Fundamental Limits of Multipath Aided Serial Search Acquisition

Watcharapan Suwansantisuk, *Student Member, IEEE,* and Moe Z. Win, *Fellow, IEEE*

(*Invited Tutorial*)

***Abstract*— Spread-spectrum systems with large transmission bandwidth present significant challenges from the standpoint of achieving acquisition before the communication commences. This tutorial summarizes our recent results on acquisition system that exploits multipath to aid the synchronization. In particular, we consider the serial-search strategy and the optimal search procedure to examine the uncertainty space. We derive a lower bound and an upper bound for the mean acquisition times (MATs) over all possible search procedures. These expressions give fundamental limits of the MATs. We then prove that a fixed-step serial search, a form of non-consecutive serial search, achieves a near-optimal MAT. We prove that the conventional serial search, in which consecutive cells are tested serially, results in the maximum MAT. Since these results are valid for all signal-to-noise ratio values, fading distributions, and details of the detection layer, the tutorial provides search strategies that acquisition receivers should employ and search strategies that acquisition receivers should avoid.**

I. Introduction

One of the most important tasks of a spread spectrum receiver is sequence acquisition [1]–[4]. During this task, the receiver coarsely aligns the sequence of the received signal with the sequence of the locally generated reference. Reliable transmission may begins only after the receiver completes sequence acquisition. This tutorial summarizes our results on sequence acquisition from the papers [5]–[10].

To acquire the received signal, the receiver search for a location of sequence phase within a required range of accuracy. Like most search problems, sequence acquisition are associated with two important, known parameters. The first parameter, the total number of phases or cells that the receiver needs to test (denoted by N_{unc}) is proportional to the transmission bandwidth. The second parameter, the number of correct phases or in-phase cells (denoted by N_{hit}) is proportional to the number of resolvable paths. In wide-bandwidth transmission systems, N_{unc} can be very large, and acquisition of received signals in a reasonable amount of time becomes a challenging task.

To mathematize an acquisition problem, one indexes the cells from 1 to N_{unc} without loss of generality. The uncertainty index set, $\mathcal{U} \triangleq \{1, 2, 3, \dots, N_{\text{unc}}\}$, denote a collection of cells for the receiver to test. The index set corresponding to the in-phase cells, $\mathcal{H}_{\text{hit}}$, denotes a collection of in-phase cells. We note that elements of set $\mathcal{H}_{\text{hit}}$ are unknown to the receiver and that $\mathcal{H}_{\text{hit}} \subseteq \mathcal{U}$. From a mathematical point of view, sequence acquisition can be regarded as a task of searching for an element that belongs to set $\mathcal{H}_{\text{hit}}$.

In a dense multipath channel, multiple resolvable paths tend to arrive in a cluster [11]–[14]. For such a channel, in-phase cells are consecutive, modulo-N_{unc}, in the uncertainty index set:[1] $\mathcal{H}_{\text{hit}} \triangleq \{b_0 \oplus i \,|\, 0 \le i \le N_{\text{hit}} - 1\}$. Here, an integer, $b_0 \in \mathcal{U}$, is a realization of random variable B, which represents the location of the cell corresponding to the first resolvable path. Regardless of the structure of set $\mathcal{H}_{\text{hit}}$, a commonly used performance measure of acquisition receivers is the mean acquisition time (MAT), the average duration required for the receiver to achieve acquisition.

The task of improving the MAT, in general, can be realized at the detection layer or at the search layer of the receiver. At the detection layer, a receiver may dedicate more resources, such as correlators, to form a decision variable [17]–[22]. At the search layer, a receiver may use a special search pattern such as an expanding zigzag window [23] or a non-consecutive [19], [24]–[26] serial search. This tutorial presents an approach to improve the MAT at the search layer by employing an intelligent search order.

A search order refers to a specific order for the receiver to search an uncertainty index set. Mathematically, a search order is a bijection from set $\mathcal{U}$ into set $\mathcal{U}$, $\pi\colon \mathcal{U} \to \mathcal{U}$. From a design perspective, a receiver that employs a search order π tests the cells in the order

$$\pi(k), \pi(k+1), \dots, \pi(N_{\text{unc}}), \pi(1), \pi(2), \dots \ ,$$

where the first cell that the receiver interrogates, $\pi(k)$, can be any cell. Therefore, the set of all possible search orders is given by $\mathcal{P} = \{\pi \,|\, \pi\colon \mathcal{U} \to \mathcal{U} \text{ is a bijection and } \pi(1) = 1\}$. We note that the condition $\pi(1) = 1$ simply reduces some redundant search orders from the set of search orders.

Examples of search orders that have been used in the literature include the conventional serial search (CSS or π^1) and the fixed-step serial search with step size N_{J} (FSSS or $\pi^{N_{\text{J}}}$). A receiver that employs the CSS tests consecutive cells serially [17], [18], while a receiver that employs the FSSS with

The authors are with the Laboratory for Information and Decision Systems (LIDS), Massachusetts Institute of Technology, Cambridge, MA 02139 USA, e-mail: wsk@mit.edu, moewin@mit.edu

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[1]The symbol $\oplus$ denotes the modulo N_{unc} addition defined by $x \oplus y \triangleq x + y - lN_{\text{unc}}$ for some unique integer l such that $x + y - lN_{\text{unc}} \in \mathcal{U}$.

step size $N_{\rm J}$ skips a fixed $N_{\rm J}$ cells after each test [5], [19], [24]. This tutorial summarizes an approach to bound the MAT, $\mathbb{E}\{T_{\rm ACQ}(\pi)\}$, for any search order π, and presents search orders that achieve these bounds. Here, the random variable $T_{\rm ACQ}(\pi)$ denotes the MAT as a function of the search order π. The key results are as follows:

- the derivation of fundamental limits of MAT,
- a proof that the FSSS with the step size $N_{\rm hit}$ effectively achieves the minimum MAT in wide bandwidth transmission systems, and
- a proof that the CSS exhibits the maximum MAT.

We focus on the most commonly used search strategy, namely the serial search [15]–[19], [23], [24], [27].

This tutorial is organized as follows. Section II presents the system model and the basic definitions for the acquisition system. Section III obtains the absorption time expression, a quantity that is closely related to the MAT, and presents important properties of the absorption time. Section IV uses properties of the absorption time to derive a lower bound and an upper bound on the MAT. Section V determines a search order that effectively achieves the minimum MAT. Section VI obtains search orders that achieve the maximum MAT. Section VII concludes the tutorial and summarizes important results.

II. System Model and Basic Definitions

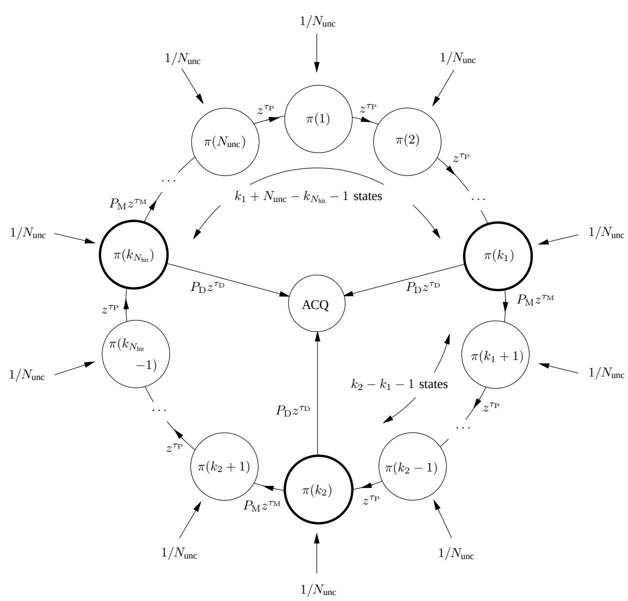

Fig. 1. A flow diagram for the serial search with the search order π contains $N_{\rm unc}+1$ states. The state labeled ACQ is the absorbing state. The states in thick circles are H_1-states. The remaining states are H_0-states.

Fig. 1 depicts a flow diagram which corresponds to a serial-search strategy with a generic search order π. There are $N_{\rm unc}+1$ states in total: one absorbing state (ACQ), $N_{\rm hit}$ states of type H_1, and $N_{\rm unc} - N_{\rm hit}$ states of type H_0. The ACQ state represents the event of successful acquisition. Each of the $N_{\rm hit}$ states of type H_1 corresponds to an in-phase cell, while each of the remaining $N_{\rm unc} - N_{\rm hit}$ states of type H_0 corresponds to a non-in-phase cell. The polynomial path gain parameters $P_{\rm D} z^{\tau_{\rm D}}$, $P_{\rm M} z^{\tau_{\rm M}}$, and $z^{\tau_{\rm P}}$ can be interpreted as effective detection layer parameters.[2]

The structure of the flow diagram describes the arrangement of the in-phase and non-in-phase cells. This structure plays an important role in the acquisition system as it strongly influences the absorption time and the MAT. The structure of a flow diagram can be specified by its *description*.

Definition 1 (Description): A *description* is a tuple (π, b) of the search order π and the location b of the first in-phase cell. The set of all possible descriptions is denoted by $\mathcal{D} = \mathcal{P} \times \mathcal{U}$.

The description (π, b) characterizes the structure of a flow diagram. In particular, π specifies the order of the non-absorbing states, while b determines the set $\mathcal{H}_{\rm hit}(b)$ of states that have transition edges to the absorbing state. We now focus our attention on a widely-used class of flow diagrams [15], [16], [19], [23], [24], which we refer to as a *non-preferential* flow diagram (see Fig. 1).

Definition 2 (Non-preferential flow diagram): The flow diagram is *non-preferential* if it has the following properties:

1) Probability of starting the search at any non-absorbing state is equally likely;
2) Every path to the absorbing state has the same path gain;
3) Every path from an H_0-state has the same path gain; and
4) Every path from an H_1-state to the adjacent non-absorbing state has the same path gain.

A non-preferential flow diagram serves as a basic model for analyzing the performance of an acquisition system operating in dense multipath environments. The use of a non-preferential flow diagram is reasonable when the power dispersion profile (PDP) is decaying slowly or constant over an interval.

III. Absorption Time

An absorption time is the average time to arrive at the absorption state. In this section, we investigate the relationship between the absorption time and the MAT and prove some important properties of the absorption time. In the next section, we will use properties of the absorption time to derive limits on the MAT.

A. Conventional Approach

For a given search order π, the MAT is given by

$$\mathbb{E}\{T_{\rm ACQ}(\pi)\} = \sum_{b=1}^{N_{\rm unc}} f(\pi, b) \cdot \Pr\{B = b\}, \qquad (1)$$

where $f(\pi, b) \triangleq \mathbb{E}\{T_{\rm ACQ}(\pi) \,|\, B = b\}$ denotes the absorption time of the flow diagram with a description (π, b). A conventional approach of computing the absorption time yields the

[2] We stressed that parameters $P_{\rm D}$, $P_{\rm M}$, $\tau_{\rm D}$, $\tau_{\rm M}$, and $\tau_{\rm P}$ are transformations of probabilities of detections, probability of false alarms, dwell times, and penalties times. The transformations are given explicitly in [5, p. 32].

expression

$$f(\pi, b) = \frac{1}{N_{\text{unc}}} \left. \frac{dh(z)}{dz} \right|_{z=1} \tag{2}$$

where the $h(z)$ is a polynomial that depends on the path gain parameters (see [6, eq. 12] for the expression of $h(z)$). Equation (2) follows from a loop-reduction technique, which is used to find MATs in [15], [16], [19], [23], [24].

The expression in (1) implies that the MAT of any given search order π_0 can be bounded by

$$\min_{(\pi,b)\in\mathcal{D}} f(\pi, b) \le \mathbb{E}\left\{T_{\text{ACQ}}(\pi_0)\right\} \le \max_{(\pi,b)\in\mathcal{D}} f(\pi, b). \tag{3}$$

However, the expression $f(\pi, b)$ in (2) does not reveal its dependence on the search order π explicitly. As a result, it is unclear how one can solve efficiently—if at all—the optimization problems $\min_{(\pi,b)\in\mathcal{D}} f(\pi, b)$ and $\max_{(\pi,b)\in\mathcal{D}} f(\pi, b)$.

A direct approach that finds the minimum or maximum MAT by searching exhaustively over the set $\mathcal{P}$ is impractical. It can be shown that the exhaustive search requires at least $N_{\text{unc}}!$ arithmetic operations to complete. To test a small number $N_{\text{unc}} = 100$ of cells with a fictional machine that has a clock speed of 10^{20} Hz and can perform 1 arithmetic operation per cycle, an acquisition system would take more than 10^{130} years to complete the exhaustive search. Clearly, the direct approach is extremely inefficient.

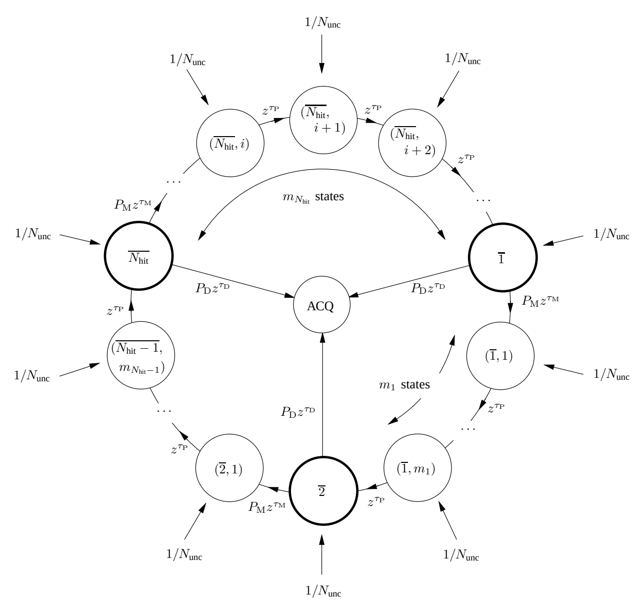

Fig. 2. The spacing rule $\mathbf{m} = \left[m_1\, m_2\, \ldots\, m_{N_{\text{hit}}}\right]^T$ characterizes the structure of the flow diagram.

B. *Transforming into the Spacing Rule Domain*

The difficulty associated with the direct optimization can be alleviated by transforming the *descriptions* into the domain of *spacing rules*.

Definition 3 (Spacing rule): A *spacing rule* of a non-preferential flow diagram with N_{hit} H_1-states and $(N_{\text{unc}} - N_{\text{hit}})$ H_0-states is an element of the set

$$\mathcal{S}_{\text{d}} = \left\{ \left[m_1\, m_2\, \ldots\, m_{N_{\text{hit}}}\right]^T \,\middle|\, \sum_{i=1}^{N_{\text{hit}}} m_i = N_{\text{unc}} - N_{\text{hit}}; \right. \\ \left. \forall i, m_i \text{ is integer} \right\}. \tag{4}$$

A spacing rule $\mathbf{m} \triangleq \left[m_1\, m_2\, \ldots\, m_{N_{\text{hit}}}\right]^T$ characterizes the structure of a flow diagram (see Fig. 2). In particular, the flow diagram has an H_1-state, which is followed by m_1 H_0-states, which are followed by another H_1-state, which is followed by m_2 H_0-states, and so on. Note that the sum $\sum_{i=1}^{N_{\text{hit}}} m_i$ must equal the number $N_{\text{unc}} - N_{\text{hit}}$ of H_0-states.

Let $v(\mathbf{m})$ be the absorption time for the flow diagram corresponding to a spacing rule $\mathbf{m}$. Since both description $(\pi, b) \in \mathcal{D}$ and spacing rule $\mathbf{m} \in \mathcal{S}_{\text{d}}$ characterize the structure of the flow diagram, which determines the absorption time, we have $f(\pi, b) = v(\mathbf{s}(\pi, b))$.[3] Therefore, (3) is equivalent to

$$\min_{\mathbf{m}\in\mathcal{S}_{\text{d}}} v(\mathbf{m}) \le \mathbb{E}\left\{T_{\text{ACQ}}(\pi_0)\right\} \le \max_{\mathbf{m}\in\mathcal{S}_{\text{d}}} v(\mathbf{m}). \tag{5}$$

Note that $\min_{\mathbf{m}\in\mathcal{S}_{\text{d}}} v(\mathbf{m})$ and $\max_{\mathbf{m}\in\mathcal{S}_{\text{d}}} v(\mathbf{m})$ are integer programming problems [28].

C. *Closed-Form Expression of $v(\mathbf{m})$*

In this subsection, we present the closed-form expression of the absorption time $v(\mathbf{m})$ for a spacing rule $\mathbf{m} \in \mathcal{S}_{\text{d}}$.

Theorem 1 (Absorption Time): The absorption time of the flow diagram with the spacing rule $\mathbf{m} \in \mathcal{S}_{\text{d}}$ is given by

$$v(\mathbf{m}) = \frac{1}{2}\mathbf{m}^T\mathbf{H}\mathbf{m} + C, \tag{6a}$$

where, for $1 \le i, j \le N_{\text{hit}}$,

$$\mathbf{H}_{ij} = \frac{\tau_{\text{P}}}{N_{\text{unc}}\left(1 - P_{\text{M}}^{N_{\text{hit}}}\right)}\left[P_{\text{M}}^{N_{\text{hit}}-|i-j|} + P_{\text{M}}^{|i-j|}\right], \tag{7}$$

$$C = \left(1 - \frac{N_{\text{hit}}}{N_{\text{unc}}}\right)\cdot\left(\frac{1+P_{\text{M}}}{1-P_{\text{M}}}\right)\frac{\tau_{\text{P}}}{2} + \frac{P_{\text{M}}}{1-P_{\text{M}}}\tau_{\text{M}} + \tau_{\text{D}}, \tag{8}$$

with $0^0 \triangleq 1$.

Proof: Omitted for brevity (see [5, pp. 38–40]). □

In the subsequent analysis, we will allow $\mathbf{m}$ in (6) to take non-integer values. In particular, let

$$\mathcal{S}_{\text{c}} = \left\{ \left[x_1\, x_2\, \ldots\, x_{N_{\text{hit}}}\right]^T \,\middle|\, \sum_{i=1}^{N_{\text{hit}}} x_i = N_{\text{unc}} - N_{\text{hit}}; \forall i, x_i \in \mathbb{R}_{0,+} \right\} \tag{9}$$

denote the convex hull of $\mathcal{S}_{\text{d}}$ and consider the function $\overline{v}\colon \mathcal{S}_{\text{c}} \to \mathbb{R}$ to be the natural extension of $v\colon \mathcal{S}_{\text{d}} \to \mathbb{R}$. That is, we evaluate $\overline{v}(\mathbf{x})$ by simply allowing $v(\cdot)$ in (6) to take the values $\mathbf{x} \in \mathcal{S}_{\text{c}}$. Because $\mathcal{S}_{\text{d}} \subset \mathcal{S}_{\text{c}}$, the MAT for any search order π_0 satisfies the following bounds:

$$\min_{\mathbf{x}\in\mathcal{S}_{\text{c}}} \overline{v}(\mathbf{x}) \le \mathbb{E}\left\{T_{\text{ACQ}}(\pi_0)\right\} \le \max_{\mathbf{x}\in\mathcal{S}_{\text{c}}} \overline{v}(\mathbf{x}). \tag{10}$$

Next, we prove important properties of $\overline{v}(\mathbf{x})$. These properties are crucial for the development in the forthcoming section.

[3] Here, $\mathbf{s}\colon \mathcal{D} \to \mathcal{P}$ is a mapping from $\mathcal{D}$ to $\mathcal{P}$.

Theorem 2 (Convexity, Rotational Invariance, Relaxations): Assume that $P_{\mathrm{M}} < 1$, so that $\overline{v}(\cdot)$ is finite.

1) Function $\overline{v}(\cdot)$ is strictly convex on $\mathcal{S}_{\mathrm{c}}$.
2) For all $[x_1\, x_2\, \ldots\, x_{N_{\mathrm{hit}}}]^T \in \mathcal{S}_{\mathrm{c}}$, $\overline{v}([x_1\, x_2\, \ldots\, x_{N_{\mathrm{hit}}}]^T) = \overline{v}([x_2\, x_3\, \ldots\, x_{N_{\mathrm{hit}}}\, x_1]^T)$.
3) The vector $\mathbf{x}^* \triangleq \left[\frac{N_{\mathrm{unc}}}{N_{\mathrm{hit}}} - 1\ \frac{N_{\mathrm{unc}}}{N_{\mathrm{hit}}} - 1\ \ldots\ \frac{N_{\mathrm{unc}}}{N_{\mathrm{hit}}} - 1\right]^T$ is a unique solution to the minimization problem $\min_{\mathbf{x}\in\mathcal{S}_{\mathrm{c}}} \overline{v}(\mathbf{x})$.
4) The vector $[N_{\mathrm{unc}} - N_{\mathrm{hit}}\, 0\, \ldots\, 0\, 0]^T$ is a solution to the maximization problem $\max_{\mathbf{m}\in\mathcal{S}_{\mathrm{c}}} \overline{v}(\mathbf{m})$.

Proof:

1) It can be shown that $\mathbf{H}$ is positive definite, which implies that $\overline{v}(\cdot)$ is strictly convex since $\overline{v}(\cdot)$ is in a quadratic form.
2) This property follows immediately from the explicit expression for $\overline{v}(\cdot)$ in (6) and from the fact that $H_{i,j} = H_{i\boxplus 1, j\boxplus 1}$ for any $1 \le i, j \le N_{\mathrm{hit}}$. Here, the symbol $\boxplus$ denotes the modulo N_{hit} addition defined by $a \boxplus b \triangleq a + b - lN_{\mathrm{hit}}$ for some unique integer l such that $1 \le a + b - lN_{\mathrm{hit}} \le N_{\mathrm{hit}}$.
3) Weierstrass' theorem implies that an optimal solution $\mathbf{x}^*$ exits [29, p. 541]. Strict convexity and rotational invariance of $\overline{v}(\cdot)$ imply that the optimal solution is unique and satisfies $x_1^* = x_2^* = x_3^* = \cdots = x_{N_{\mathrm{hit}}}^* = N_{\mathrm{unc}}/N_{\mathrm{hit}} - 1$.
4) For $1 \le i \le N_{\mathrm{hit}}$, let $\mathbf{e}_i$ denote a standard basis vector in $\mathbb{R}^{N_{\mathrm{hit}}}$. Note that any $\mathbf{x} \in \mathcal{S}_{\mathrm{c}}$ can be written as $\mathbf{x} = \sum_{i=1}^{N_{\mathrm{hit}}} \lambda_i \cdot [(N_{\mathrm{unc}} - N_{\mathrm{hit}})\mathbf{e}_i]$, for some $\lambda_i \ge 0$ that sum to unity. Strict convexity and rotational invariance of $\overline{v}(\cdot)$ imply that $\overline{v}(\mathbf{x}) < ((N_{\mathrm{unc}} - N_{\mathrm{hit}})\mathbf{e}_1)$.

□

IV. Fundamental Limits on the MAT

In this section, we use properties of the absorption time to obtain fundamental limits on the MAT.

Theorem 3 (Fundamental Limits): Any search order $\pi_0 \in \mathcal{P}$ satisfies the bounds

$$T_{\mathrm{min}}^{\mathrm{L}} \le \mathbb{E}\{T_{\mathrm{ACQ}}(\pi_0)\} \le T_{\mathrm{max}}, \tag{11}$$

where

$$T_{\mathrm{min}}^{\mathrm{L}} \triangleq \left(\frac{N_{\mathrm{unc}}}{N_{\mathrm{hit}}} - 1\right)\left(\frac{1+P_{\mathrm{M}}}{1-P_{\mathrm{M}}}\right)\frac{\tau_{\mathrm{P}}}{2} + \frac{P_{\mathrm{M}}}{1-P_{\mathrm{M}}}\tau_{\mathrm{M}} + \tau_{\mathrm{D}},$$

$$T_{\mathrm{max}} \triangleq \frac{(N_{\mathrm{unc}} - N_{\mathrm{hit}})^2}{N_{\mathrm{unc}}} \cdot \left(\frac{1+P_{\mathrm{M}}^{N_{\mathrm{hit}}}}{1-P_{\mathrm{M}}^{N_{\mathrm{hit}}}}\right)\frac{\tau_{\mathrm{P}}}{2} + \left(1 - \frac{N_{\mathrm{hit}}}{N_{\mathrm{unc}}}\right)\cdot\left(\frac{1+P_{\mathrm{M}}}{1-P_{\mathrm{M}}}\right)\frac{\tau_{\mathrm{P}}}{2} + \frac{P_{\mathrm{M}}}{1-P_{\mathrm{M}}}\tau_{\mathrm{M}} + \tau_{\mathrm{D}}.$$

Proof: The theorem follows immediately from the bounds in (10), the Minimum Relaxation property (part 4 of Thm. 2), the Maximum Relaxation property (part 3 of Thm. 2), and the explicit expression of $v(\cdot)$ (Thm. 1). □

The bounds $T_{\mathrm{min}}^{\mathrm{L}}$ and T_{max} are explicit in terms of the detection layer parameters, and suggest that intelligent search procedures improve the MAT of a receiver employing serial search strategy.

V. Achieving the Lower Bound

In this section, we determine a search order that effectively achieves a near-optimal MAT in wide bandwidth transmission systems. To do so, we introduce the concept of η-optimal search order.

Definition 4 (η-Optimal Search Order): Let $\eta(N_{\mathrm{hit}}, N_{\mathrm{unc}})$ be a function only of N_{hit} and N_{unc}. A search order π is η-optimal, if

$$\frac{\mathbb{E}\{T_{\mathrm{ACQ}}(\pi)\} - T_{\mathrm{min}}^{\mathrm{L}}}{T_{\mathrm{min}}^{\mathrm{L}}} \le \eta(N_{\mathrm{hit}}, N_{\mathrm{unc}}) \tag{12}$$

and $\eta(N_{\mathrm{hit}}, N_{\mathrm{unc}}) \to 0$ as the ratio $N_{\mathrm{hit}}/N_{\mathrm{unc}} \to 0$.

By definition, the function $\eta(N_{\mathrm{unc}}, N_{\mathrm{hit}})$ does not depend on the detection layer parameters P_{D}, P_{M}, τ_{D}, τ_{M}, and τ_{P}.[4] Hence, an η-optimal search order π_η can be used to effectively achieve the minimum MAT in many operating environments. Therefore, the particularities at the detection layer such as fading statistics, combining methods for the decision variables, and detector architectures, do not significantly affect the optimality of π_η.

Signal acquisition is a challenging task when the total number of in-phase cells is significantly smaller than the total number of cells in $\mathcal{U}$. In this situation, which is typical for UWB systems, the acquisition time can be intolerably long and there are many scenarios and applications that necessitate the use of faster acquisition techniques. Note that $N_{\mathrm{hit}}/N_{\mathrm{unc}}$ is small and approaches zero as the demand for faster acquisition is intensified. Our goal is to find η-optimal solutions since they are almost as good as the optimal one in a situation where rapid acquisition is of utmost importance.

Theorem 4 (Near-Optimality): If N_{hit} and N_{unc} are relatively prime, then the search order $\pi^{N_{\mathrm{hit}}}$ is η-optimal with $\eta = \frac{2N_{\mathrm{hit}}}{N_{\mathrm{unc}} - N_{\mathrm{hit}}}$. Furthermore, we have the inequalities

$$T_{\mathrm{min}}^{\mathrm{L}} \le \mathbb{E}\{T_{\mathrm{ACQ}}(\pi^*)\} \le \mathbb{E}\left\{T_{\mathrm{ACQ}}(\pi^{N_{\mathrm{hit}}})\right\} \le \left(1 + \frac{2N_{\mathrm{hit}}}{N_{\mathrm{unc}} - N_{\mathrm{hit}}}\right) T_{\mathrm{min}}^{\mathrm{L}}, \tag{13}$$

in which π^* denotes an optimal search order that minimizes the MAT.

Proof: Omitted for brevity (see [5, pp. 47–48]). □

VI. Achieving the Upper Bound

The next theorem proves that the upper bound T_{max} in (11) is achievable and implies that the conventional serial search should be avoided, because it results in the maximum MAT.

Theorem 5 (Maximum MAT):

1) If the receiver uses the CSS π^1 or the FSSS $\pi^{N_{\mathrm{unc}}-1}$, it will result in the maximum MAT.
2) If the number N_{hit} of in-phase cells satisfies $2 \le N_{\mathrm{hit}} \le N_{\mathrm{unc}} - 2$ and the receiver's MAT is equal to T_{max}, then

[4] One can utilize simple bounds on the probabilities and the durations, $0 \le P \le 1$ and $0 \le \tau$, to obtain $\eta(N_{\mathrm{unc}}, N_{\mathrm{hit}})$.

the receiver must have used the CSS π^1 or the FSSS $\pi^{N_{\text{unc}}-1}$.

Proof: Omitted for brevity (see [5, pp. 48–52]). □

In typical scenarios, N_{hit} is in the range $2 \leq N_{\text{hit}} \leq N_{\text{unc}} - 2$. In these scenarios, the receiver exhibits the maximum MAT if and only if it uses the CSS or the FSSS with the step size $N_{\text{unc}} - 1$. Therefore, the receiver can immediately improve the MAT by choosing another search order other than the worst search orders π^1 and $\pi^{N_{\text{unc}}-1}$.

VII. Conclusion

This tutorial summarizes our recent results, providing a methodology for exploiting multipath, typically considered deleterious for efficient communications, to aid the acquisition. We consider a class of serial search strategies and model an acquisition procedure by a flow diagram, containing N_{unc} total cells and N_{hit} in-phase cells.

We present the fundamental limits of achievable MATs. We introduce a notion of η-optimality and show that the fixed-step serial search (FSSS) with the step size N_{hit} is η-optimal. Thus, this search order can be effectively used to achieve a near-optimal MAT in wide bandwidth transmission systems, operating in dense multipath channels. We also presents the search orders that result in the maximum MAT. It turns out that the conventional serial search (CSS) and the FSSS with step size $N_{\text{unc}} - 1$ exhibit the maximum MAT. Our results are valid for all signal-to-noise ratio values, detection layer decision rules, and fading distributions.

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