The Strong Interference Channel with Unidirectional Cooperation

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***Abstract*— We consider the interference channel in which messages sent at one encoder are known to the other encoder, but not vice versa. Such a channel model allows for unidirectional node cooperation: the encoder that knows both messages can exploit that information to improve the achievable rates. We seek to derive conditions under which the capacity region of this channel coincides with the capacity region of the channel in which both messages are required at both receivers. We wish to compare the obtained conditions with the strong interference conditions in the interference channel with independent messages, as well as in the interference channel with both private and common messages.**

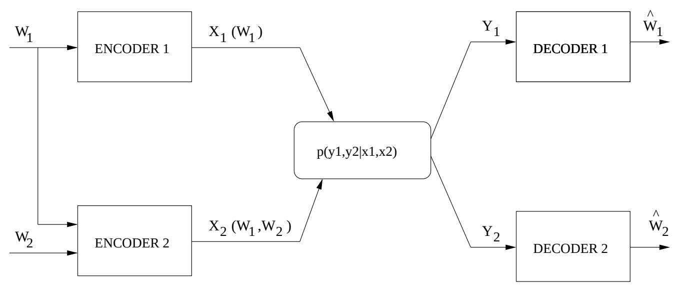

Fig. 1. Interference channel with unidirectional cooperation.

I. Introduction

A discrete memoryless interference channel [1] consists of two input alphabets $\mathcal{X}_1, \mathcal{X}_2$, two output alphabets $\mathcal{Y}_1, \mathcal{Y}_2$ and the conditional probability distribution $p(y_1, y_2|x_1, x_2)$. This channel model assumes that the two channel inputs are independent, thus precluding any form of cooperation between encoders. However, cooperation among encoders can improve the achievable rates in the channel. For a Gaussian network with two transmitters and two receivers, improvements in the achievable rates due to node cooperation were demonstrated in [2]–[6]. In [2], the transmitters fully cooperate by exchanging their intended messages and then jointly encode them using dirty paper coding. Other cooperation schemes were analyzed in [3]–[5].

In a discrete memoryless channel, a problem in which encoders partially cooperate was proposed by Willems for a multiple access channel (MAC) [7]. To model the transmitter cooperation, two communication links with finite capacities are introduced between the two encoders. The amount of information exchanged between the two transmitters is bounded by the capacities of the communication links. The proposed discrete channel model enables investigation of transmitter cooperation gains. When cooperating over links with finite capacities, encoders obtain partial information about each other's messages. This information is referred to as a *common* message as it is known to both encoders after cooperation. In addition, each encoder will still have independent information referred to as a *private* message, as this message remains unknown to the other encoder.

In this paper, we consider the interference channel in which full information about messages sent at one encoder is available to the other encoder, but not vice versa. Such a channel model allows for unidirectional node cooperation; the encoder that knows both messages can exploit that information to improve the achievable rates. The communication system is shown in Figure 1. Without cooperation, this channel reduces to the interference channel [1], [8] for which the capacity region is known in the case of *strong interference* [9] satisfying

$$I(X_1; Y_1|X_2) \leq I(X_1; Y_2|X_2) \quad (1)$$

$$I(X_2; Y_2|X_1) \leq I(X_2; Y_1|X_1) \quad (2)$$

for all product distributions on the inputs X_1 and X_2. The capacity region in this case coincides with the capacity region of the two-sender, two-receiver channel in which both messages are decoded at both receivers, as determined by Ahlswede [10].

In this work, our goal is to derive conditions equivalent to (1)-(2) under which there is no penalty in decoding both messages at both decoders in the interference channel with unidirectional cooperation. We can then compare the obtained conditions to the strong interference conditions determined in [9] and [11].

II. Channel model

We consider a memoryless interference channel [1] that consists of finite sets $\mathcal{X}_1, \mathcal{X}_2, \mathcal{Y}_1, \mathcal{Y}_2$ and a conditional probability distribution $p(y_1, y_2|x_1, x_2)$. Symbols $(x_1, x_2) \in \mathcal{X}_1 \times \mathcal{X}_2$ are channel inputs and $(y_1, y_2) \in \mathcal{Y}_1 \times \mathcal{Y}_2$ are the corresponding channel outputs. Each encoder t, $t = 1, 2$, wishes to send a message $W_t \in \{1, \ldots, M_t\}$ to decoder t in N channel uses. The channel is memoryless and time-invariant in the sense that

$$p(y_{1,n}, y_{2,n}|\mathbf{x}_1^n, \mathbf{x}_2^n, \mathbf{y}_1^{n-1}, \mathbf{y}_2^{n-1}) = p(y_{1,n}, y_{2,n}|x_{1,n}, x_{2,n}) \quad (3)$$

where $\mathbf{x}_t^n = \left[x_{t,1}, \quad \ldots, \quad x_{t,n}\right]$. Here we follow the convention of dropping subscripts of probability distributions if the arguments of the distributions are lower case versions of the corresponding random variables. To simplify notation, we drop the superscript when $n = N$. Indexes W_1 and W_2 are independently generated. Each encoder t, $t = 1, 2$, wishes to send a message $W_t \in \{1, \ldots, M_t\}$ to decoder t in N channel uses. It is assumed that message W_1 is also known at encoder 2, thus allowing for unidirectional cooperation.

An (M_1, M_2, N, K, P_e) code for the channel consists of two encoding functions generating codewords

$$\mathbf{x}_1 = f_1(W_1) \tag{4}$$

$$\mathbf{x}_2 = f_2(W_1, W_2) \tag{5}$$

two decoding functions

$$\hat{W}_t = g_t(\mathbf{Y}_t) \quad t = 1, 2 \tag{6}$$

and a maximum error probability

$$P_e = \max\{P_{e,1}, P_{e,2}\} \tag{7}$$

where, for $t = 1, 2$

$$P_{e,t} = \sum_{(w_1,w_2)} \frac{1}{M_1 M_2} P\left[g_t(\mathbf{Y}_t) \neq (w_t) | (w_1, w_2) \text{ sent}\right]. \tag{8}$$

A rate pair (R_1, R_2) is achievable if, for any $\epsilon > 0$, there is an (M_1, M_2, N, P_e) code such that

$$P_e \leq \epsilon \text{ and } M_i \geq 2^{NR_i} \quad i = 1, 2.$$

The capacity region of the *interference channel with unidirectional cooperation* is the closure of the set of all achievable rate pairs (R_1, R_2).

III. Achievability: The Compound MAC With Common Information

The encoder cooperation results in a common message known to both encoders, in addition to the private messages. The capacity region of the interference channel with encoder cooperation is then closely related to the capacity region of the multi-access channel in which private and common messages are transmitted, refered to as the MAC with common information [12]. More generally, the interference channel with cooperation relates to the MAC with arbitrarily correlated sources [13].

Unlike the MAC, the interference channel model assumes two receivers. When each receiver wishes to decode both private and common messages, the resulting channel becomes a compound MAC with common information. We wish to determine the conditions under which the capacity region of the interference channel with unidirectional cooperation coincides with the capacity region of the compound MAC with common information, denoted $\mathcal{C}_{\text{CMAC}}$, and given by [14]

$$\begin{aligned}\mathcal{C}_{\text{CMAC}} = \bigcup \Big\{ (R_0', R_1', R_2') : \\ R_1' &\leq \min_t I(X_1; Y_t | X_2, U) \\ R_2' &\leq \min_t I(X_2; Y_t | X_1, U) \\ R_1' + R_2' &\leq \min_t I(X_1, X_2; Y_t | U) \\ R_0' + R_1' + R_2' &\leq \min_t I(X_1, X_2; Y_t) \Big\}\end{aligned} \tag{9}$$

where the union is over all $p(u, x_1, x_2, y_1, y_2)$ that factor as $p(u)p(x_1|u)p(x_2|u)p(y_1, y_2|x_1, x_2)$.

For the interference channel with unidirectional cooperation, encoder 2 knows the entire message W_1, and thus we view R_1 as the common rate, corresponding to the common rate R_0' in the compound MAC. For the same reason, user 1 in the compound MAC has private message rate $R_1' = 0$. It follows that we can choose $U = X_1$ and the region (9) becomes

$$\begin{aligned}\mathcal{C}_{\text{MAC}} = \bigcup \Big\{ (R_1, R_2) : \\ R_2 &\leq \min_t I(X_2; Y_t | X_1) \\ R_1 + R_2 &\leq \min_t I(X_1, X_2; Y_t) \Big\}\end{aligned} \tag{10}$$

where the union is over all $p(x_1, x_2, y_1, y_2)$.

Consider next the strong interference channel with unidirectional cooperation. The achievability of the rates (10) in the case in which both messages are required at the receivers guarantees that these rates are also achieved when a weaker constraint of decoding of a single message is imposed at the receivers. Hence the proof of achievability of rates (10) is immediate. Our goal is to determine the strong interference conditions under which we can prove the converse.

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