-Forward (AF) and Decode-Forward (DF). It was shown that the AF protocol achieves the same outage rate as when there is no cooperation and only the direct link is used. On the other hand the DF protocol exploits full diversity gain. But still there is a gap between the outage capacity of DF protocol and the upper bound on the outage rate of the relay channel. Then the performance of a new protocol called Bursty Amplify-Forward (BAF) protocol was investigated. It was shown that, somewhat surprisingly, this simple protocol closes the gap and achieves the optimal outage capacity of the relay channel, in the limit of low SNR and low probability of outage. The summary of the results stated

Scenario	Outage Rate(nats/s)
Non Cooperative	$\epsilon\ \mathsf{SNR}_{\mathsf{sd}}$
Amplify-Forward (AF)	$\epsilon\ \mathsf{SNR}_{\mathsf{sd}}$
Decode-Forward (DF)	$\sqrt{\frac{2\mathsf{SNR}_{\mathsf{sd}}\mathsf{SNR}_{\mathsf{rd}}\mathsf{SNR}_{\mathsf{sr}}}{2\mathsf{SNR}_{\mathsf{rd}}+\mathsf{SNR}_{\mathsf{sr}}}}\ \epsilon$
Bursty Amplify-Forward (BAF)	$\sqrt{\frac{2\mathsf{SNR}_{\mathsf{sd}}\mathsf{SNR}_{\mathsf{rd}}\mathsf{SNR}_{\mathsf{sr}}}{\mathsf{SNR}_{\mathsf{rd}}+\mathsf{SNR}_{\mathsf{sr}}}}\ \epsilon$
Upper Bound On the Outage Capacity	$\sqrt{\frac{2\mathsf{SNR}_{\mathsf{sd}}\mathsf{SNR}_{\mathsf{rd}}\mathsf{SNR}_{\mathsf{sr}}}{\mathsf{SNR}_{\mathsf{rd}}+\mathsf{SNR}_{\mathsf{sr}}}}\ \epsilon$
Outage Capacity	$\sqrt{\frac{2\mathsf{SNR}_{\mathsf{sd}}\mathsf{SNR}_{\mathsf{rd}}\mathsf{SNR}_{\mathsf{sr}}}{\mathsf{SNR}_{\mathsf{rd}}+\mathsf{SNR}_{\mathsf{sr}}}}\ \epsilon$
Outage Capacity per Unit Cost	$\sqrt{\frac{2g_{\mathsf{sd}}g_{\mathsf{rd}}g_{\mathsf{sr}}}{g_{\mathsf{rd}}+g_{\mathsf{sr}}}}\ \epsilon$

TABLE I

THE RESULTS ON THE APPROXIMATE OUTAGE RATES (NATS/S) AT LOW SNR AND LOW PROBABILITY OF OUTAGE ϵ.

in [11] are shown in Table I. In this table g_{sd}, g_{rd} and g_{sr} are the variances of channel gains from the source to destination, relay to destination and source to relay and $\mathsf{SNR}_{\mathsf{sd}}$, $\mathsf{SNR}_{\mathsf{rd}}$ and $\mathsf{SNR}_{\mathsf{sr}}$ are the average received SNRs from the source to destination, relay to destination and source to relay. The main results are also stated in the following two theorems,

Theorem 1. *In the limit of low SNR and low outage probability, the* ϵ-outage *capacity ,* $C_{\epsilon_{relay}}$ *of the FD- relay channel(in nats/s) is*

$$C_{\epsilon_{relay}} \approx \sqrt{\frac{2\mathsf{SNR}_{\mathsf{sd}}\mathsf{SNR}_{\mathsf{rd}}\mathsf{SNR}_{\mathsf{sr}}}{\mathsf{SNR}_{\mathsf{rd}}+\mathsf{SNR}_{\mathsf{sr}}}}\ \epsilon \tag{1}$$

where $\mathsf{SNR}_{\mathsf{sd}}$, $\mathsf{SNR}_{\mathsf{rd}}$ *and* $\mathsf{SNR}_{\mathsf{sr}}$ *are the average received SNRs from the source to destination, relay to destination and source to relay.*

And if we define the ϵ*-outage* capacity per unit energy of the FD- relay channel to be the maximum number of bits that one can transmit with outage probability ϵ, per unit energy spent at the source and unit energy spent at the relay we have

Theorem 2. *In the limit of low outage probability the* ϵ-outage *capacity per unit energy,* $\mathbf{C}_{\epsilon,\ relay}$*, of the FD- relay channel (in nats/s/J) is*

$$\mathbf{C}_{\epsilon,\ relay} \approx \sqrt{\frac{2g_{\mathsf{sd}}g_{\mathsf{rd}}g_{\mathsf{sr}}}{g_{\mathsf{rd}}+g_{\mathsf{sr}}}}\ \epsilon \tag{2}$$

where g_{sd}, g_{rd} *and* g_{sr} *are the average channel gains from the source to destination, relay to destination and source to relay. The noise variance of all channels have been assumed to be 1.*

However as the channel estimation is quite challenging in the low SNR regime in this paper we ask the following natural question: how much is the channel knowledge beneficial or crucial in this regime? To address this question we look at two extremes. One extreme is the case that neither the transmitter nor the receiver knows the channel. We show that the outage capacity in the interested regime is the same as before. The optimal scheme in this case is to use bursty pulse position modulation (PPM) encoding at the source and the energy estimator at the destination while the relay is just amplifies and forwards the received signal. Therefore we can achieve the same outage performance even in the absence of CSI at the receiver.

In the other extreme we look at the case that the CSI is available at both the transmitter and receiver (full CSI). In this case the source and the relay can beam-form to the destination to obtain better outage performance. To understand how beneficial this additional information can be, we derive the outage capacity in the interested regime. We show that the mixed protocol of bursty amplify-forward + beamforming is the optimal strategy in this case. We also show that for some typical cases the gain from this additional knowledge is small as the source tends to allocate less power for beam-forming and more power to broadcast the information.

II. MODEL

In this paper we consider a simple relay network consisting of a source (S), a relay (R) and a destination (D). We impose a practical constraint on the relay that does not allow the relay to receive and transmit signals simultaneously at the same time and the same frequency band, known as the half-duplex constraint. There are two major models in the literature that satisfy this constraint: fixed and random division strategies. In the fixed division strategy the relay receives and transmits data on different frequency-bands/time-slots (frequency division/time division). In the random division strategy the relay randomly decides to listen to data or transmit at each time slot. In this paper we consider the fixed division strategy and the discrete-time frequency division (FD) model for the fading relay channel with AWGN noise is shown in Figure 1. We focus on the case that the channel from the source to the relay and from the relay to the destination is split into two bands. The path gains h_{sd}, h_{rd} and h_{sr} are subject to independent Rayleigh fading with variances g_{sd}, g_{rd} and g_{sr} respectively. The received signal at the relay at time $i \geq 1$ is

$$Y_{R_i} = h_{\mathsf{sr}}X_{1_i} + Z_{R_i}$$

The received signals at time i at the destination from the first and the second frequency bands are denoted by Y_{1_i} and Y_{2_i} respectively, where $Y_{1_i} = h_{\mathsf{sd}}X_{1_i} + Z_{1_i}$ and $Y_{2_i} = h_{\mathsf{rd}}X_{R_i} + h_{\mathsf{sd}}X_{2_i} + Z_{2_i}$. Also $\{Z_{R_i}\}$, $\{Z_{1_i}\}$ and $\{Z_{2_i}\}$ are assumed to be independent (over time and with each other) $\mathcal{CN}(0,1)$ noises. An average transmitted power constraint equal to P at both the source and the relay is assumed. We also define $\mathsf{SNR} := \mathsf{P}/1$ as the SNR per (complex) degree-of-freedom. Therefore the average received SNRs from the source to destination ($\mathsf{SNR}_{\mathsf{sd}}$), relay to destination ($\mathsf{SNR}_{\mathsf{rd}}$) and source to relay ($\mathsf{SNR}_{\mathsf{sr}}$) are equal to

$$\mathsf{SNR}_{\mathsf{sd}} = g_{\mathsf{sd}}\mathsf{SNR}$$
$$\mathsf{SNR}_{\mathsf{rd}} = g_{\mathsf{sr}}\mathsf{SNR}$$
$$\mathsf{SNR}_{\mathsf{sr}} = g_{\mathsf{rd}}\mathsf{SNR}$$

We consider the *slow fading* situation where the delay requirement is short compared to the coherence time of the channel. Thus we can assume that the channel gains are random but

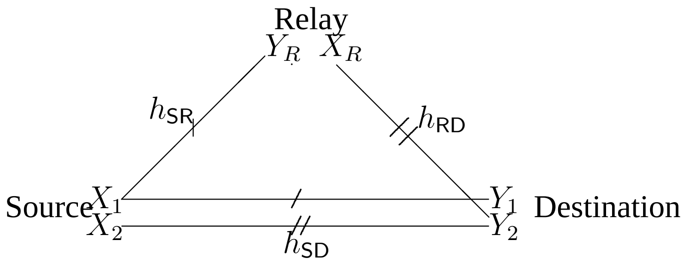

Fig. 1. The frequency division communication model that satisfies the half duplex constraint.

fixed for all time. We also assume that the relay knows channel gain h_{sr} and the destination knows the channel gains h_{sd} and h_{rd}. In this paper we compute the mutual information and rates in nats/s.

III. The Effect of Channel Knowledge

Channel estimation is quite challenging in the low SNR regime therefore it is important to understand how much the channel knowledge is beneficial or crucial to the outage capacity of the fading relay channel in the interested regime. We study two extremes in this section. One extreme is the case that neither the transmitter nor the receiver knows the channel (non coherent model). We show even without the channel knowledge available at the destination one can achieve the same outage capacity as before using a bursty pulse position modulation (PPM) scheme. On the other hand in the other extreme that both the transmitter and receiver know the channel (Full CSI model) we show that the outage capacity can be increased slightly while the channel estimation becomes very hard. We also discuss that the bursty amplify-forward scheme combined with beam-forming can achieve the outage capacity in this case.

A. Outage Capacity of Non-Coherent Fading Relay Channel

In this section we show that even without the channel knowledge available at the receiver as well as the transmitter, we can achieve the same outage capacity as the case that CSI is available at the receiver (1). The achievable scheme is using bursty pulse position modulation coding and the amplify forward scheme at the relay. The detection at the destination is based on energy detection, i.e. the position with highest energy is decoded at the destination.

Let $\phi_1,\ldots,\phi_M$ be M orthonormal signals of the form $\phi_i = (0,\ldots,0,1,0,\ldots,0)$, which is a length M vector with non-zero value at the i-th position ($i = 1,\ldots,M$).

To transmit message m ($m = 1,\ldots,M$), the source will broadcast the message $x_m = A\phi_m$ followed by zeros in $L > M$ time slots. The relay and the destination will respectively receive y_R and y_1. In the next L time slots, the source remains silent and the relay will transmit the first M time unit of y_R that contains information (normalized by $\sqrt{\frac{A^2}{A^2|h_{\mathsf{sr}}|^2+M}}$ to satisfy the average power and remains silent afterwards and the destination will receive y_2. In order to satisfy the average power constraint we should have $A^2 = 2LP$. To decode, the destination will compute $y = (|y_{1,1}|^2+|y_{2,1}|^2/\hat{\sigma}^2,\ldots,|y_{1,M}|^2+|y_{2,M}|^2/\hat{\sigma}^2)$, where $\hat{\sigma}^2$ is the estimated variance of the indirect path and $y_{1,m}$ and $y_{2,m}$ are respectively the projection of the first M elements of y_1 and y_2 onto ϕ_m $(m = 1,\ldots,M)$:

$$y_{1,m} = y_1(1,\ldots,M)\phi_m^t, \quad m = 1,\ldots,M \tag{3}$$
$$y_{2,m} = y_2(1,\ldots,M)\phi_m^t, \quad m = 1,\ldots,M \tag{4}$$

The destination will decode the unique message m if the $m-th$ component of y is maximum. But for making the analysis simpler we will use another decoding technique that requires the destination to pick a threshold, τ, and to decode the unique message m if the $m-th$ component of y is uniquely larger than the threshold τ. It is obvious that the probability of error using this genie aided scheme can not be less than the first strategy (picking the maximum).

By symmetry lets assume that message 1 has been transmitted by the source, then for fixed channel gains we have,

$$\begin{aligned}
y_{1,1} &= Ah_{\mathsf{sd}} + z_{1,1}\\
&\sim \mathcal{CN}(Ah_{\mathsf{sd}},1)\\
y_{1,m} &= z_{1,m}, \quad 1 < m \le M\\
&\sim \mathcal{CN}(0,1)\\
y_{2,1} &= \frac{A^2h_{\mathsf{sr}}h_{\mathsf{rd}}}{\sqrt{A^2|h_{\mathsf{sr}}|^2+M}} + \frac{Ah_{\mathsf{rd}}}{\sqrt{A^2|h_{\mathsf{sr}}|^2+M}}z_{R,1} + z_{2,1}\\
&\sim \mathcal{CN}\left(\frac{A^2h_{\mathsf{sr}}h_{\mathsf{rd}}}{\sqrt{A^2|h_{\mathsf{sr}}|^2+M}}, \frac{A^2|h_{\mathsf{rd}}|^2}{A^2|h_{\mathsf{sr}}|^2+M}+1\right)\\
y_{2,m} &= \frac{Ah_{\mathsf{rd}}}{\sqrt{A^2|h_{\mathsf{sr}}|^2+M}}z_{R,m} + z_{2,m}, \quad 1 < m \le M\\
&\sim \mathcal{CN}\left(0, \frac{A^2|h_{\mathsf{rd}}|^2}{A^2|h_{\mathsf{sr}}|^2+M}+1\right)
\end{aligned}$$

where $z_{1,1}, z_{1,2}, z_{2,1}, z_{2,2}, z_{R,1}$ and $z_{R,2}$ are distributed like $\mathcal{CN}(0,1)$.

There are two cases that the decoding fails:

$$|y_{1,1}|^2 + \frac{|y_{2,1}|^2}{\hat{\sigma}^2} < \tau \tag{5}$$

or there exists one $1 < i \le M$ such that

$$|y_{1,i}|^2 + \frac{|y_{2,i}|^2}{\hat{\sigma}^2} > \tau \tag{6}$$

We select the variance estimator to be

$$\hat{\sigma}^2 = \frac{\sum_{i=1}^{M}|y_{2,i}|^2}{M} \overset{\text{M Large}}{\approx} \frac{A^2|h_{\mathsf{rd}}|^2}{A^2|h_{\mathsf{sr}}|^2+M}\left(\frac{A^2|h_{\mathsf{sr}}|^2}{M}+1\right)+1 \tag{7}$$

Now to make the probability of first event small we make sure that the mean of the random variable $|y_{1,1}|^2+|y_{2,1}|^2/\hat{\sigma}^2$

is far from τ.

$$\mathbb{E}[|y_{1,1}|^2 + \frac{|y_{2,1}|^2}{\hat{\sigma}^2}] \approx$$
$$A^2|h_{\mathsf{sd}}|^2 + \frac{A^4|h_{\mathsf{sr}}|^2|h_{\mathsf{rd}}|^2}{A^2|h_{\mathsf{sr}}|^2 + A^2|h_{\mathsf{rd}}|^2 + M + \frac{A^4|h_{\mathsf{sr}}|^2|h_{\mathsf{rd}}|^2}{M}} +$$
$$+ \frac{A^2|h_{\mathsf{sr}}|^2 + A^2|h_{\mathsf{rd}}|^2 + M}{A^2|h_{\mathsf{sr}}|^2 + A^2|h_{\mathsf{rd}}|^2 + M + \frac{A^4|h_{\mathsf{sr}}|^2|h_{\mathsf{rd}}|^2}{M}}$$
$$> A^2|h_{\mathsf{sd}}|^2 + \frac{A^4|h_{\mathsf{sr}}|^2|h_{\mathsf{rd}}|^2}{A^2|h_{\mathsf{sr}}|^2 + A^2|h_{\mathsf{rd}}|^2 + M + \frac{A^4|h_{\mathsf{sr}}|^2|h_{\mathsf{rd}}|^2}{M}}$$

To make the mean of the random variable $|y_{1,1}|^2 + \frac{|y_{2,1}|^2}{\hat{\sigma}^2}$ far from τ it is sufficient to have

$$\tau << A^2|h_{\mathsf{sd}}|^2 + \frac{A^4|h_{\mathsf{sr}}|^2|h_{\mathsf{rd}}|^2}{A^2|h_{\mathsf{sr}}|^2 + A^2|h_{\mathsf{rd}}|^2 + M + \frac{A^4|h_{\mathsf{sr}}|^2|h_{\mathsf{rd}}|^2}{M}} \quad (8)$$

Before considering the second case we state a lemma,

Lemma 1. *If u and v are exponential random variables with mean μ_u and μ_v respectively then*

$$\mathbb{P}\{u + v > \tau\} = \frac{\mu_u}{\mu_u - \mu_v} e^{-\frac{\tau}{\mu_u}} - \frac{\mu_v}{\mu_u - \mu_v} e^{-\frac{\tau}{\mu_v}} \quad (9)$$

Proof: See [12].

The second case of error consists of $M - 1$ events. Each event occurs when a sum of two exponential random variables (with means 1 and $\mu = \frac{\frac{A^2|h_{\mathsf{rd}}|^2}{A^2|h_{\mathsf{sr}}|^2+M}+1}{\hat{\sigma}^2} < 1$) is greater than τ. Therefore by union bound and lemma 1 we have

$$\mathbb{P}\{\exists i : |y_{1,i}|^2 + \frac{|y_{2,i}|^2}{\hat{\sigma}^2} > \tau\} < (M-1)\mathbb{P}\{|y_{1,2}|^2 + \frac{|y_{2,2}|^2}{\hat{\sigma}^2} > \tau\} < \frac{1}{1-\mu} e^{\ln M - \tau}$$

Now in order to make the probability of this event small we should have a large negative exponent, i.e.

$$\ln M << \tau \quad (10)$$

To be able to pick the threshold τ to simultaneously satisfy (8) and (10) we should have

$$\ln M << A^2|h_{\mathsf{sd}}|^2 + \frac{A^4|h_{\mathsf{sr}}|^2|h_{\mathsf{rd}}|^2}{A^2|h_{\mathsf{sr}}|^2 + A^2|h_{\mathsf{rd}}|^2 + M + \frac{A^4|h_{\mathsf{sr}}|^2|h_{\mathsf{rd}}|^2}{M}} \quad (11)$$

As we are interested in the regime that $R = \frac{\ln M}{2L} \to 0$, $P \to 0$ and $\frac{R}{P} = \frac{\log M}{2LP} \to 0$, we pick M and L large enough such that:

$$\ln M << M << 2LP = A^2 \quad (12)$$

Therefore as long as $\min\{A^2|h_{\mathsf{sr}}|^2, A^2|h_{\mathsf{rd}}|^2\} < M$,

$$\frac{A^4|h_{\mathsf{sr}}|^2|h_{\mathsf{rd}}|^2}{A^2|h_{\mathsf{sr}}|^2 + A^2|h_{\mathsf{rd}}|^2 + M + \frac{A^4|h_{\mathsf{sr}}|^2|h_{\mathsf{rd}}|^2}{M}} \approx \min\{A^2|h_{\mathsf{sr}}|^2, A^2|h_{\mathsf{rd}}|^2\} \quad (13)$$

and we can satisfy (11) if

$$\ln M << A^2|h_{\mathsf{sd}}|^2 + A^2 \min\{|h_{\mathsf{sr}}|^2, |h_{\mathsf{rd}}|^2\} \quad (14)$$

or

$$\frac{R}{P} = \frac{\ln M}{2LP} = \frac{\ln M}{A^2} < |h_{\mathsf{sd}}|^2 + \min\{|h_{\mathsf{sr}}|^2, |h_{\mathsf{rd}}|^2\} \quad (15)$$

which is the same expression as the max-flow min-cut upper bound being greater than the rate that we try to communicate.

And if $\min\{A^2|h_{\mathsf{sr}}|^2, A^2|h_{\mathsf{rd}}|^2\} > M$ then

$$\frac{A^4|h_{\mathsf{sr}}|^2|h_{\mathsf{rd}}|^2}{A^2|h_{\mathsf{sr}}|^2 + A^2|h_{\mathsf{rd}}|^2 + M + \frac{A^4|h_{\mathsf{sr}}|^2|h_{\mathsf{rd}}|^2}{M}} \approx M \quad (16)$$

and we can obviously satisfy (11). Therefore with this protocol we can achieve all rates up to the max-flow min-cut upper bound of the network.

B. Outage Capacity of the Fading Relay Channel with Full CSI

In this part we investigate the outage capacity of the FD-relay channel shown in Figure 1. The model is the same as before except for the fact that channel state information is available at both transmitter and receiver (full CSI). One might think once the transmitter knows the channel it can always adjust the power such that no outage occurs, this is a valid idea if we can average the power on different realizations of the channel. How ever in a slow fading scenario that the channel varies very slowly over time it is a practical constraint to have average power constraint during a single realization of the channel. Therefore in this scenario the transmitter can not avoid the outage and outage capacity if an interesting measure to look at.

1) The Upper Bound on the Outage Capacity: In this section we use the general max-flow min-cut bound for the network shown in Figure 1 to find an upper bound on the $\epsilon - outage$ capacity of the FD relay channel with full CSI using in the limit of low SNR and low probability of outage. The details can be found in [12] and the bounds are

$$C_{relay}(h_{\mathsf{sd}}, h_{\mathsf{rd}}, h_{\mathsf{sr}}) \leq \min_{\substack{0\leq\beta\leq 1 \\ -1\leq\rho\leq 1}} \{|h_{\mathsf{sd}}|^2(\beta + (1-\beta)(1-\rho^2)) + |h_{\mathsf{sr}}|^2\beta , |h_{\mathsf{sd}}|^2 + |h_{\mathsf{rd}}|^2 + 2|h_{\mathsf{sd}}||h_{\mathsf{rd}}|\rho\sqrt{1-\beta}\}P \quad (17)$$

Also we have the following bound on the outage probability with full CSI:

$$P_{out,relay} \geq \min_{\substack{0\leq\beta\leq 1 \\ -1\leq\rho\leq 1}} \mathbb{P}\{|h_{\mathsf{sd}}|^2(\beta + (1-\beta)(1-\rho^2)) + |h_{\mathsf{sr}}|^2\beta , |h_{\mathsf{sd}}|^2 + |h_{\mathsf{rd}}|^2 + 2|h_{\mathsf{sd}}||h_{\mathsf{rd}}|\rho\sqrt{1-\beta} < \frac{R}{SNR}\} \quad (18)$$

2) The Achievable Scheme: BAF + Beamforming: Here we show that for any choice of β and ρ it is possible to achieve the max-flow min-cut bound on the outage probability shown in (18) in the limit of low SNR and low outage probability. To achieve the bound we use the described BAF protocol (source talks fraction of α of the time) with the difference that here the source uses both frequency bands to transmit the new data and in the second frequency band some of the power is allocated to beamform with the help of the relay.

The idea is that for given β and ρ, we construct X_1 using random Gaussian code generation with power $\frac{\beta P}{\alpha}$. Now a part of X_2 should be used to transmit new data and a part of it is used to beamform with the relay. The details are referred to [12] and the achievable rate is

$$\begin{aligned} R_{\text{BAF+B}} &\approx \alpha \ln(1 + |h_{\mathsf{sd}}|^2 \frac{\beta P}{\alpha} + |h_{\mathsf{sd}}|^2 (1-\rho)^2 (1-\beta) \frac{P}{\alpha} + \\ &\quad + |h_{\mathsf{sd}}|^2 (1-\beta) \rho^2 \frac{P}{\alpha} + |h_{\mathsf{rd}}|^2 \frac{P}{\alpha} + 2|h_{\mathsf{sd}}||h_{\mathsf{rd}}| \rho \sqrt{1-\beta} \frac{P}{\alpha}) \\ &\approx |h_{\mathsf{sd}}|^2 P + |h_{\mathsf{rd}}|^2 P + 2|h_{\mathsf{sd}}||h_{\mathsf{rd}}| \rho \sqrt{1-\beta} P \end{aligned} \quad (19)$$

Which is the same as the multiple access cut in (17).

To have a sense of this additional gain lets look at the case that all the channel gains are rayleigh fading with variances 1 ($g_{\mathsf{sd}} = g_{\mathsf{sr}} = g_{\mathsf{rd}} = 1$). If we solve the maximization problem in this case we get

$$\beta \approx 0.94$$
$$\rho \approx 1$$
$$R_{\text{BAF+B}} \approx 1.04\sqrt{\epsilon}\ \mathsf{SNR}$$

Now if we compared this rate to the outage capacity of the corresponding relay channel without CSI at the transmitter ($\approx \sqrt{\epsilon}\ \mathsf{SNR}$) we notice that the additional gain from having CSI at the transmitter is quite low (just 4%). This can be intuitively explained by noticing that the source prefers to allocate more power to the first frequency band which both the destination and the relay can receive data from to increase diversity than the second frequency band (for beam-forming with the relay).

IV. CONCLUSION

In this paper we looked at the outage performance of the FD fading relay channel. We were able to find the ϵ-outage capacity and the ϵ-outage capacity per unit cost of this relay channel in the limit of low SNR and low probability of outage. We also showed that this optimal outage rate is achieved by Bursty Amplify-Forward protocol.

As the channel estimation is quite challenging in the low SNR regime we look at a non coherent scenario that neither the transmitter nor the receiver know the channel state. We showed that there is a scheme that uses bursty pulse position modulation for encoding and a type of energy detection for decoding and achieves the same rate as before (with the same outage probability). Hence the outage capacity of non coherent scenario is the same as the coherent scenario. We also investigate another extreme that the channel state information is available at both the transmitter and the receiver (full CSI). We show that this additional information will just slightly increase the outage capacity while the communication protocol gets quite complicated. The optimal scheme in this case is a combination of beam-forming and bursty amplify-forward protocols.

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