A Comparison of Two Achievable Rate Regions for the Interference Channel

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***Abstract*— A recent result for the general interference channel is presented. The encoding technique is a generalization of the technique employed by Han and Kobayashi. Coupled with an improvement in the computation of the error probabilities, fewer constraints are necessary to define the achievable rate region. We further compare the two achievable rate regions and discuss whether the two achievable rate regions are equivalent.**

I. Introduction

An interference channel (IC) models the situation where M unrelated senders try to communicate their separate information to M different receivers via a common channel. There is no cooperation between any of the receivers or senders. Hence, transmission of information from each sender to its corresponding receiver interferes with the communication between the other senders and their receivers. In this paper, we consider only the two-user IC.

The study of the IC was first initiated by Shannon [1], and was further studied by Ahlswede [2]. Carleial [3] established several fundamental results, and also determined an improved achievable rate region for the IC. Han and Kobayashi introduced a superior decoder and established the best achievable rate region to date for the general IC [4]. Except for the Gaussian IC under strong interference ([5], [6], [4]), a class of discrete additive degraded IC [7], a class of deterministic IC [8] and the discrete memoryless IC with strong interference [9], the capacity of the general IC remains unknown to date.

The purpose of this paper is to compare two achievable rate regions for the general IC. In Section II, we first give a description of the mathematical model for the discrete memoryless IC. In Section III, we review the best achievable rate region for the general IC to date by Han and Kobayashi. We call this the Han-Kobayashi region. Next, in Section IV, we derive an achievable rate region for the general IC which contains the Han-Kobayashi region. We term this the Chong-Motani-Garg region. Finally, in Section V, we provide comparisons between the Han-Kobayashi region and the Chong-Motani-Garg region.

II. Mathematical Preliminaries

A. Discrete Memoryless Interference Channel

A two-user discrete interference channel consists of four finite sets $\mathcal{X}_1$, $\mathcal{X}_2$, $\mathcal{Y}_1$, $\mathcal{Y}_2$, and conditional probability distributions $p(.,.|x_1,x_2)$ on $\mathcal{Y}_1 \times \mathcal{Y}_2$, where $(x_1,x_2) \in \mathcal{X}_1 \times \mathcal{X}_2$. The interference channel is said to be memoryless if

$$p\left(y_1^n, y_2^n|x_1^n, x_2^n\right) = \prod_{i=1}^{n} p\left(y_{1i}^n, y_{2i}^n|x_{1i}^n, x_{2i}^n\right). \tag{1}$$

A $\left(2^{nR_1}, 2^{nR_2}, n\right)$ code for an interference channel with independent information consists of two encoders

$$f_1 : \left\{1, ..., 2^{nR_1}\right\} \rightarrow \mathcal{X}_1^n$$
$$f_2 : \left\{1, ..., 2^{nR_2}\right\} \rightarrow \mathcal{X}_2^n$$

and two decoding functions

$$g_1 : \mathcal{Y}_1^n \rightarrow \left\{1, ..., 2^{nR_1}\right\}$$
$$g_2 : \mathcal{Y}_2^n \rightarrow \left\{1, ..., 2^{nR_2}\right\}. \tag{2}$$

The average probability of error is defined as the probability the decoded message is not equal to the transmitted message, i.e.,

$$P_e^{(n)} = \Pr\left(g_1\left(Y_1^n\right) \neq W_1 \text{ or } g_2\left(Y_2^n\right) \neq W_2\right) \tag{3}$$

where (W_1, W_2) are assumed to be uniformly distributed over $2^{nR_1} \times 2^{nR_2}$.

A rate pair (R_1, R_2) is said to be achievable for the interference channel if there exists a sequence of $\left(2^{nR_1}, 2^{nR_2}, n\right)$ codes with $P_e^{(n)} \rightarrow 0$. We also denote by $A_\epsilon^{(n)}$ the set of jointly ϵ-typical sequences.

B. Gaussian Interference Channel

The discrete-time additive white Gaussian IC, shown in Fig. 1, is described by

$$Y_1 = c_{11}X_1 + c_{21}X_2 + Z_1 \tag{4}$$
$$Y_2 = c_{12}X_1 + c_{22}X_2 + Z_2 \tag{5}$$

where the input and output signals are real, the coefficients c_{ij} are real constants, and the noise terms Z_1 and Z_2 are zero-mean Gaussian random variables. Also, the mean value of X_1^2 and X_2^2 cannot exceed P_1 and P_2 respectively. In [3], it was shown that any Gaussian IC can be reduced to its standard form, where $c_{11}^2 = c_{22}^2 = 1$ and $\mathbb{E}\left[Z_1^2\right] = \mathbb{E}\left[Z_2^2\right] = 1$. The capacity of the Gaussian IC is not known, except for the case of no interference, where $c_{21}^2 = c_{12}^2 = 0$, for the case of strong interference, where $c_{21}^2 \geq 1$ and $c_{12}^2 \geq 1$ and a mixture of these two cases, where $c_{12}^2 = 0$ and $c_{21}^2 \geq 1$ or $c_{21}^2 = 0$ and $c_{12}^2 \geq 1$.

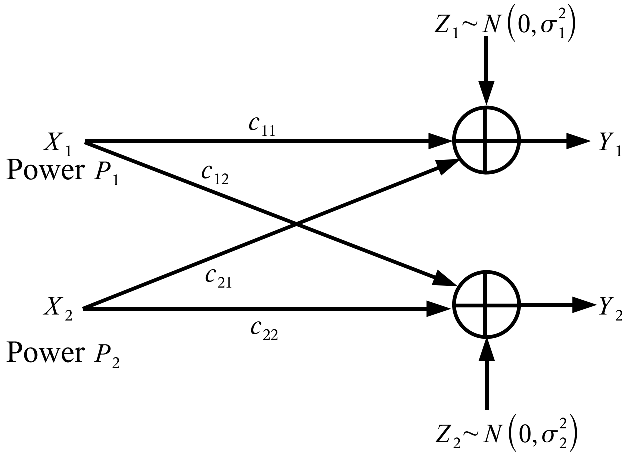

Fig. 1. The Gaussian interference channel

III. Han-Kobayashi Rate Region

In [4], the authors established an achievable rate region for the general interference channel. They considered 5 auxiliary random variables Q, U_1, W_1, U_2 and W_2, defined on arbitrary finite sets $\mathcal{Q}$, $\mathcal{U}_1$, $\mathcal{W}_1$, $\mathcal{U}_2$ and $\mathcal{W}_2$ respectively. X_1 and X_2 are the input random variables defined on the input alphabet sets $\mathcal{X}_1$ and $\mathcal{X}_2$ respectively, while Y_1 and Y_2 are the output random variables defined on the output alphabet sets $\mathcal{Y}_1$ and $\mathcal{Y}_2$ respectively.

Each of the encoders splits its respective rates into two parts, where the auxiliary random variables U_1 and U_2 serve as cloud centers that can be distinguished by both receivers. Hence, U_1 represents information intended for receiver 1 but that can also be decoded by receiver 2, while W_1 represents information intended for receiver 1 but that cannot be decoded by receiver 2. The same applies to auxiliary random variables U_2 and W_2.

This is basically an application of the superposition coding technique of Cover [10] and was first applied by Carleial [3] to the Gaussian IC. However, Carleial made use of a sequential decoder, decoding U_1 and U_2 first before decoding W_1 and W_2. Han and Kobayashi applied a more powerful decoding technique known as simultaneous decoding. Receiver 1 decodes U_1 and W_1 simultaneously, while receiver 2 decodes U_2 and W_2 simultaneously. Moreover, they introduced a time-sharing parameter Q instead of using the convex-hull operation. This time-sharing parameter Q will also include the time division multiplex/frequency division multiplex strategy introduced by Carleial [3]. We next describe this achievable rate region.

Let $\mathcal{P}^*$ be the set of probability distributions $P^*(.)$ that factor as

$$\begin{aligned} &P^*(q,u_1,w_1,u_2,w_2,x_1,x_2)\\ =&p(q)\,p(u_1|q)\,p(w_1|q)\,p(u_2|q)\,p(w_2|q)\\ &\times p(x_1|u_1,w_1,q)\,p(x_2|u_2,w_2,q) \end{aligned} \tag{6}$$

Theorem 1: For a fixed $P^* \in \mathcal{P}^*$, let $\mathcal{R}_{\text{HK}}(P^*)$ be the set of (R_1,R_2) satisfying

$$R_1 \le R_{11} + R_{12} \tag{7}$$
$$R_2 \le R_{21} + R_{22} \tag{8}$$

where

$$R_{11} \le I(W_1;Y_1|U_1U_2Q) \tag{9}$$
$$R_{12} \le I(U_1;Y_1|W_1U_2Q) \tag{10}$$
$$R_{21} \le I(U_2;Y_1|W_1U_1Q) \tag{11}$$
$$R_{11}+R_{12} \le I(U_1W_1;Y_1|U_2Q) \tag{12}$$
$$R_{11}+R_{21} \le I(W_1U_2;Y_1|U_1Q) \tag{13}$$
$$R_{12}+R_{21} \le I(U_1U_2;Y_1|W_1Q) \tag{14}$$
$$R_{11}+R_{12}+R_{21} \le I(U_1W_1U_2;Y_1|Q) \tag{15}$$
$$R_{22} \le I(W_2;Y_2|U_2U_1Q) \tag{16}$$
$$R_{21} \le I(U_2;Y_2|W_2U_1Q) \tag{17}$$
$$R_{12} \le I(U_1;Y_2|W_2U_2Q) \tag{18}$$
$$R_{21}+R_{22} \le I(U_2W_2;Y_2|U_1Q) \tag{19}$$
$$R_{22}+R_{12} \le I(W_2U_1;Y_2|U_2Q) \tag{20}$$
$$R_{21}+R_{12} \le I(U_2U_1;Y_2|W_2Q) \tag{21}$$
$$R_{21}+R_{22}+R_{12} \le I(U_2W_2U_1;Y_2|Q) \tag{22}$$

Then the following set

$$\mathcal{R}_{\text{HK}} = \bigcup_{P^* \in \mathcal{P}^*} \mathcal{R}_{\text{HK}}(P^*) \tag{23}$$

is an achievable rate region for the discrete memoryless IC.

The bounds (7)-(22) can be simplified by using Fourier-Motzkin elimination [4, Theorem 4.1]. Moreover, X_1 is a deterministic function of U_1, W_1 and Q and X_2 is a deterministic function of U_2, W_2 and Q, i.e.,

$$X_1 = f_1(U_1,W_1,Q) \tag{24}$$
$$X_2 = f_2(U_2,W_2,Q). \tag{25}$$

Then, we can describe the Han-Kobayashi region $\mathcal{R}_{\text{HK}}$ as follows.

Theorem 2: For a fixed $P^* \in \mathcal{P}^*$, let $\mathcal{R}_{\text{HK}}(P^*)$ be the set of (R_1,R_2) satisfying

$$R_1 \le \rho_1 \tag{26}$$
$$R_2 \le \rho_2 \tag{27}$$
$$R_1+R_2 \le \rho_{12} \tag{28}$$
$$2R_1+R_2 \le \rho_{10} \tag{29}$$
$$R_1+2R_2 \le \rho_{20} \tag{30}$$

where

$$\rho_1 = \sigma_1 + I(X_1;Y_1|U_1U_2Q) \quad (31)$$

$$\rho_2 = \sigma_2 + I(X_2;Y_2|U_1U_2Q) \quad (32)$$

$$\rho_{12} = \sigma_{12} + I(X_1;Y_1|U_1U_2Q) + I(X_2;Y_2|U_1U_2Q) \quad (33)$$

$$\begin{aligned}\rho_{10} = {} & 2\sigma_1 + 2I(X_1;Y_1|U_1U_2Q) + I(X_2;Y_2|U_1U_2Q) \\ & - [\sigma_1 - I(U_1;Y_2|U_2Q)]^+ \\ & + \min\left\{\begin{array}{c} I(U_2;Y_2|U_1Q), \\ I(U_2;Y_2|Q) + [I(U_1;Y_2|U_2Q) - \sigma_1]^+, \\ I(U_2;Y_1|U_1Q), I(U_1U_2;Y_1|Q) - \sigma_1 \end{array}\right\}\end{aligned} \quad (34)$$

$$\begin{aligned}\rho_{20} = {} & 2\sigma_2 + 2I(X_2;Y_2|U_1U_2Q) + I(X_1;Y_1|U_1U_2Q) \\ & - [\sigma_2 - I(U_2;Y_1|U_1Q)]^+ \\ & + \min\left\{\begin{array}{c} I(U_1;Y_1|U_2Q), \\ I(U_1;Y_1|Q) + [I(U_2;Y_1|U_1Q) - \sigma_2]^+, \\ I(U_1;Y_2|U_2Q), I(U_1U_2;Y_2|Q) - \sigma_2 \end{array}\right\}\end{aligned} \quad (35)$$

$$\sigma_1 = \min\{I(U_1;Y_1|U_2Q), I(U_1;Y_2|X_2Q)\} \quad (36)$$

$$\sigma_2 = \min\{I(U_2;Y_2|U_1Q), I(U_2;Y_1|X_1Q)\} \quad (37)$$

$$\sigma_{12} = \min\left\{\begin{array}{c} I(U_1U_2;Y_1|Q), I(U_1U_2;Y_2|Q), \\ I(U_1;Y_1|U_2Q) + I(U_2;Y_2|U_1Q), \\ I(U_1;Y_2|U_2Q) + I(U_2;Y_1|U_1Q) \end{array}\right\} \quad (38)$$

Here, $[x]^+ = 0$ if $x \le 0$, $[x]^+ = x$ if $x > 0$. Then, we have

$$\mathcal{R}_{\text{HK}} = \bigcup_{P^* \in \mathcal{P}^*} \mathcal{R}_{\text{HK}}(P^*). \quad (39)$$

Even though the resulting bounds for Theorem 2 may seem very complicated, we give a simplified version in Section V. This is to ease comparison with the Chong-Motani-Garg region which will be derived in the next section.

IV. Chong-Motani-Garg Rate Region

We now establish a potentially new achievable rate region for the interference channel. We make the observation that since both receivers are not interested in the message of the non-intended transmitters, constraints (11) and (18) are unnecessary to drive the probability of error to zero.

Moreover, instead of 5 auxiliary random variables, we only consider 3 auxiliary random variables Q, U_1 and U_2 defined on arbitrary finite sets $\mathcal{Q}$, $\mathcal{U}_1$ and $\mathcal{U}_2$. Again, the auxiliary random variables U_1 and U_2 will serve as cloud centers that can be distinguished by both receivers. For transmitter 1, instead of generating two independent codebooks with codewords $\mathbf{U}_1^n(j)$ and $\mathbf{W}_1^n(k)$, for each codeword $\mathbf{U}_1^n(j)$, we generate a codebook with codewords $\mathbf{W}_1^n(j,k)$, where $j \in \{1,2,...,2^{nR_{12}}\}$ and $k \in \{1,2,...,2^{nR_{11}}\}$. Due to our code construction, the constraints (10), (14), (17) and (21) are unnecessary to drive the probability of error to zero.

Let $\mathcal{P}_1^*$ be the set of probability distributions $P_1^*(.)$ that factor as

$$\begin{aligned} & P_1^*(q,u_1,u_2,x_1,x_2) \\ = & p(q)\, p(u_1x_1|q)\, p(u_2x_2|q) \end{aligned} \quad (40)$$

Theorem 3: For a fixed $P_1^* \in \mathcal{P}_1^*$, let $\mathcal{R}_{\text{CMG}}(P_1^*)$ be the set of (R_1,R_2) satisfying

$$R_1 \le R_{11} + R_{12} \quad (41)$$

$$R_2 \le R_{21} + R_{22} \quad (42)$$

where

$$R_{11} \le I(X_1;Y_1|U_1U_2Q) \quad (43)$$

$$R_{11} + R_{21} \le I(U_2X_1;Y_1|U_1Q) \quad (44)$$

$$R_{11} + R_{12} \le I(X_1;Y_1|U_2Q) \quad (45)$$

$$R_{11} + R_{12} + R_{21} \le I(U_2X_1;Y_1|Q) \quad (46)$$

$$R_{22} \le I(X_2;Y_2|U_1U_2Q) \quad (47)$$

$$R_{22} + R_{12} \le I(U_1X_2;Y_2|U_2Q) \quad (48)$$

$$R_{21} + R_{22} \le I(X_2;Y_2|U_1Q) \quad (49)$$

$$R_{12} + R_{21} + R_{22} \le I(U_1X_2;Y_2|Q) \quad (50)$$

Then the set given by

$$\mathcal{R}_{\text{CMG}} = \bigcup_{P_1^* \in \mathcal{P}_1^*} \mathcal{R}_{\text{CMG}}(P_1^*) \quad (51)$$

is an achievable rate region for the discrete memoryless IC.

A. Proof of Theorem 3

Codebook Generation: Generate a codeword Q^n of length n, generating each element i.i.d according to $\prod_{i=1}^n p(q)$. For the codeword $\mathbf{Q}^n$, generate $2^{nR_{12}}$ independent codewords $\mathbf{U}_1^n(j)$, $j \in \{1,2,...,2^{nR_{12}}\}$, generating each element i.i.d according to $\prod_{i=1}^n p(u_{1i}|q_i)$. For the codeword $\mathbf{Q}^n$, and each of the codeword $\mathbf{U}_1^n(j)$, generate $2^{nR_{11}}$ i.i.d codewords $\mathbf{X}_1^n(j,k)$, $k \in \{1,2,...,2^{nR_{11}}\}$, generating each element i.i.d according to $\prod_{i=1}^n p(x_{1i}|q_i,u_{1i}(j))$. For the codeword $\mathbf{Q}^n$, generate $2^{nR_{21}}$ independent codewords $\mathbf{U}_2^n(l)$, $l \in \{1,2,...,2^{nR_{21}}\}$, generating each element i.i.d according to $\prod_{i=1}^n p(u_{2i}|q_i)$. For the codeword $\mathbf{Q}^n$, and each of the codeword $\mathbf{U}_2^n(l)$, generate $2^{nR_{22}}$ i.i.d codewords $\mathbf{X}_2^n(l,m)$, $m \in \{1,2,...,2^{nR_{22}}\}$, generating each element i.i.d according to $\prod_{i=1}^n p(x_{2i}|q_i,u_{2i}(l))$.

Encoding: For encoder 1, to send the codeword pair (j,k), send the corresponding codeword $\mathbf{X}_1^n(j,k)$. For encoder 2, to send the codeword pair (l,m), send the corresponding codeword $\mathbf{X}_2^n(l,m)$.

Decoding: Receiver 1 determines the unique $(\hat{j},\hat{k})$ and a $\hat{l}$ such that

$$\left(\mathbf{U}_1^n(\hat{j}), \mathbf{X}_1^n(\hat{j},\hat{k}), \mathbf{U}_2^n(\hat{l}), \mathbf{Y}_1^n\right) \in A_\epsilon^{(n)}. \quad (52)$$

Receiver 2 determines the unique $(\hat{\hat{l}},\hat{\hat{m}})$ and a $\hat{\hat{j}}$ such that

$$\left(\mathbf{U}_2^n(\hat{\hat{l}}), \mathbf{X}_2^n(\hat{\hat{l}},\hat{\hat{m}}), \mathbf{U}_1^n(\hat{\hat{j}}), \mathbf{Y}_2^n\right) \in A_\epsilon^{(n)}. \quad (53)$$

Analysis of the Probability of Error: We consider only the decoding error probability for receiver 1. The same analysis applies for receiver 2. By the symmetry of the random code construction, the conditional probability of error does not depend on which pair of indices is sent. Thus the conditional probability of error is the same as the unconditional probability of error. So, without loss of generality, we assume that $(j,k) = (1,1)$ and $(l,m) = (1,1)$ was sent.

We have an error if the correct codewords, $\{\mathbf{U}_1^n(1), \mathbf{X}_1^n(1,1), \mathbf{U}_2^n(1)\}$ are not jointly typical with the received sequence. An error is also declared if incorrect codewords $\left\{\mathbf{U}_1^n\left(\hat{j}\right), \mathbf{X}_1^n\left(\hat{j},\hat{k}\right), \mathbf{U}_2^n\left(\hat{l}\right)\right\}$ where $\hat{j} \neq 1$ or $\hat{k} \neq 1$ are jointly typical with the received codeword. However, no error is declared if $\left\{\mathbf{U}_1^n(1), \mathbf{X}_1^n(1,1), \mathbf{U}_2\left(\hat{l} \neq 1\right)\right\}$ are jointly typical with the received sequence. Define the following event

$$E_{jkl} = \left\{\left(\mathbf{U}_1^n(j), \mathbf{X}_1^n(j,k), \mathbf{U}_2^n(l), \mathbf{Y}_1^n\right) \in A_\epsilon^{(n)}\right\}. \quad (54)$$

Then by the union of events bound,

$$\begin{aligned} P_e^{(n)} &= P\left(E_{111}^c \bigcup \cup_{(j,k)\neq(1,1)} E_{jkl}\right) \\ &\leq P\left(E_{111}^c\right) + \sum_{j\neq1,k=1,l=1} P\left(E_{j11}\right) \\ &\quad + \sum_{j\neq1,k=1,l\neq1} P\left(E_{j1l}\right) + \sum_{j=1,k\neq1,l=1} P\left(E_{1k1}\right) \\ &\quad + \sum_{j=1,k\neq1,l\neq1} P\left(E_{1kl}\right) + \sum_{j\neq1,k\neq1,l=1} P\left(E_{jk1}\right) \\ &\quad + \sum_{j\neq1,k\neq1,l\neq1} P\left(E_{jkl}\right) \end{aligned}$$

$$\begin{aligned} P_e^{(n)} \leq\ & P\left(E_{111}^c\right) + 2^{nR_{12}} 2^{-n(I(X_1;Y_1|U_2Q)-4\epsilon)} \\ &+ 2^{n(R_{12}+R_{21})} 2^{-n(I(U_2X_1;Y_1|Q)-4\epsilon)} \\ &+ 2^{nR_{11}} 2^{-n(I(X_1;Y_1|U_1U_2Q)-4\epsilon)} \\ &+ 2^{n(R_{11}+R_{21})} 2^{-n(I(U_2X_1;Y_1|U_1Q)-4\epsilon)} \\ &+ 2^{n(R_{11}+R_{12})} 2^{-n(I(X_1;Y_1|U_2Q)-4\epsilon)} \\ &+ 2^{n(R_{11}+R_{12}+R_{21})} 2^{-n(I(U_2X_1;Y_1|Q)-4\epsilon)}. \quad (55) \end{aligned}$$

Since $\epsilon > 0$ is arbitrary, the conditions of Theorem 3 imply that each term tends to 0 as $n \to \infty$. The above bound shows that the average probability of error, averaged over all choices of codebooks in the random code construction, is arbitrarily small. Hence there exists at least one code $\mathcal{C}^*$ with arbitrarily small probability of error.

Since we can choose a fixed P_1^* such that

$$\begin{aligned} &P_1^*(q, u_1, u_2, x_1, x_2) \\ &= \sum_{w_1\in\mathcal{W}_1, w_2\in\mathcal{W}_2} P^*(q, u_1, u_2, w_1, w_2, x_1, x_2) \end{aligned} \quad (56)$$

we readily see that $\mathcal{R}_{\mathrm{HK}}(P^*) \subseteq \mathcal{R}_{\mathrm{CMG}}(P_1^*)$, and hence $\mathcal{R}_{\mathrm{HK}} \subseteq \mathcal{R}_{\mathrm{CMG}}$. The bounds (41)-(50) can be again be simplified using Fourier-Motzkin elimination [11, Theorem 3].

Theorem 4: For a fixed $P_1^* \in \mathcal{P}_1^*$, let $\mathcal{R}_{\mathrm{CMG}}(P_1^*)$ be the set of (R_1, R_2) satisfying

$$R_1 \leq I(X_1; Y_1|U_2Q) \quad (57)$$

$$R_2 \leq I(X_2; Y_2|U_1Q) \quad (58)$$

$$R_1 + R_2 \leq I(X_1U_2; Y_1|Q) + I(X_2; Y_2|U_1U_2Q) \quad (59)$$

$$R_1 + R_2 \leq I(X_1; Y_1|U_1U_2Q) + I(X_2U_1; Y_2|Q) \quad (60)$$

$$R_1 + R_2 \leq I(X_1U_2; Y_1|U_1Q) + I(X_2U_1; Y_2|U_2Q) \quad (61)$$

$$\begin{aligned} 2R_1 + R_2 \leq\ & I(X_1U_2; Y_1|Q) + I(X_1; Y_1|U_1U_2Q) \\ &+ I(X_2U_1; Y_2|U_2Q) \end{aligned} \quad (62)$$

$$\begin{aligned} R_1 + 2R_2 \leq\ & I(X_2; Y_2|U_1U_2Q) + I(X_2U_1; Y_2|Q) \\ &+ I(X_1U_2; Y_1|U_1Q) \end{aligned} \quad (63)$$

Then we have

$$\mathcal{R}_{\mathrm{CMG}} = \bigcup_{P_1^*\in\mathcal{P}_1^*} \mathcal{R}_{\mathrm{CMG}}(P_1^*). \quad (64)$$

V. Comparing the Han-Kobayashi and Chong-Motani-Garg Rate Regions

To facilitate comparison of the Han-Kobayashi rate region with the Chong-Motani-Garg rate region, we simplify the bounds in Theorem 2. In fact, many of the bounds are redundant and can be further reduced as in the following theorem.

Theorem 5: For a fixed $P_1^* \in \mathcal{P}_1^*$, let $\mathcal{R}_{\mathrm{HK}}(P_1^*)$ be the set of (R_1, R_2) satisfying

$$\begin{aligned} R_1 \leq\ & \min\{I(U_1; Y_1|U_2Q), I(U_1; Y_2|X_2Q)\} \\ &+ I(X_1; Y_1|U_1U_2Q) \end{aligned} \quad (65)$$

$$\begin{aligned} R_2 \leq\ & \min\{I(U_2; Y_2|U_1Q), I(U_2; Y_1|X_1Q)\} \\ &+ I(X_2; Y_2|U_1U_2Q) \end{aligned} \quad (66)$$

$$R_1 + R_2 \leq I(X_1U_2; Y_1|Q) + I(X_2; Y_2|U_1U_2Q) \quad (67)$$

$$R_1 + R_2 \leq I(X_1; Y_1|U_1U_2Q) + I(X_2U_1; Y_2|Q) \quad (68)$$

$$R_1 + R_2 \leq I(X_1U_2; Y_1|U_1Q) + I(X_2U_1; Y_2|U_2Q) \quad (69)$$

$$\begin{aligned} 2R_1 + R_2 \leq\ & I(X_1U_2; Y_1|Q) + I(X_1; Y_1|U_1U_2Q) \\ &+ I(X_2U_1; Y_2|U_2Q) \end{aligned} \quad (70)$$

$$\begin{aligned} R_1 + 2R_2 \leq\ & I(X_2; Y_2|U_1U_2Q) + I(X_2U_1; Y_2|Q) \\ &+ I(X_1U_2; Y_1|U_1Q) \end{aligned} \quad (71)$$

Finally, we have

$$\mathcal{R}_{\mathrm{HK}} = \bigcup_{P_1^*\in\mathcal{P}_1^*} \mathcal{R}_{\mathrm{HK}}(P_1^*). \quad (72)$$

The proof for the bounds on R_1+R_2 can be found in [11]. In addition, given the constraints (65)-(69), it can be easily shown that all the other constraints in Theorem 2, except for (70)-(71), are redundant. Comparing the Han-Kobayashi region given by Theorem 5 with the Chong-Motani-Garg region given by Theorem 4, we again see that $\mathcal{R}_{\mathrm{HK}} \subseteq \mathcal{R}_{\mathrm{CMG}}$. Moreover, the only difference lies in the bound for R_1 and R_2.

In [11], Kramer asked whether there are ICs for which there exists $P_1^* \in \mathcal{P}_1^*$ such that $\mathcal{R}_{\mathrm{HK}}(P_1^*) \subsetneq \mathcal{R}_{\mathrm{CMG}}(P_1^*)$. For the Gaussian IC, when we set $|\mathcal{Q}| = 1$, we can easily determine

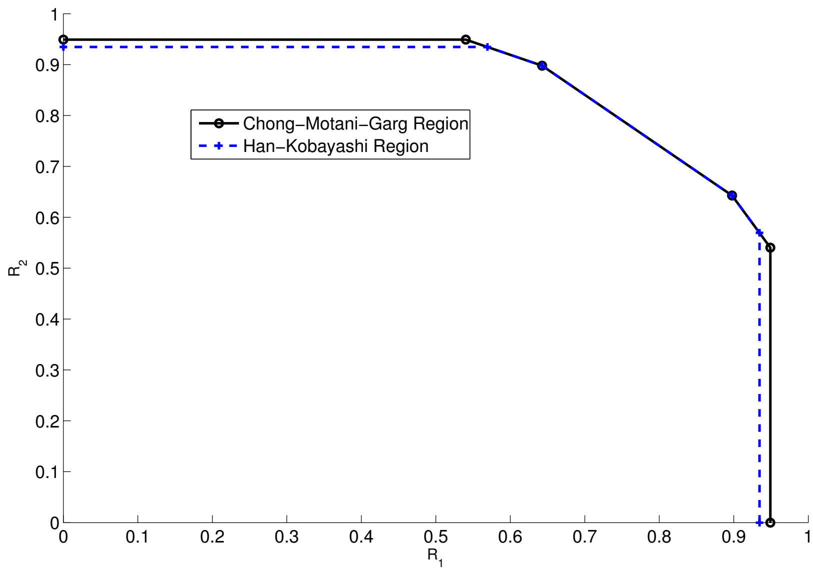

Fig. 2. Two rate regions for the Gaussian IC when $P_1 = P_2 = 6$, $c_{12}^2 = c_{21}^2 = 0.4$ and $\alpha = \beta = 0.5$

parameters where $\mathcal{R}_{\mathrm{HK}}(P_1^*) \subsetneq \mathcal{R}_{\mathrm{CMG}}(P_1^*)$. We assume U_1, U_2, X_1 and X_2 are Gaussian random variables where

$$\frac{\mathbb{E}\left[U_1^2\right]}{\mathbb{E}\left[X_1^2\right]} = \alpha, \qquad \frac{\mathbb{E}\left[U_2^2\right]}{\mathbb{E}\left[X_2^2\right]} = \beta \tag{73}$$

such that $\alpha \in [0,1]$, $\beta \in [0,1]$, $\mathbb{E}\left[X_1^2\right] = P_1$ and $\mathbb{E}\left[X_2^2\right] = P_2$. From Fig. 2, we see that when $P_1 = P_2 = 6$, $c_{12}^2 = c_{21}^2 = 0.4$ and $\alpha = \beta = 0.5$, $\mathcal{R}_{\mathrm{HK}}(P_1^*) \subsetneq \mathcal{R}_{\mathrm{CMG}}(P_1^*)$.

However, this is only for particular choices of $P_1^* \in \mathcal{P}_1^*$. Our numerical simulations seem to indicate that $\mathcal{R}_{\mathrm{HK}} = \mathcal{R}_{\mathrm{CMG}}$ for the Gaussian IC. Currently, we have still been unable to prove that $\mathcal{R}_{\mathrm{HK}} = \mathcal{R}_{\mathrm{CMG}}$ or that $\mathcal{R}_{\mathrm{HK}} \subsetneq \mathcal{R}_{\mathrm{CMG}}$ for the general IC, or even for the Gaussian IC. We conjecture that $\mathcal{R}_{\mathrm{HK}} = \mathcal{R}_{\mathrm{CMG}}$ for the Gaussian IC, but that there may be other ICs where $\mathcal{R}_{\mathrm{HK}} \subsetneq \mathcal{R}_{\mathrm{CMG}}$.

VI. Conclusion

We have reviewed two rate regions for the general IC and made comparisons between the Han-Kobayashi rate region and the Chong-Motani-Garg rate region. We have reduced the bounds for the two rate regions to their simplest forms for comparison. Even though the Chong-Motani-Garg rate region will always include the Han-Kobayashi rate region, it is not known whether the two rate regions are equivalent for all ICs.

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