Ensemble Analysis on Minimum Span of Stopping Sets

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***Abstract*— In this paper, the minimum span of stopping sets of regular LDPC codes are defined and analyzed. The minimum span of an LDPC code is closely related to immunity to burst erasures when an LDPC code is decoded with belief propagation (BP) for erasure channels. The minimum span of stopping sets is the smallest span of a non-empty stopping set. If a single erasure burst of length shorter than the minimum span, the erasure burst can be corrected with BP. In order to study the nature of the minimum span, we use ensemble analysis for the bipartite graph ensemble.**

I. Introduction

Recent progress on the studies of LDPC codes and decoding algorithms shows that LDPC codes are not only theoretically interesting but also practically promising error correcting codes. An LDPC code is now considered to be a candidate of an error correction scheme for high speed wireless channel, wired channel (such as 10G-baseT) and magnetic recording systems.

In such an application, we have to take care of burst errors occurred on the channel as well as random (Gaussian) noise. For example, fading with time-correlation in wireless communication causes burst errors, which severely limits the throughput of the system. In magnetic recording system such as hard disk system, burst errors due to thermal asperity, media defect and off-track should be corrected using an error correcting code to attain high reliability of recording.

Some works on LDPC codes for burst channels have been made. There are two approaches: One approach is to devise a decoding algorithm for burst channel. Another is to construct LDPC codes which have high burst error correcting capability. The works by Garcia-Frias [1][2][3], Wadayama [4], Ratzer [5] belong to the first category. They presented iterative decoding algorithms suitable for burst channels.

The works by Hosoya et al.[6] and Yang and Ryan[7] lie in the second category. In order to construct a code with burst error immunity, Hosoya et al. proposed a method to obtain a new parity check matrix from an original parity check matrix by permuting its columns. Yang and Ryan proposed Lmax algorithm which efficiently evaluates maximum correctable erasure burst length of a given LDPC code.

In some situations, burst errors can be detected before decoding. Namely, in such a case, a burst detector can inform the location of burst errors to the decoder. For example, in magnetic recording system, burst errors caused by thermal asperity can be detected by observing the magnitude of the received signals. When we know the channel state information (CSI), i.e., the location of burst errors, we can regard the burst errors as burst erasures. Another example is packet loss channel which is a channel model for a packet based network. In such a network, burst erasures occur at a router when the network traffic is high.

The introduction of stopping sets by Di et al.[9] opened a way to finite length performance analysis on LDPC codes over erasure channels. A stopping set is a subset of variable nodes which dominates the error performance of belief propagation(BP)-based decoder over an erasure channel.

In this paper, the minimum span of stopping sets of regular LDPC codes are defined and analyzed. The minimum span of an LDPC code is closely related to immunity to burst erasures when an LDPC code is decoded with BP for erasure channels. The minimum span of stopping sets is the smallest span of a non-empty stopping set. If a single erasure burst of length shorter than the minimum span, the erasure burst can be corrected with BP. In order to study the nature of the minimum span, we use ensemble analysis for the bipartite graph ensemble.

II. Preliminaries

In this section, some notations required in this paper will be introduced and several known results on ensemble average of stopping sets will be reviewed.

A. Bipartite graph ensemble

In this paper, we consider an ensemble of regular bipartite graphs with n-variable nodes and m-check nodes [6]. The variable node and check node degrees are denoted by d_v and d_c, respectively. A graph G belonging to $\mathcal{E}$ is associated with equal probability $P(G) = 1/|\mathcal{E}|$. We call such an ensemble a *bipartite graph ensemble* and it is denoted by $\mathcal{E}$.

The set of variable nodes and check nodes are denoted by V and C, respectively. In this paper, a node is represented by a integer such that $V = \{1, 2, 3, \ldots, n\}$, $C = \{1, 2, 3, \ldots, m\}$. Let $E = \{(v, c) : v \in V, c \in C\}$ be a set of edges which is called an edge set.

A graph $G = (V \cup C, E) \in \mathcal{E}$ naturally corresponds to an $m \times n$ binary parity check matrix which is the adjacent matrix of G. The matrix is denoted by $H(G)$.

B. Probability on stopping sets

Stopping sets of a bipartite graph are basis of finite length performance analysis of LDPC codes under erasure channels. The definition of stopping sets is as follows.

Definition 1: Let S be a subset of V. The subset S is a stopping set if and only if every check nodes connected to S has at least two edges connected to S.

The most important property of stopping sets is that they dominates the correctability of BP-based decoding algorithm under erasure channels. If an erasure pattern covers a stopping set, a BP-based decoding procedure fails.

Di et al. [9], Orlitsky et al. [10] presented ensemble analysis on stopping set distribution. The following lemma is due to Orlitsky et al.[10].

Lemma 1: Assume a bipartite graph ensemble $\mathcal{E}$. Let $U \subset V$. The probability that U is a stopping set is given by

$\sum_{G \in \mathcal{E}} P(G) I[U \text{ is a stopping set}]$

$$= \frac{coef(((1+x)^{d_c} - d_c x)^m, x^{d_v |U|})}{\binom{d_v n}{d_v |U|}},$$

where $coef(f(x), x^e)$ means the coefficient of $f(x)$ corresponding to the term x^e. The function $I[predicate]$ is 1 if the predicate is true; otherwise $I[predicate]$ takes 0. □

Since this probability depends only on the size of U, we define $Q(w) (w \in [0, n])$, for convenience, by

$$Q(w) = \frac{coef(((1+x)^{d_c} - d_c x)^m, x^{d_v w})}{\binom{d_v n}{d_v w}}, \qquad (1)$$

which mean the probability of a subset of V with size w is a stopping set. The notation $[a, b]$ denotes the set of consecutive integers from a to b.

III. Minimum Span of Stopping Sets

A. Definitions

In the memoryless erasure channel case, the smallest size of of a stopping set has prime importance. In the context of burst erasure channel, the minimum span of stopping set dominates the block error probability.

Suppose that an instance of a bipartite graph ensemble, $G = (V \cup C, E)$, is given. Let $S = \{s_1, s_2, \ldots, s_t\} \subset V$ be a non-empty subset of V. The span of S is defined by $\ell(S) = 1 + \max_{s_i, s_j \in S, s_i \neq s_j} |s_i - s_j|$. If the subset S is a non-empty stopping set, $\ell(S)$ is called the *span of a stopping set*.

Based on the the span of a stopping set, we can define *minimum span of stopping sets* as follows;

Definition 2: For a given graph G, the minimum span of stopping sets $\mu(G)$ is defined by $\mu(G) = \min_{S \in SS(G)} \ell(S)$, where $SS(G)$ is the set of all non-empty stopping sets of G:

$$SS(G) = \{S \subset V : S \text{ is a non-empty stopping set}\}. \qquad (2)$$

B. Immunity to a single burst erasure

Consider that a sender transmits a codeword $X = (x_1, x_2, \ldots, x_n)$ to an erasure channel and then a receiver obtains $Y = (y_1, y_2, \ldots, y_n)$ as a received word. The set of indices such that y_i is the erasure is denoted by $Z(Y) = \{i \in [1, n] : y_i = e\}$, where the symbol e represents an erasure. Assume that the span of $Z(Y)$ is L; namely $\ell(Z(Y)) = L$. We say that this erasure pattern Y is a *single erasure burst of length* L.

The following lemma shows relation between the minimum span and correctability of a single erasure burst.

Lemma 2: Any single erasure burst of length smaller than $\mu(G)$ is correctable with BP.

(Proof) From the definition of $\mu(G)$, we know that no non-empty stopping sets whose span is smaller than $\mu(G)$ exist. In other words, the span of any non-empty stopping set is larger than or equal to $\mu(G)$. The means that no single erasure burst of length smaller than $\mu(G)$ can cover a non-empty stopping set. A BP decoding procedure fails only when the set of erasure positions contains at least one non-empty stopping set. Thus, any single erasure burst of length smaller than $\mu(G)$ is correctable with BP. □

This problem to find $\mu(G)$ of a given code has been already studied by Yang and Ryan[8]. Yang and Ryan proposed a measure of burst immunity called *maximum resolvable length of erasure burst*, which is equal to $\mu(G) - 1$.

IV. A Bound on minimum span

In this section, we will discuss an lower bounds on the minimum span. The paper by Hosoya et al. [7] proposed that neighboring variable nodes should be distributed as uniform as possible in order to improve the burst error immunity of LDPC codes. The lower bound on $\mu(G)$ presented next justifies their observation.

We here define the *minimum separation* of neighboring variable nodes in the following way:

Definition 3: The minimum separation of $G \in \mathcal{E}$ is defined by

$$\Delta(G) = \min_{c \in C} \min\{|v_1 - v_2| + 1 : v_1, v_2 \in \mathcal{N}(c), v_1 \neq v_2\}, \qquad (3)$$

where $\mathcal{N}(c)$ is the set of variable nodes which are connected to the check node $c \in C$.

The next lemma presents a lower bound on $\mu(G)$ based on the minimum separation $\Delta(G)$.

Lemma 3: Assume that $\Delta(G) > 1$. A lower bound on $\mu(G)$ based on the minimum separation is given by $\mu(G) \geq \Delta(G)$.

(Proof) It is clear that the span of any stopping set is larger than or equal to $\Delta(G)$. □

V. Ensemble analysis on minimum span

In this section, ensemble analysis on minimum span of stopping sets is given. Ensemble average argument is powerful technique to prove some finite length/asymptotic properties of minimum span.

A. Ensemble analysis

Let an *interval* S be a set of consecutive variable nodes such that $S_i = \{i-L+1, i-L+2, \ldots, i-1, i\}$, where $i \in [L, n]$ and $|S_i| = L$. For a given graph $G = (V \cup C, E) \in \mathcal{E}$, we here consider the number of non-empty stopping sets which are included in the interval S_i:

$$N_{S_i}(G) = \sum_{w=1}^{L} \sum_{\substack{U \subset S_i \\ |U|=w}} I[U \text{ is a stopping set}]. \tag{4}$$

The following lemma gives a way to bound the probability such that an interval contains non-empty stopping sets.

Lemma 4: The probability such that the interval S_i contains at least one non-empty stopping set is upper bounded by

$$Pr[N_{S_i}(G) \geq 1] \leq \sum_{w=1}^{L} \binom{L}{w} Q(w), \tag{5}$$

where $Pr[N_{S_i}(G) \geq 1]$ is given by

$$Pr[N_{S_i}(G) \geq 1] = \sum_{G \in \mathcal{E}} P(G) I[N_{S_i}(G) \geq 1]. \tag{6}$$

The parameter L is the size of S_i. (Proof) In oder to prove the claim of the lemma, we first define the average number of non-empty stopping sets which are included in S_i:

$$\tilde{N}_L = \sum_{G \in \mathcal{E}} P(G) N_{S_i}(G). \tag{7}$$

Namely, $\tilde{N}_L$ is the ensemble average of $N_{S_i}(G)$. By using the definition of $N_{S_i}(G)$ and then changing the order of summation, the equation (7) can be rewritten in the following way:

$$\begin{aligned}\tilde{N}_L &= \sum_{G \in \mathcal{E}} P(G) \sum_{w=1}^{L} \sum_{\substack{U \subset S_i \\ |U|=w}} I[U \text{ is a stopping set}] \\ &= \sum_{w=1}^{L} \sum_{\substack{U \subset S_i \\ |U|=w}} \sum_{G \in \mathcal{E}} P(G) I[U \text{ is a stopping set}]. \end{aligned} \tag{8}$$

Lemma 1 can be used to simplify the above equation. We obtain

$$\begin{aligned}\tilde{N}_L &= \sum_{w=1}^{L} \left(\sum_{\substack{U \subset S_i \\ |U|=w}} 1 \right) \sum_{G \in \mathcal{E}} P(G) I[U \text{ is a stopping set}] \\ &= \sum_{w=1}^{L} \binom{L}{w} Q(w). \end{aligned} \tag{9}$$

By using Markov inequality, we finally have the inequality

$$\begin{aligned} Pr[N_{S_i}(G) \geq 1] &\leq \left(\sum_{G \in \mathcal{E}} P(G) N_{S_i}(G) \right) /1 \qquad (10) \\ &= \tilde{N}_L. \qquad (11) \end{aligned}$$

□

The next theorem is the main result of this section.

Theorem 1: Let L be a integer in the range $[2, m+1]$. The probability such that $\mu(G)$ is smaller than or equal to L is upper bounded by

$$\begin{aligned} Pr[\mu(G) \leq L] \leq & \sum_{w=1}^{L} \left(1 + \frac{(n-L)w}{L}\right) \binom{L}{w} \\ & \times \frac{coef(((1+x)^{d_c} - d_c x)^m, x^{d_v w})}{\binom{d_v n}{d_v w}}, \end{aligned} \tag{12}$$

where $Pr[\mu(G) \leq L]$ is given by

$$Pr[\mu(G) \leq L] = \sum_{G \in \mathcal{E}} P(G) I[\mu(G) \leq L]. \tag{13}$$

(Proof) Let $M_{S_i}(G)$ be the number of non-empty stopping sets which are included in the interval S_i *and contains the variable node* i:

$$M_{S_i}(G) = \sum_{w=1}^{L} \sum_{\substack{U \subset S_i \\ |U|=w, i \in U}} I[U \text{ is a stopping set}]. \tag{14}$$

Note that there is a small difference in the definitions of $N_{S_i}(G)$ and $M_{S_i}(G)$. The stopping sets counted in the definition of $M_{S_i}(G)$ always contain the variable node i. On the other hand, no such a constraint is imposed in the definition of $N_{S_i}(G)$.

By using almost same argument in the proof of Lemma 4, we can evaluate the ensemble average of $M_{S_i}(G)$, which is denoted by $\tilde{M}_L$:

$$\begin{aligned} \tilde{M}_L &= \sum_{G \in \mathcal{E}} P(G) M_{S_i}(G) \qquad (15) \\ &= \sum_{w=1}^{L} \binom{L-1}{w-1} Q(w). \qquad (16) \end{aligned}$$

We are now ready for evaluating $Pr[\mu(G) \leq L]$. The probability $Pr[\mu(G) \leq L]$ can be rewritten in the following way:

$$\begin{aligned} & Pr[\mu(G) \leq L] \\ &= \sum_{G \in \mathcal{E}} P(G) I[\mu(G) \leq L] \\ &= \sum_{G \in \mathcal{E}} P(G) I\left[(N_{S_L}(G) \geq 1) \cup (\bigcup_{i=L+1}^{n} (M_{S_i}(G) \geq 1)) \right]. \end{aligned}$$

Using the inequality (Γ_i is a predicate which takes true or false)

$$I[\Gamma_1 \cup \Gamma_2 \cup \cdots \cup \Gamma_K] \leq I[\Gamma_1] + I[\Gamma_2] + \cdots + I[\Gamma_K], \tag{17}$$

the above equation can be simplified in the following way:

$$
\begin{aligned}
&Pr[\mu(G) \le L] \\
&\quad \le \sum_{G \in \mathcal{E}} P(G) \\
&\quad \times \left(I[N_{S_L}(G) \ge 1] + \sum_{i=L+1}^{n} I[M_{S_i}(G) \ge 1] \right) \\
&\quad = Pr[N_{S_L}(G) \ge 1] + \sum_{i=L+1}^{n} Pr[M_{S_i}(G) \ge 1]. \quad (18)
\end{aligned}
$$

In order to bound the right-hand side of the last equality, we can use Lemma 4 and a variant of Lemma 4 for $M_{S_i}(G)$. Finally, we have

$$
\begin{aligned}
&Pr[\mu(G) \le L] \\
&\quad \le \tilde{N}_L + (n-L)\tilde{M}_L \qquad (19) \\
&\quad = \sum_{w=1}^{L} \left(\binom{L}{w} Q(w) + (n-L) \binom{L-1}{w-1} Q(w) \right), \quad (20)
\end{aligned}
$$

which is the same inequality in the claim. □

B. Numerical results

In order to evaluate the tightness of the results obtained by ensemble analysis, an experiment has been made. In the experiment, 1000-instances of a bipartite graph ensemble have been randomly generated with uniform probability to each instance. Figure 1 presents the histogram of minimum spans of the sampled graphs. Three code lengths $n = 60, 120, 240$ are considered. The averages and maximum $\mu(G)$ of sampled graphs are summarized in Table I. For example, we can see that a code of length 240 (with rate 1/2) can correct any single erasure burst of length 87.5 in average.

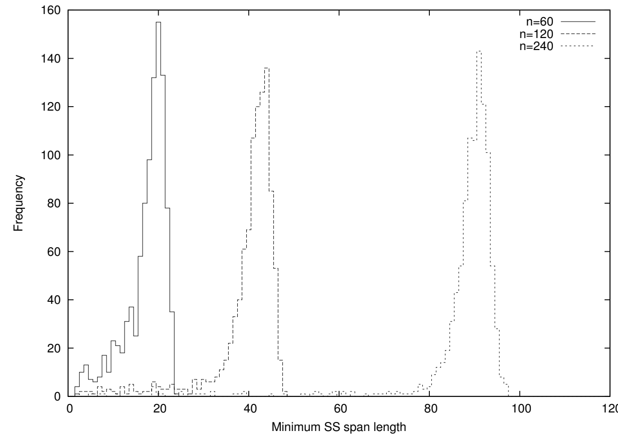
TABLE I
MINIMUM SPAN OF STOPPING SETS $\mu(G)$ OF SAMPLED GRAPHS

Length n	Average	Maximum
60	17.4	24
120	39.7	48
240	87.5	97

Let $f_L (L \in [1, m])$ be the number of sampled graphs G with $\mu(G) = L$. Let $P_L = \sum_{i=1}^{L} (f_i / F)$, where F denotes the number of sampled graphs ($F = 1000$). The probability P_L can be considered as an approximation of $Pr[\mu(G) \le L]$. Figure 2 compares the values P_L and the upper bound of $Pr[\mu(G) \le L]$. It can be observed that the upper bound goes up along with the curve of P_L as L grows.

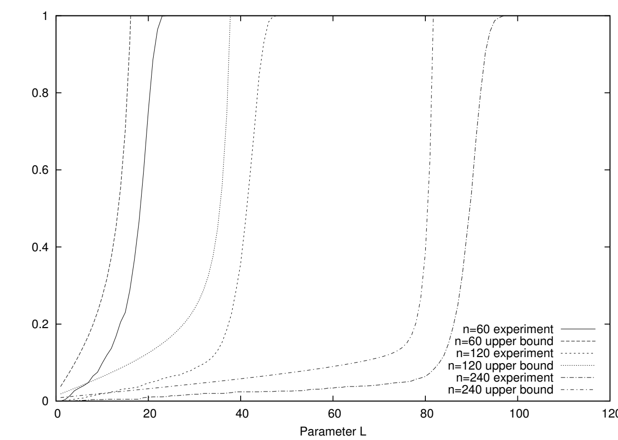
VI. Asymptotic Analysis

In this subsection, asymptotic properties of the upper bound on $Pr[\mu(G) \le L]$ will be discussed.



Randomly generated 1000-samples, $d_v = 3, d_c = 6, m = n/2$

Fig. 1. Histogram of minimum span of stopping sets



Randomly generated 1000-samples, $d_v = 3, d_c = 6, m = n/2$

Fig. 2. Probability $Pr[\mu(G) \le L]$ and P_L

A. Asymptotic bound on minimum span

Letting $L = \gamma n (0 < \gamma \le 1)$, we have

$$
Pr[\mu(G) \le \gamma n] \le \sum_{w=1}^{\gamma n} \left(1 + \frac{(1-\gamma)w}{\gamma} \right) \binom{\gamma n}{w} Q' \left(\frac{w}{n} \right) \quad (21)
$$

from Theorem 1, where

$$
Q'(\alpha) = \frac{coef(((1+x)^{d_c} - d_c x)^m, x^{d_v \alpha n})}{\binom{d_v n}{d_v \alpha n}}. \quad (22)
$$

We here consider an upper bound of

$$
\lim_{n \to \infty} \frac{1}{n} \log_2 Pr[\mu(G) \le \gamma n]. \quad (23)
$$

The next theorem gives an upper bound on the above limit.

Theorem 2: The inequality

$$
\lim_{n \to \infty} \frac{1}{n} \log_2 Pr[\mu(G) \le \gamma n] \le B(\gamma), \quad (24)
$$

holds for a bipartite graph ensemble $\mathcal{E}$, where $B(\gamma)$ is defined by

$$B(\gamma) = \sup_{0<\alpha\le\gamma} \left[\frac{d_v}{d_c} F(d_c, \alpha, x_0) - d_v h(\alpha) + \gamma h\left(\frac{\alpha}{\gamma}\right) \right]. \tag{25}$$

where $F(d_c, \alpha, x_0)$ is given by

$$F(d_c, \alpha, x_0) = \log_2 \left(\frac{(1 + x_0)^{d_c} - d_c x_0}{x_0^{\alpha d_c}} \right). \tag{26}$$

The function $h(p)$ is the binary entropy function defined by $h(p) = -p \log_2 p - (1-p)\log_2(1-p) (0 \le p \le 1)$. The real value x_0 is the only positive solution of

$$\frac{x((1+x)^{d_c-1} - 1)}{(1+x)^{d_c} - d_c x} = \alpha. \tag{27}$$

(Proof) Starting from the upper bound shown in Theorem 1, we have the following sequence of inequalities:

$$\begin{aligned}
&\tfrac{1}{n}\log_2 Pr[\mu(G) \le \gamma n] \\
&\le \frac{1}{n}\log_2 \left(\sum_{w=1}^{\gamma n} \left(1 + \frac{(1-\gamma)w}{\gamma}\right) \binom{\gamma n}{w} Q'\left(\frac{w}{n}\right) \right) \\
&\le \frac{1}{n}\log_2 \left((n - \gamma n + 1) \sum_{w=1}^{\gamma n} \binom{\gamma n}{w} Q'\left(\frac{w}{n}\right) \right) \\
&\le \frac{1}{n}\log_2 \left((n - \gamma n + 1)\gamma n \max_{w=1}^{\gamma n} \binom{\gamma n}{w} Q'\left(\frac{w}{n}\right) \right) \\
&\le \frac{1}{n}\log_2 \left((n - \gamma n + 1)\gamma n \sup_{0<\alpha\le\gamma} \binom{\gamma n}{\alpha n} Q'(\alpha) \right) \\
&\le \frac{1}{n}\log_2 \left((n - \gamma n + 1)\gamma n \right) \\
&+ \quad \frac{1}{n}\log_2 \left(\sup_{0<\alpha\le\gamma} \binom{\gamma n}{\alpha n} Q'(\alpha) \right),
\end{aligned} \tag{28}$$

where α is a rational number such that αn is an integer. It is apparent that the limit of the first term of the right-hand side of the last inequality goes to zero as n goes to infinity:

$$\lim_{n\to\infty} \frac{1}{n}\log_2 \left((n - \gamma n + 1)\gamma n \right) = 0. \tag{29}$$

The limit of the second term is given by

$$\begin{aligned}
&\lim_{n\to\infty} \frac{1}{n}\log_2 \sup_{0<\alpha\le\gamma} \binom{\gamma n}{\alpha n} Q'(\alpha) \\
= \quad &\sup_{0<\alpha\le\gamma} \left[\gamma h\left(\frac{\alpha}{\gamma}\right) + \lim_{n\to\infty} \frac{1}{n}\log_2 Q'(\alpha) \right].
\end{aligned}$$

Theorem 2 in [10] shows that

$$\lim_{n\to\infty} \frac{1}{n}\log_2 Q'(\alpha) = \frac{d_v}{d_c} F(d_c, \alpha, x_0) - d_v h(\alpha), \tag{30}$$

where x_0 is the only positive solution of

$$\frac{x((1+x)^{d_c-1} - 1)}{(1+x)^{d_c} - d_c x} = \alpha. \tag{31}$$

$\square$

B. Critical minimum span rate

The *critical minimum span rate* of stopping sets for a bipartite graph ensemble, γ^*, is defined by $\gamma^* = \inf\{\gamma : B(\gamma) > 0, 0 < \gamma \le 1\}$. The critical minimum span rate can be regarded as a counterpart of the typical minimum distance. Table II presents γ^* of some bipartite graph ensembles. From

TABLE II
CRITICAL MINIMUM SPAN RATE OF STOPPING SETS

d_v	d_c	γ^*
3	6	0.366
4	8	0.323
5	10	0.286
3	12	0.174

Table II, we can observe that an ensemble with larger d_v has smaller critical minimum span rate under the condition d_v/d_c is constant. For example, the $(d_v = 3, d_c = 6)$-ensemble has $\gamma^* = 0.37$ and the $(d_v = 4, d_c = 8)$-ensemble has $\gamma^* = 0.32$.

As presented above, the $(d_v = 3, d_c = 6)$-ensemble has $\gamma^* = 0.37$. Roughly speaking, it means that a burst erasure of length up to 37 % of code length can be corrected in average. We have $0.37 \times 120 = 88.8$ for the case of $n = 120$. The experimental value of the average minimum span is 87.5 in Table 1. We can see good agreement between theory and experiment.

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