# Queue Length Stability of Maximal Greedy Schedules in Wireless Networks

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***Abstract*— We consider wireless networks with interference constraints. The network consists of a set of links and a set of users who generate packets that traverse these links. Each user is associated with a route consisting of a sequence of links. The links are subject to the usual interference constraints: (i) if link $l$ interferes with link $k$, then link $k$ also interferes with link $l$, and (ii) two links that interfere with each other cannot transmit simultaneously. The interference set of a link is defined to be the set of links that interfere with the link, along with the link itself. A greedy scheduler is one which selects an arbitrary set of links to transmit subject only to the interference constraint. We use a traffic regulator at each link along the route of each flow which shapes the traffic of the flow. We prove that the network is queue-length stable under any maximal greedy scheduling policy provided that the total arrival rate in the interference set of each link is less than one.**

## I. Introduction

In this paper, we consider distributed scheduling algorithms for resource allocation in general multi-hop wireless networks. It has been well-known for a while that scheduling policies which use queue length information can stabilize very general queueing networks [12], [13], [11]. However, such scheduling policies have to be implemented in a centralized fashion. In this paper, we consider the design of simpler decentralized scheduling policies, albeit at the loss of some throughput in the network.

We consider a fairly general interference model for the wireless network. Associated with each link is a set of other links which interfere with the given link. The only assumption we make is that the interference relationship is symmetric, i.e., if link $l$ interferes with link $k$, then $l$ also interferes with $k$. We are interested in maximal greedy policies which select links for scheduling such that the interference constraints are satisfied. The policies are greedy in the sense the schedule can be chosen in the following fashion: start with any link in the network and add links to the schedule, one link at a time, subject to interference constraints. If the scheduler proceeds to implement this algorithm until no more links can be added to the schedule, then it is called maximal. Such maximal greedy policies require no significant computation and potentially can be implemented in a decentralized manner.

## II. Stability of Maximal Greedy Schedules

### A. Single-Hop Routes

For ease of exposition, we first consider a network where each user's route consists of a link. The stability result for this model will then be used to derive a stability result for general multi-hop networks. Consider a network consisting of $L$ links operating in a discrete time-slotted manner. Associated with each link is an interference set $\mathcal{E}_l$ which satisfies the following two properties: (a) $l \in \mathcal{E}_l$, and (b) if $k \in \mathcal{E}_l$, then $l \in \mathcal{E}_k$. There can be at most one departure from each interference set in any time slot. We will state this more precisely later.

Associated with each link is a stochastic arrival process $\{A_l(n)\}$, where $A_l(n)$ is the number of packet arrivals in slot $n$ to link $l$. We assume that the arrival processes are stationary and let $\lambda_l = E(A_l(n))$. The number of departures from link $l$ at time $n$ is denoted by $D_l(n)$, and $D_l(n)$ is assumed to be less than or equal to $c_l$ packets. In other words, link $l$ can serve at the most $c_l$ packets per time slot. Let $q_l(n)$ be the number of packets at link $l$ at the beginning of time slot $n$. We assume that the sequence of events in each time slot is as follows: (i) schedule consistent with the interference constraint is chosen, (ii) departures occur next, and (iii) arrivals occur last.

Let $d_l(n)$ be an indicator function indicating whether link $l$ is scheduled in time slot $l$ or not. We make the assumption that link $l$ is eligible for scheduling only if it has at least $c_l$ waiting packets. Thus, $D_l(n) = c_l d_l(n)$. Now, we define a maximal greedy scheduling policy. A policy is said to be a maximal greedy policy if the departures under this policy satisfy the following constraint: if $q_l(n) \geq c_l$, then $\sum_{k\in\mathcal{E}_l} d_l(n) = 1$. This ensures that when a link has at least $c_l$ packets, either the link is scheduled or if it not scheduled, then the reason that it is not scheduled is that another link from its interference set is scheduled.

We now comment on prior work on this problem. The policy that we consider in this paper is a natural extension of the maximal schedules considered for high-speed switches in [14], [4] and, for Bluetooth-like wireless networks, in [7]. Assuming $c_l = 1$ for all $l$, the model considered in this paper has been considered previously in [16] and [3]:

- Let $\mathcal{C}$ denote the capacity region of the network, i.e., the set of all arrival rates for which there is a scheduling policy that renders the queues stable. Let $\alpha = \max_l |\mathcal{E}_l|$. In [16], it was shown that, under any maximal greedy policy, the queue lengths in the network are stable if the arrival rates lie inside $\frac{1}{\alpha}\mathcal{C}$. In other words, the capacity reduction due to maximal greedy policies is upper-bounded by $1/\alpha$.
- In [3], it was shown that if $\sum_{k\in\mathcal{E}_l} \lambda_l \leq 1$, then the network is rate stable under any maximal greedy scheduling policy, i.e, the departure rate from each link is equal to the arrival rate into the link. If only rate stability is required, this is a much stronger result than the one in [16]. In this paper, for the case where the link capacities are in general greater than or equal to 1, we will prove that if $\sum_{k\in\mathcal{E}_l} \lambda_l/c_l < 1$, then the network is queue length stable.

We assume that $Cov(A_k(n), A_l(n)) = \sigma^2_{kl} < \infty$. The proof is presented only for the case where

$$A(n) := \{A_1(n), A_2(n), \ldots, A_L(n)\}$$

are i.i.d. across $n$. In other words, the arrival process is i.i.d. across time slots, but may be dependent across links. The extension to more general Markovian arrival processes is straightforward. In the following Lyapunov analysis, one has to then consider the drift over multiple time slots; otherwise, the proof is similar.

Define the state of the system to be

$$Q(n) := (q_1(n), q_2(n), \ldots, q_L(n)),$$

where the dynamics of $q_l(n)$ are given by

$$q_l(n+1) = q_l(n) + A_l(n) - D_l(n).$$

Assume that $\{D_l(n)\}$ is chosen according to some probability distribution given $Q(n)$, i.e., $P(\{D_l(n)\}|Q(n))$ is given. Thus, $Q(n)$ is a countable-state-space Markov chain. Note that $P(\{D_l(n)\}|Q(n))$ can be arbitrary. In particular, $\{D_l(n)\}$ could be any sequence of schedules consistent with the interference constraints. In a real-life network, the maximal schedule may by selected by a random access protocol. In this case, it may be reasonable to assume that the all feasible maximal schedules are equally likely at each time instant. However, our analysis applies to more general models as well; in fact, it holds for any rule used to choose the set of active links, as long as the resulting schedule is a maximal schedule.

Define

$$V(n) = \sum_l \frac{q_l(n)}{c_l}\left(\sum_{k\in\mathcal{E}_l}\frac{q_k(n)}{c_k}\right). \quad (1)$$

The above Lyapunov function is very similar to the Lyapunov function considered in [4] to study maximal matching in high-speed switches. The Lyapunov function in [4] is of the form

$$\sum_l \frac{q_l(n)}{c_l}\left(\frac{q_l(n)}{c_l} + \sum_{k\in\mathcal{E}_l}\frac{q_k(n)}{c_k}\right).$$

It has an additional $q_l/c_l$ term. This would result in a slightly weaker condition than the one we prove. Now

$$\begin{aligned}
&V(n+1) - V(n)\\
&= \sum_l \frac{(q_l(n+1)-q_l(n))}{c_l}\left(\sum_{k\in\mathcal{E}_l}\frac{q_k(n)}{c_k}\right)\\
&\quad + \sum_l \frac{q_l(n)}{c_l}\left(\sum_{k\in\mathcal{E}_l}\frac{(q_k(n+1)-q_k(n))}{c_k}\right)\\
&\quad + \sum_l \frac{(q_l(n+1)-q_l(n))}{c_l}\left(\sum_{k\in\mathcal{E}_l}\frac{(q_k(n+1)-q_k(n))}{c_k}\right)\\
&= 2\sum_l \frac{q_l(n)}{c_l}\left(\sum_{k\in\mathcal{E}_l}\frac{(q_k(n+1)-q_k(n))}{c_k}\right)\\
&\quad + \sum_l \frac{(q_l(n+1)-q_l(n))}{c_l}\left(\sum_{k\in\mathcal{E}_l}\frac{(q_k(n+1)-q_k(n))}{c_k}\right)\\
&= 2\sum_l \frac{q_l(n)}{c_l}\left(\sum_{k\in\mathcal{E}_l}(a_k(n)-d_k(n))\right)\\
&\quad + \sum_l (a_l(n)-d_l(n))\left(\sum_{k\in\mathcal{E}_l}(a_k(n)-d_k(n))\right),
\end{aligned}$$

where $a_k(n) := A_k(n)/c_k$. In the above derivation, we have used the fact that

$$\begin{aligned}
&\sum_l \frac{(q_l(n+1)-q_l(n))}{c_l}\left(\sum_{k\in\mathcal{E}_l}\frac{q_k(n)}{c_k}\right)\\
&= \sum_l \frac{q_l(n)}{c_l}\left(\sum_{k\in\mathcal{E}_l}\frac{(q_k(n+1)-q_k(n))}{c_k}\right).
\end{aligned}$$

Define $E_X(\cdot) = E(\cdot|X)$. Using the bounded second-moment assumption and the fact that the number of departures from each interference set is bounded, we get

$$\begin{aligned}
&E_{Q(n)}\left(V(n+1)-V(n)\right)\\
&\leq 2\sum_l \frac{q_l(n)}{c_l}\left(\sum_{k\in\mathcal{E}_l}\frac{\lambda_k(n)}{c_k} - \sum_{k\in\mathcal{E}_l} d_k(n)\right) + B\\
&= 2\sum_{l:q_l(n)>0} \frac{q_l(n)}{c_l}\left(\sum_{k\in\mathcal{E}_l}\frac{\lambda_k(n)}{c_k} - \sum_{k\in\mathcal{E}_l} d_k(n)\right) + B\\
&\leq 2\sum_{l:q_l(n)>0} \frac{q_l(n)}{c_l}\left(\sum_{k\in\mathcal{E}_l}\frac{\lambda_k(n)}{c_k} - 1\right) + B_1\\
&\leq -2\epsilon \sum_{l:q_l(n)>0} \frac{q_l(n)}{c_l} + B_1,
\end{aligned}$$

where $B, B_1 > 0$ are some constants and $\epsilon = 1 - \max_l \sum_{k\in\mathcal{E}_l}\frac{\lambda_k}{c_k}$. Thus, we have the following theorem.

*Theorem 1:* For any set of distributions $P(\{D_l(n)\}|Q(n))$, the Markov chain $Q(n)$ is stable-in-the-mean, i.e.,

$$\lim\sup_{N\to\infty}\frac{1}{N}\sum_{n=1}^{N}\sum_{l=1}^{L} E(q_l(n)) < \infty,$$

if

$$\sum_{k\in\mathcal{E}_l} \frac{\lambda_k}{c_k} < 1, \qquad \forall l.$$

Further, if $P(\{D_l(n)\}|Q(n))$ is independent of $n$, then $Q(n)$ is a time-homogenous Markov chain and it is positive recurrent.

Proof: Stability-in-the-mean follows from [6] and positive recurrence follows from Foster's theorem [1]. $\diamond$

In what follows, if the network is either stable-in-the-mean or positive recurrent, we call it queue-length stable.

### *B. Multi-Hop Routes*

In this section, we extend the result in the last section to wireless networks shared by many *users*, each of whom has a *traffic flow* traversing the network through possibly multiple hops. Specifically, we assume there are $S$ users each of whose traffic takes a fixed path through the network. The path information is contained by the routing matrix $\mathbf{H} = [H_l^s, 1 \le l \le L; 1 \le s \le S]$, where $H_l^s$ is an indicator function and is determined as follows

$$H_l^s = \begin{cases} 1 & \textit{if } l \in \mathcal{L} \textit{ is on the path of user } s; \\ 0 & \textit{otherwise}, \end{cases} \tag{2}$$

where $\mathcal{L} = \{1, 2, \ldots, L\}$. Further to indicate the fact that a path is a directed route, we define two functions $P^s(l)$ and $N^s(l)$ to be the previous and next link, respectively, from link $l$ on user $s$'s route. Of course, the previous link and last link are undefined for the first and last link, respectively, on a route. Each user $s$'s arrival process $A_s(n)$ is a random process with mean arrival rate $\lambda^s$. Similar to the previous section, we assume that $A_s(n)$ is i.i.d. across time slots, but we also make the further assumption that the arrival processes of different users are independent. Let $L_m$ denote the maximum number of hops traversed by any user in the network.

As before, let $D_l(n)$ be the number of departures from link $1 \le l \le L$ in time slot $n$. Further, $D_l^s(n)$ is the number of departures of user $s$ over link $l$ in time slot $n$, i.e.,

$$D_l(n) = \sum_s H_l^s D_l^s(n).$$

Define $q_l^s$ to be the queue length of user $s$ on link $l$ and $q_l$ without the superscript $s$ denote the total queue length at $l$, i.e., $q_l = \sum_s q_l^s H_l^s$.

One way to achieve queue stability in a multi-hop network system is to introduce *regulators* in the system. We have investigated such problems in our previous works [15], [2] for networks with a simple interference constraint. In this paper, we show that the same idea is also applicable to the general model considered in this paper, and prove that the sufficient condition for single-hop stability case essentially remains the same even with multi-hop routes.

A regulator is introduced, for each flow using a link, such that the burstiness of the packets belonging to each user is regulated before entry into the node. A $\lambda$-regulator associated with link $l$ is a logical device with a maximum service rate $\lambda$, i.e., it generates packets for the node at its output at a maximum rate of $\lambda$. Specifically, assuming $c_l$ to be the capacity of link $l$, at each time slot, a $\lambda$-regulator associated with link $l$ checks its buffer size and if it exceeds link capacity $c_l$, it transfers $c_l$ bits to the user's queue with probability $\frac{\lambda}{c_l}$. Otherwise, it transfers nothing. We use the notation $R_l^s(n)$ to denote the number of departing packets from the regulator of user $s$ on link $l$. The idea of a regulator was originally suggested in the context of re-entrant manufacturing lines in [5].

Denote the arrival rate vector consisting of the arrival rates of all the users by $[\lambda_s]_{1\le s\le S}$. We choose the regulators of a user $s$ according to the following rule: for the first hop (node) along the path of user $s$, we use a $\lambda_s$-regulator at its input queue for user $s$; for $k^{\text{th}}$ hop ($k \ge 2$), we use a $(\lambda_s + (k-1)\epsilon)$-regulator.

We define the combination of the regulators and maximal greedy scheduling to be a regulated greedy scheduling algorithm. The main result of this section is given below:

*Theorem 2:* For a multi-hop wireless network, if the rate vector $\{\lambda_s; 1 \le s \le S\}$ satisfies

$$\sum_{k\in\mathcal{E}_l} \frac{\sum_s \lambda_s H_k^s}{c_k} < 1 \tag{3}$$

for any link $l$ in the network, the network is queue length stable under regulated greedy scheduling if $\epsilon$ is chosen sufficiently small. $\diamond$

**Proof:** Let $p_l^s$ denote the length of the regulator queue on link $l$ for user $s$. It is easy to see that the whole system $(\mathbf{q}(n), \mathbf{p}(n))$ is a Markov Chain. We define the following Lyapunov function for the system:

$$V(\mathbf{q}, \mathbf{p}) = V_1(\mathbf{q}) + \xi V_2(\mathbf{p}, \mathbf{q}), \tag{4}$$

where $V_1(\mathbf{q})$ is a natural modification of the Lyapunov function in (1). Specifically, due to fact that we are considering multiple flows in each link, we define

$$V_1(\mathbf{q}) = \sum_l \frac{\sum_s q_l^s H_l^s}{c_l} \left( \sum_{k\in\mathcal{E}_l} \frac{\sum_s q_k^s H_k^s}{c_k} \right).$$

On the other hand, $V_2$ is defined as follows

$$V_2(\mathbf{p}, \mathbf{q}) = \frac{1}{2} \sum_{l\in\mathcal{L}} \sum_s (p_{N^s(l)}^s + q_l^s)^2.$$

$\xi$ is a positive parameter.

The queue update equations for this system are

$$\begin{aligned} q_l^s(n+1) &= q_l^s(n) - D_l^s(n) + R_l^s(n) && (5) \\ p_l^s(n+1) &= (p_l^s(n) - R_l^s(n))^+ + D_{P^s(l)}^s(n), && (6) \end{aligned}$$

where $R_l^s(n)$ is the output of the regulator that immediately preceds link $l$ on path of source $s$. When link $l$ is the first link on the path of source $s$, then $D_{P^s(l)} = A_s(n)$. We also use the notation $r_l^s(n)$ to denote the departure normalized by the associated link capacity $c_l$, i.e.,

$$r_l^s(n) = \frac{R_l^s(n)}{c_l}.$$

Thus, $r_l^s(n)$ is either 0 or 1. Due to our definition of the regulator, $R_l^s$ can only be non-zero when $p_l^s$ exceeds $c_l$. Hence, we can remove the projection in (6) as well and we have

$$p_l^s(n+1) = p_l^s(n) - R_l^s(n) + D^s_{P^s(l)}(n).$$

We have to upper bound

$$E\left[V(\mathbf{q}(n+1), \mathbf{p}(n+1)) - V(\mathbf{q}(n), \mathbf{p}(n)) | \mathbf{q}(n), \mathbf{p}(n)\right]$$

by a negative number for all states $(\mathbf{q}, \mathbf{p})(n)$ except possibly in a bounded region, where the drift should simply be finite.

We consider the contribution of $V_1(\mathbf{q})$ and $V_2(\mathbf{p}, \mathbf{q})$ separately for now. Thus, we first look at

$$\begin{aligned}
&\Delta V_1(\mathbf{q}) = V_1(\mathbf{q}(n+1)) - V_1(\mathbf{q}(n)) \\
&= 2\sum_l \frac{q_l(n)}{c_l}\left(\sum_{k\in\mathcal{E}_l}\frac{(q_k(n+1)-q_k(n))}{c_k}\right) \\
&+ \sum_l \frac{(q_l(n+1)-q_l(n))}{c_l}\left(\sum_{k\in\mathcal{E}_l}\frac{(q_k(n+1)-q_k(n))}{c_k}\right) \\
&= 2\sum_l \frac{q_l(n)}{c_l}\left(\sum_{k\in\mathcal{E}_l}\sum_s (r_k^s(n) - d_k^s(n))\right) \\
&+ \sum_l\sum_s (r_l^s(n) - d_l^s(n))\left(\sum_{k\in\mathcal{E}_l}\sum_s (r_k^s(n) - d_k^s(n))\right).
\end{aligned}$$

Similar to the proof in Theorem 1, the second term can be bounded by a constant. Further, given the fact that $E_{Q(n),P(n)}[r_k^s(n)] \leq \frac{\lambda^s + L_m\epsilon}{c_k}$, (due to the definition of the regulators), we have

$$E_{Q(n),P(n)}\left(V_1(n+1) - V_1(n)\right) \leq -2\eta \sum_{1\leq l\leq L}\frac{q_l(n)}{c_l} + B,$$

Next, consider the contribution of $V_2$ to the drift. Note that

$$\begin{aligned}
q_l^s(n+1) + p^s_{N^s(l)}(n+1) &= q_l^s(n) + p^s_{N^s(l)}(n) \\
&\quad + R_l^s(n) - R^s_{N^s(l)}(n). \quad (7)
\end{aligned}$$

We can bound the drift due to $V_2$ as follows:

$$\begin{aligned}
&\Delta V_2(\mathbf{p}, \mathbf{q}) \\
&= E\left[V_2(\mathbf{p}(n+1), \mathbf{q}(n+1)) - V_2(\mathbf{p}(n), \mathbf{q}(n)) | \mathbf{q}(n), \mathbf{p}(n)\right] \\
&= E\left[\sum_{1\leq l\leq L}\sum_s (p^s_{N^s(l)}(n+1) + q_l^s(n+1))^2 \right.\\
&\quad \left. - \sum_{1\leq l\leq L}\sum_s (p^s_{N^s(l)}(n) + q_l^s(n))^2 | \mathbf{q}(n), \mathbf{p}(n)\right] \\
&= E\left[\sum_{1\leq l\leq L}\sum_s \left(p^s_{N^s(l)}(n) + q_l^s(n)\right)\left(p^s_{N^s(l)}(n+1)\right.\right. \\
&\quad \left.\left. + q_l^s(n+1) - p^s_{N^s(l)}(n) - q_l^s(n)\right) | \mathbf{q}(n), \mathbf{p}(n)\right] \\
&\quad + \frac{1}{2}E\left[\sum_{1\leq l\leq L}\sum_s \left(p^s_{N^s(l)}(n+1) + q_l^s(n+1)\right.\right. \\
&\quad \left.\left. - p^s_{N^s(l)}(n) - q_l^s(n)\right)^2 | \mathbf{q}(n), \mathbf{p}(n)\right]
\end{aligned}$$

The second term above can be bounded by a constant $C_3$ independent of $\mathbf{p}(n)$ and $\mathbf{q}(n)$ as follows:

$$\begin{aligned}
&E\left[\sum_{1\leq l\leq L}\sum_s \left(p^s_{N^s(l)}(n+1) + q_l^s(n+1)\right.\right. \\
&\quad \left.\left. - p^s_{N^s(l)}(n) - q_l^s(n)\right)^2 \Big| \mathbf{q}(n), \mathbf{p}(n)\right] \\
&\leq 2\sum_{1\leq l\leq L}\sum_s ((\lambda_s + L_m\epsilon))^2 + ((\lambda_s + L_m\epsilon))^2 = C_3. \quad (8)
\end{aligned}$$

The first term can be bounded by

$$\begin{aligned}
&E\left[\sum_{1\leq l\leq L}\sum_s \left(p^s_{N^s(l)}(n) + q_l^s(n)\right)\left(p^s_{N^s(l)}(n+1)\right.\right. \\
&\quad \left.\left. + q_l^s(n+1) - p^s_{N^s(l)}(n) - q_l^s(n)\right) | \mathbf{q}(n), \mathbf{p}(n)\right] \\
&= \sum_{1\leq l\leq L}\sum_s \left(p^s_{N^s(l)}(n) + q_l^s(n)\right) E\left[R_l^s(n)\right. \\
&\quad \left. - R^s_{N^s(l)}(n) | \mathbf{q}(n), \mathbf{p}(n)\right] \\
&= \sum_{1\leq l\leq L}\sum_s \left(p^s_{N^s(l)}(n) + q_l^s(n)\right)\left[\bar{R}_l^s I_{p_l^s(n)\geq c_l}\right. \\
&\quad \left. - \bar{R}^s_{N^s(l)} I_{p^s_{N^s(l)}(n)\geq c_{N^s(l)}}\right] \\
&\leq \sum_{1\leq l\leq L}\sum_s \left(p^s_{N^s(l)}(n) + q_l^s(n)\right)\left[\bar{R}_l^s\right. \\
&\quad \left. - \bar{R}^s_{N^s(l)} I_{p^s_{N^s(l)}(n)\geq c_{N^s(l)}}\right],
\end{aligned}$$

where $\bar{R}_l^s$ is the average departure rate of regulator $R_l^s$, if $p_l^s$ exceeds $c_l$. From our design of the regulators, we know that

$$\bar{R}^s_{N^s(l)} = \bar{R}_l^s + \epsilon,$$

for any $l$ on the path ofuser $s$. From this, we have

$$\begin{aligned}
\Delta V_2(\mathbf{p}, \mathbf{q}) \leq &\sum_{1\leq l\leq L}\sum_s \left(p^s_{N^s(l)}(n) + q_l^s(n)\right) \times \\
&\left(\bar{R}_l^s - \bar{R}^s_{N^s(l)} I_{p^s_{N^s(l)}(n)\geq c_{N^s(l)}}\right) + C_2. \quad (9)
\end{aligned}$$

By combining (8) and (9), we have

$$\begin{aligned}
&E\left[V(\mathbf{q}(n+1), \mathbf{p}(n+1)) - V(\mathbf{q}(n), \mathbf{p}(n)) | \mathbf{q}(n), \mathbf{p}(n)\right] \\
&\leq -\eta\sum_{1\leq l\leq L}\frac{q_l(n)}{c_l} + \xi\sum_{1\leq l\leq L}\sum_s \left(p^s_{N^s(l)}(n) + q_l^s(n)\right) \\
&\quad \times \left(\bar{R}_l^s - \bar{R}^s_{N^s(l)} I_{p^s_{N^s(l)}(n)\geq c_{N^s(l)}}\right) + C_3 \\
&= -\sum_{1\leq l\leq L}\sum_s \left[\frac{\eta}{c_l} - \xi\left(\bar{R}_l^s - \bar{R}^s_{N^s(l)} I_{p^s_{N^s(l)}(n)\geq c_{N^s(l)}}\right)\right] \\
&\quad \times q_l^s(n) + \xi\sum_{1\leq l\leq L}\sum_s p^s_{N^s(l)}(n)\left(\bar{R}_l^s - \right. \\
&\quad \left. \bar{R}^s_{N^s(l)} I_{p^s_{N^s(l)}(n)\geq c_{N^s(l)}}\right) + C_3
\end{aligned}$$

$$
\begin{aligned}
\leq \quad & -\sum_{1\leq l\leq L}\sum_{s}\left[\frac{\eta}{c_l}-\xi c_l\right]q_l^s(n)+\xi\sum_{1\leq l\leq L}\sum_{s}p^s_{N^s(l)}(n)\\
& \times\left(\bar{R}_l^s-\bar{R}^s_{N^s(l)}I_{p^s_{N^s(l)}(n)\geq c_{N^s(l)}}\right)+C_3\\
= \quad & -\sum_{1\leq l\leq L}\sum_{s}\left[\frac{\eta}{c_l}-\xi c_l\right]q_l^s(n)\\
& -\epsilon\xi\sum_{1\leq l\leq L}\sum_{s}p^s_{N^s(l)}(n)+C_4,
\end{aligned}
$$

where $C_4$ is another constant. We can easily choose $\xi$ (independent of $\mathbf{p}(n)$ and $\mathbf{q}(n)$, of course) here such that

$$\frac{\eta}{c_l}-\xi c_l\geq C_0>0$$

for any $l$ (we assume $c_l>0$ for any $l$; links with zero capacity can be removed from the network without affecting the capacity region), and thus

$$
\begin{aligned}
& E\left[V(\mathbf{q}(n+1),\mathbf{p}(n+1))-V(\mathbf{q}(n),\mathbf{p}(n))|\mathbf{q}(n),\mathbf{p}(n)\right]\\
\leq \quad & -\sum_{1\leq l\leq L}\sum_{s}C_0 q_l^s(n)-\epsilon\xi\sum_{1\leq l\leq L}\sum_{s}p_l^s(n)+C_4.
\end{aligned}
$$

This concludes the proof of this theorem. $\diamond$

## III. Resource Allocation

In the previous section, we assumed that the arrival rate satisfies the sufficient condition for stability given by (3). In this section, we discuss how one can ensure that the arrival rates can be controlled to satisfy the sufficient condition. Associate a utility function $U_s(\lambda_s)$ with each user $s$, where $U_s(\cdot)$ is an increasing, twice differentiable concave function. Suppose that the $\{\lambda_s\}$ are chosen to solve the following optimization problem:

$$\max_{\{\lambda_s\geq 0\}}\sum_s U_s(\lambda_s) \tag{10}$$

subject to

$$\sum_{k\in\mathcal{E}_l}\sum_{s:H_k^s=1}\frac{\lambda_s}{c_k}\leq 1-(L_m+1)\epsilon, \qquad \forall l,$$

where $\epsilon$ is chosen to be sufficiently small. This problem can be solved in a distributed manner to obtain the $\{\lambda_s\}$; see [10] for a general overview of congestion control and [2] for an application to wireless networks with a special interference constraint. Here we will briefly present how such a congestion-control algorithm can be implemented for our model. Each source chooses its arrival rate at each time instant according to the following algorithm which depends on the congestion price $\tilde{p}_l(n)$ generated by each link:

$$U'(\lambda_s(n))=\sum_{l:H_l^s=1}\tilde{p}_l(n), \tag{11}$$

and each link implements the following algorithm to compute the congestion price $\tilde{p}_l(n)$ :

$$\tilde{p}_l(n+1)=\left[\tilde{p}_l(n)+\delta\left(\sum_{k\in\mathcal{E}_l}\sum_{s:H_k^s=1}\frac{\lambda_s}{c_k}-1-(L_m+1)\epsilon\right)\right]^+.$$

Using techniques in [8], this algorithm can be shown to converge to the optimal solution of (10) if $\delta$ is chosen to be sufficiently small. We assume here that the congestion information is available instantaneously to each source; algorithms and analysis for more general delay models can be obtained by appropriately modifying the results in [2]. Now, these arrival rates, can be used to design the regulator parameters, which along with greedy maximal scheduling ensures the stability of the network. For an alternative solution to the resource allocation problem for the special case of max-min fairness, we refer the reader to [9].

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