Capacity and Cooperation in Wireless Networks

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***Abstract*— We consider fundamental capacity limits in wireless networks where nodes can cooperate in transmission and reception. Cooperation can take many forms, including virtual multiple-input multiple-output transmission, different relaying strategies, and conferencing. The optimal cooperation strategy depends on the network topology, the channel SNR, and the channel side information available at the nodes. We discuss the lessons learned so far along with the remaining open questions. The challenges in developing new cooperation techniques and applying them to a broader class of channels are also outlined.**

I. INTRODUCTION

Wireless networks consist of a large number of geographically-dispersed nodes. Each node in the network can communicate with any other node, although the link quality between nodes separated by a large distance may be poor, and utilizing such a link may cause excessive interference to other nodes. In many wireless networks, nodes cooperate in transmission and/or reception. In particular, nodes close to the transmitter can cooperate to form a transmitter cluster, and/or nodes close to the receiver can cooperate to form a receiver cluster, as illustrated in Fig. 1. Cooperation can be used to form a virtual multiple-antenna transmitter and/or receiver, or the cooperating nodes can be used as relays.

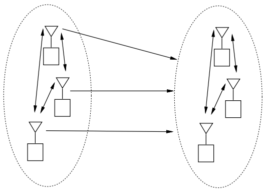


Fig. 1. Cooperating transmitters and receivers.

The idea of cooperative diversity was pioneered in [1], [2], where the transmitters repeat detected symbols from each other to increase their rate region. In [3] transmitters forward parity bits to achieve cooperation diversity. Orthogonal cooperative protocols were shown to achieve full spatial diversity order in [4]. Information-theoretic achievable rate regions and capacity bounds under transmitter and/or receiver cooperation were derived in [5]–[8]. For relay networks it was shown in [9] that decode-and-forward is close to optimal when the relay is near the source, whereas compress-and-forward is better when the relay is near the destination.

Multiple-input multiple-output (MIMO) channel capacity exhibits multiplexing gain linear in the number of antennas. However, in [10] it was shown that at an asymptotically high signal-to-noise ratio (SNR), cooperative systems do not enjoy full multiplexing gain. Similarly, in [11] it was shown that cooperative large relay networks fail to achieve the linear capacity scaling order (in the number of antennas) found in MIMO systems. However, we will see in Section III that under some conditions on SNR, network geometry, and number of antennas, a cooperative system performs at least as well as a MIMO system with isotropic inputs.

Cooperative communications between wireless network nodes has also been considered in the context of multiuser channels. Specifically, it was shown in [12] that receiver cooperation enlarges the broadcast channel (BC) rate region. The achievable rates of a BC with conferencing receivers were presented in [13]. When both receivers wish to decode a common message, the performance gain from iterative cooperation was illustrated in [14] on binary erasure channels with conferencing receivers. In particular, [13], [14] show that iterative cooperation outperforms one-shot cooperation in the case of a common message for the receivers.

In this paper we explore different cooperation strategies and their relative benefits for different network topology, channel side information, and channel SNR assumptions, drawing from our capacity results in [15]–[19]. Section II considers cooperation in the general setting of transmitter and receiver clusters. Section III investigates capacity of transmitter and receiver cooperation in relay channels. The benefits of iterative cooperation in conferencing relay channels are investigated in Section IV. A summary of lessons learned and open questions regarding the capacity benefits of cooperation, as well as extending cooperation ideas to include partial decoding and broadcasting, are given in Sections V and VI, respectively.

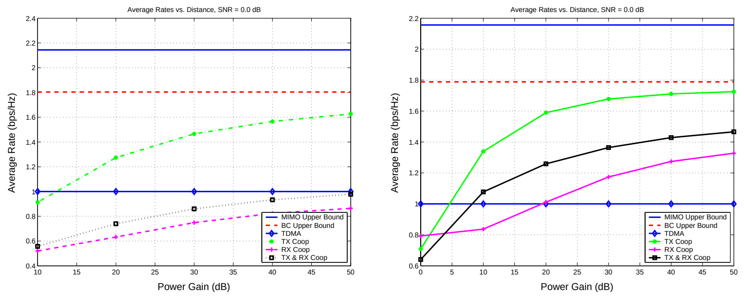
II. COOPERATION FOR TRANSMIT/RECEIVER CLUSTERS

We first consider a system with two cooperating nodes at both the transmit and receive side. Transmitter 1 wishes to communicate with Receiver 1, and Transmitter 2 wishes to communicate with Receiver 2. The transmit nodes are close together and the receive nodes are close together, with a much larger distance separating the transmit and receiver clusters, as shown in Fig. 1. In [20] the capacity of this system was studied taking into account both the power and bandwidth cost of cooperation between transmitter and receiver nodes.

Assume a channel model

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \mathbf{H} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} n_1 \\ n_2 \end{bmatrix}, \qquad \mathbf{H} = \begin{bmatrix} e^{j\theta_1} & e^{j\theta_2} \\ e^{j\theta_3} & e^{j\theta_4} \end{bmatrix}$$

where x_i is the transmitted symbol of transmitter i, y_i is the received symbol of receiver i, and n_i is zero-mean circularly symmetric complex Gaussian (ZMCSCG) white noise with



(a) Free cooperation BW.

(b) Total BW shared.

Fig. 2. Cooperation at TX, RX, and both.

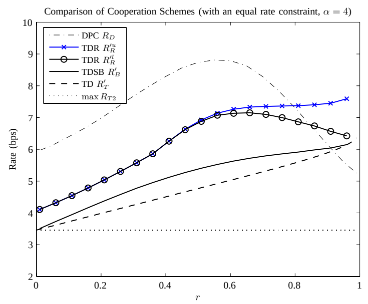


Fig. 3. DPC vs. relaying.

normalized unit variance. Since the amplitudes of all gains are equal, phase differences are required for $\mathbf{H}$ to be full rank. The channel gains and phases were assumed to be fixed and known at all nodes. Furthermore, perfect synchronization between nodes was assumed. In addition to the direct communication channel, there is also an AWGN channel (with gain $\sqrt{G}$, $G > 1$) between the two transmitters and between the two receivers that allows for cooperation. A large G indicates that the transmission distance far exceeds the distance between nodes in the transmit and receive clusters. Two assumptions were made about the system bandwidth: either the cooperation channels and transmission channel divided the system bandwidth B orthogonally in an optimal way or each cooperation channel as well as the transmission bandwidth were assigned bandwidth B, which effectively removes the bandwidth cost of cooperation (this latter case is reasonable when the transmit and receiver clusters are separated by a large distance and hence the same bandwidth can be reused at each cluster). By separating the cooperation and transmission bandwidths orthogonally, capacity can be computed using known capacity formulas for the BC, MAC, and MIMO channels [20].

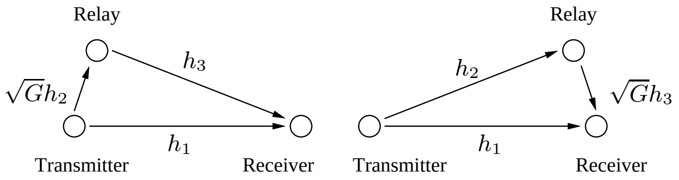
Assuming "free" cooperation bandwidth, the capacity under different forms of transmitter and receiver cooperation for an SNR of 0 dB on the transmission channel is given in Fig. 2(a). Note that the TX cooperation comes close to that of the BC upper bound for large G, which is expected since the cooperation cost is almost free (bandwidth is free and the cooperative transmit power is small for G large). However, while receiver cooperation alone beats time-division, it is much less beneficial than transmit cooperation. Transmit and receive cooperation also do not approach MIMO capacity. These results indicate that even without a bandwidth cost of cooperation, receiver cooperation does not yield much capacity gain at low SNRs.

When the total system bandwidth B is allocated between cooperation and transmission, we get the capacity shown in Fig. 2(b). Again we see that transmitter cooperation provides the most benefit, however time division beats receiver cooperation as well as the combination of transmit and receive cooperation at low SNRs [20]. At high SNRs (e.g., 10 dB), receiver cooperation is more beneficial than at low SNRs under both bandwidth assumptions, but still much less beneficial than transmit cooperation. These results indicate some intriguing properties of cooperation: that cooperation on one side can be much more beneficial than on the other side, and that SNR has a big impact on the benefits of cooperation.

Given the benefits of transmitter cooperation illustrated in these numerical results, a natural question that arises is the following: what is the best form of transmitter cooperation, and how does it depends on the network topology? This question was addressed in [15], where capacity under different forms of transmitter cooperation were compared. The network topology is similar to before: two transmitters that cooperate and two receivers that cooperate. However, we now assume that one of the nodes in the transmit cluster, Transmitter 1, can be located anywhere along a straight line separating Transmitter 2 from the receive node cluster.

As Transmitter 1 moves closer to the receiver cluster, the cost of exchanging messages with Transmitter 2 increases, and Transmitter 1 becomes more effective as a relay for Transmitter 2. Thus, cooperative schemes that incorporate relaying need to be compared against cooperative dirty-paper coding (DPC) between the two transmitters. In [15] two cooperative relay schemes were compared against dirty paper coding: time-division successive broadcasting and time-division relaying. In time-division successive broadcasting (TDSB), over some fraction of time Transmitter 2 broadcasts independent messages to Transmitter 1 (for later cooperation) and to its desired receiver, Receiver 2. In the remaining time Transmitter 1 broadcasts to its desired receiver, Receiver 1, as well as to Receiver 2 to help Transmitter 2's communication. Capacity is evaluated based on broadcast channel capacity calculations. In time-division relaying (TDR), for some fraction of time Transmitter 1 sends directly to Receiver 1, and for the remaining time Transmitter 2 sends to Receiver 2 using Transmitter 1 as a relay. The capacity of the AWGN relay channel is unknown, so capacity of the TDR scheme is bounded via the cut-set upper bound and the block-Markov (BM) decode-and-forward lower bound [21].

The sum-rate capacities of the different cooperation schemes under an equal rate constraint between each transmitter-receiver pair are shown in Fig. 3, assuming a power fall-off exponent α of 4 on the transmission links. The figure indicates



(a) Transmitter cooperation. (b) Receiver cooperation.

Fig. 4. Relay model.

that the sum-rate associated with DPC outperforms every other transmitter cooperation schemes for most values of r, the distance between the two cooperating transmitters. We also see that TDSB has a sum-rate capacity R'_B that outperforms that achieved by the non-cooperative TD scheme R'_T for all r. The TDR achievable rate, as represented by the lower bound R'^l_R, in turn outperforms TDSB, with the most significant capacity improvement for $r \approx 0.5$. Beyond $r \approx 0.5$, the TDR capacity with an equal rate constraint either decreases or offers little additional improvement. The upper bound of TDR continues to increase beyond $r \approx 0.5$; however, it is not known if the upper bound rate is achievable. These bounds indicate that under an equal rate constraint, using a transmitter in the transmit cluster as a relay node is most beneficial when this potential relay is about halfway between the transmit and receive clusters. Different values of the power fall-off exponent α indicate the same trend. Note that the bandwidth cost of cooperating on the transmit side for DPC was not taken into account—this will reduce the capacity gains of DPC relative to the other techniques.

In the next section these ideas are explored in detail for a slightly different network setup, that of a relay channel where the relay can aid either the transmitter or the receiver.

III. COOPERATION IN RELAY CHANNELS

A. Transmitter vs. receiver cooperation

We now consider a discrete-time memoryless channel where a transmitter is communicating with a relay and receiver, as illustrated in Fig. 4. We assume a static channel, unit-variance AWGN, and a network average power constraint of P. The channel power gain between the cooperating nodes is G, while the other channels have unit magnitude: $|h_i| = 1$, $i = 1, 2, 3$.

The relay can be deployed near the transmitter or near the receiver, and the questions is where the relay should be placed to maximize capacity between the transmitter and receiver. This question was investigated in [16] assuming both full or partial channel state information (CSI) as well as optimal or equal power allocation between the transmitter and relay. The capacity of the relay channel is unknown, but its capacity can be bounded using the cut-set upper bound [22, Thm. 14.10.1] and a lower bound based on any achievable transmission scheme. Two cooperation schemes were considered in [16]: the decode-and-forward scheme [21, Thm. 1] and the compress-and-forward scheme [21, Thm. 6]. In decode-and-forward transmission is done in blocks: the relay decodes the signal sent by the transmitter over one block, and in the subsequent block the relay and transmitter cooperatively send the message to the receiver. In compress and forward the relay sends a Wyner-Ziv compressed version of its received signal to the receiver.

It was shown in [9] that decode and forward (DF) is close to optimal when the relay is near the transmitter, and compress and forward (CF) is close to optimal when the relay is near the source. Thus, in [16], DF was used for transmitter cooperation with the relay, and CF was used for receiver cooperation. Under these assumptions it was shown that when all nodes have equal average transmit power along with full channel state information (CSI), transmitter cooperation outperforms receiver cooperation, whereas the opposite is true when power is optimally allocated among the nodes but only receiver phase CSI is available. In addition, when the nodes have equal average power with receiver phase CSI only, cooperation is shown to offer no capacity improvement over a non-cooperative scheme with the same average network power. When the system is under optimal power allocation with full CSI, the decode-and-forward transmitter cooperation rate is close to its cut-set capacity upper bound, and outperforms compress-and-forward receiver cooperation. Moreover, it is shown that full CSI is essential in transmitter cooperation, while optimal power allocation is essential in receiver cooperation. These results were extended to Rayleigh fading channels in [17], where similar observations hold.

B. Multiple-antenna relay vs. MIMO channel

When the relay has multiple antennas, we can compare the capacity of the cooperative system to that of a MIMO system. Consider the transmitter cooperation network in Fig. 4(a), where the transmitter, relay, and receiver has 1, $M - 1$, and M antennas, respectively. The multiple-antenna relay channel performance represents an upper bound for the case where the transmitter utilizes multiple single-antenna nodes clustered together that coordinate to form a relay.

While it is known that in the asymptotic regime, at a high SNR [10] or with a large number of cooperating nodes [11], cooperative systems lack full multiplexing gain, in [18] cooperative capacity gain was considered at moderate SNR with a fixed number of cooperating antennas. It was shown that up to a lower bound to an SNR threshold (MIMO-gain region), a cooperative system performs at least as well as a MIMO system with isotropic inputs; whereas beyond an upper bound to the SNR threshold (coordination-limited region), the cooperative system is limited by its coordination costs, and the capacity is strictly less than that of a MIMO orthogonal channel. The SNR threshold depends on the network geometry (the power gain G between the transmitter and relay) and the number of cooperating antennas M; when the relay is close to the transmitter ($G \gg 1$), the SNR threshold lower and upper bounds are approximately equal. As the cooperating nodes are closer, i.e., as G increases, the MIMO-gain region extends to

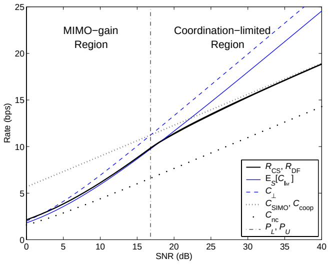


Fig. 5. SNR regions of a 2×2 cooperative system.

a higher SNR. Whereas for a populous cluster, i.e., when M is large, the coordination-limited region sets in at a lower SNR.

For example, the cooperative capacity with $M = 2$ can be contrasted with that of a 2×2 MIMO channel. The multiple-antenna relay channel cut-set bound R_{CS} and decode-and-forward rate R_{DF} are shown in Fig. 5, along with the SNR threshold lower and upper bounds P_L, P_U, the multiple-antenna capacity bounds, and for comparison the non-cooperative capacity $C_{\mathrm{nc}} = \log(1 + 2P)$, which corresponds to the case where the relay is not used, and the source is under power constraint P. We assume the relay is near the source ($G = 100$); decode-and-forward is close to capacity-achieving as expected, and plots of R_{CS}, R_{DF} appear overlapped. The figure indicates that in the MIMO-gain region when $P < P_L$, the relay rate R_{DF} outperforms the isotropic-input MIMO capacity $\mathrm{E}_S[\,C_{\mathbf{I}_M}]$. On the other hand, in the coordination-limited region, as $P > P_U$, the relay cut-set bound R_{CS} fails to parallel the orthogonal channel capacity $C_\perp$. Indeed, the cooperative capacity is bottlenecked by the SIMO channel capacity C_{SIMO}, and which scales with the SNR as $\Theta(\log P)$, instead of $\Theta(2 \log P)$.

IV. CONFERENCING IN RELAY CHANNELS

We now consider a relay channel where the relay and receiver cooperate via orthogonal conference links with finite capacity. The conference cooperation model was introduced by Willems in [23] for a multiple-access channel (MAC) with conferencing encoders. By contrast, we consider conferencing between the relay and receiver. Specifically, we consider a discrete-time memoryless channel where a transmitter is communicating with a relay and receiver, as illustrated in Fig. 6. We assume a static channel, unit-variance AWGN, perfect channel state information (CSI) everywhere, and an average total transmit power constraint P. The relative channel power gain between the relay and the receiver is g. We can assume real channel gains since the receivers can zero-phase the observed signals.

The relay and receiver cooperate by way of a conference, as defined in [23]. The conference links are assumed to have finite capacity $\alpha\mathsf{C}$ and $(1-\alpha)\mathsf{C}$, as shown in Fig. 6, where C is the total conference link capacity available between the receivers, and $\alpha \in [0, 1]$ represents the allocation of conferencing resources in each direction. A conference is permissible if the total cardinality of the conference communications (possibly sent over multiple rounds) does not exceed that allowed by the capacity of the conference link.

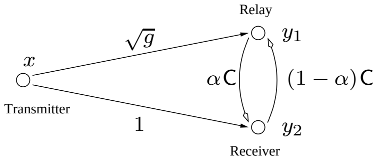
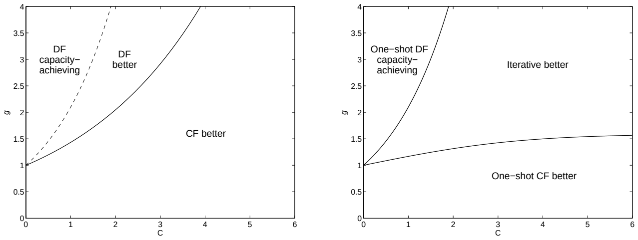


Fig. 6. Conferencing relay and receiver.



(a) One-shot conferencing.

(b) Iterative vs. one-shot.

Fig. 7. The best cooperation strategy as a function of g and C.

Within this general conferencing setup the conference itself can be one-shot (non-iterative) or iterative. A comparison of these conferencing schemes was done in [19] under different SNR and resource allocation assumptions. In particular, precise conditions that dictate which cooperation scheme achieves higher capacity were determined. It was shown that under one-shot conferencing, decode-and-forward (DF) is capacity-achieving when the relay has a strong channel relative to the conferencing capacity and power constraints. On the other hand, Wyner-Ziv compress-and-forward (CF) approaches the cut-set bound when the conference link capacity is large. A plot of the conditions under which each cooperation scheme is better and if it is capacity-achieving is shown in Fig. 7(a).

To contrast with one-shot conferencing, a two-round iterative conference was also considered. In this two-round conference, CF is done in the first round, and DF in the second. A plot of the relative performance of one-shot versus a two-round iterative conference is shown in Fig. 7(b). The figure indicates that when the relay has a weak channel, the iterative scheme is disadvantageous. However, when the relay channel is strong, iterative cooperation, with optimal allocation of conferencing resources, outperforms one-shot cooperation provided that the conference link capacity is large.

V. LESSONS LEARNED

The lessons we have learned from these studies revolve around orthogonalization, cooperative strategies, and the role of SNR. In particular, we have seen that orthogonal channelization imposes a considerable capacity loss in cooperative networks. That is because orthogonal cooperation channels take away system power and bandwidth that could otherwise be allocated to the transmission of user data. However, in a network of clusters where the cooperating nodes within a cluster are close together but far from other clusters, the cooperation channel between nodes within a cluster could operate at low transmit power and be reused spatially, thereby significantly reducing its bandwidth cost.

In transmitter cooperation, the block-Markov DF scheme achieves high spectral efficiency by staggering the cooperation message (i.e., transmission to the relay) with the user data (i.e., transmission to the receiver). Under CF receiver cooperation, the transmitter and relay do not joint-encode. In fact, the receiver treats the transmitter's signal as noise when it decodes the relay's message. From numerical results, we observe that suboptimal compression schemes (e.g., source coding without exploiting correlation in the observed signals of the relay and receiver) levy a minor capacity penalty. Thus it is conjectured that the suboptimality of CF is due to the spectral inefficiency of the receiver cooperation scheme.

We also observe that at high SNR, the rate requirement of the cooperation messages increase accordingly and the burden of the cooperation cost becomes more significant. Hence the cooperative capacity is limited by the cost of exchanging cooperation messages, and it trails the capacity of a MIMO system. However, for a given cooperative system (i.e., fixed geometry and the number of cooperating nodes), an SNR region can be established within which cooperation is efficient. Specifically, cooperation is most useful at low or moderate SNR where a cooperative system can perform at least as well as a MIMO system with isotropic inputs.

VI. EXTENSIONS AND OPEN QUESTIONS

Between the different modes of cooperation, transmitter cooperation is more well-understood. When the cooperating transmitters are close together, the DF strategy is nearly optimal. However, the optimal receiver cooperation strategy is less obvious. When the cooperating receivers are close together, even with the best-known receiver cooperation strategy of CF, the achievable rate is considerably lower than the cut-set bound. The DF and CF schemes represent two extremes in modes of cooperation. In the former case, the cooperating node decodes the transmitter's message completely, while in the latter no decoding is performed. Further work includes allowing partial-decoding at the cooperating node, possibly combined with multi-round iterative cooperation. In each round the cooperating node may decode parts of the message (with DF and CF being special cases of decoding the whole, or none of the message); we expect the optimal amount of decoding would depend on the network geometry, as well as the number of rounds of iteration allowed.

While the relay channel serves as a simple model for cooperative systems, the cooperation strategies investigated often apply to more general multiuser channels. For instance, the cooperative system model can be extended to a BC with cooperating receivers, where each of the receivers decodes its own independent message, possibly along with a common message both wish to decode. As the relay channel is a special case in which only one receiver wishes to decode the message sent by the transmitter, we expect the relay cooperation techniques, such as iterative conferencing, would be applicable as well in the general multiuser channels.

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