The Impact of Time-Reversal Modulation on the Performance of Cooperative Relaying Strategies in Wireless Networks

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Abstract*— The predominate traffic patterns in a wireless sensor network are many-to-one and one-to-many communication. Hence, the performance of wireless sensor networks is characterized by the rate at which data can be disseminated from or aggregated to a data sink. In this paper, we consider the data aggregation problem. We demonstrate that a data aggregation rate of $\left(\log n/n\right)$ can be achieved in wireless sensor networks using a generalization of cooperative beamforming called *cooperative time-reversal communication.

I. Introduction

The design and analysis of wireless sensor networks differs from that of more general data communication networks in that the predominate traffic patterns in a sensor network are many-to-one and one-to-many communication. The performance of wireless sensor networks is thus characterized by the rate at which data can be disseminated from or aggregated to a data sink. In [17], we have investigated the broadcast capacity and information dissemination rate of multihop wireless networks. In this paper, we consider the reverse problem, *data aggregation*, which concerns the maximum sustainable rate at which each sensor can transmit data to the sink under a power constraint.

Capacity bounds for the data aggregation problem have been established in [4, 9]. In [9], the capability of large-scale sensor networks to measure and transport a two-dimensional stationary random field using sensors equipped with fixed scalar quantizers was investigated. It was shown that as the density of the sensor nodes increases to infinity, the total number of bits transmitted to the sink in order to represent the field with a given level of fidelity also increases to infinity under any compression scheme. At the same time, the single-receiver transport capacity of the network remains constant as the density increases.

In [4], the more general problem of computing and communicating a symmetric function of the sensor measurements is investigated. It was shown that for a certain class of functions, called *divisible functions*, the maximum rate (or the maximum frequency) at which the function f can be computed and communicated to the sink satisfies $\left(1/\log\left(\left|\mathcal{R}\left(f,n\right)\right|\right)\right)$, where n is the number of sensors in the network and $\mathcal{R}\left(f,n\right)$ is the range of the function f. Since computation of the identity function is equivalent to the transport all raw data, and the identity function is a divisible function with $\mathcal{R}\left(f,n\right)=\left|\mathcal{X}\right|^n$ for some $\left|\mathcal{X}\right|<$, $\left(1/n\right)$ is a tight bound on the achievable throughput for each sensor.

Both of the studies discussed above assume a simplified *protocol model* for the wireless channel. This model does not take into account the time-varying or non-deterministic nature of many channels and makes simplified assumptions regarding link capacity. For example, a Raleigh fading model is often more appropriate for nodes dispersed over a large region in an environment subject to multipath propagation, and link capacity is more accurately modeled as a function of signal-to-interference-and-noise ratio (SINR). Nonetheless, the $O\left(1/n\right)$ upper bound is not at all surprising and, as we demonstrate, remains valid for more complicated channel models. Indeed, the $O\left(1/n\right)$ upper bound reflects the basic observation that for data aggregation in a multihop environment, the traffic load increases for nodes closer to the sink, and the total achievable rate is limited by the maximum rate at which the sink can receive information from its neighbors.

One approach to improving the data aggregation rate in the network is to employ cooperative communication techniques [6, 10, 11, 13, 16]. In this paper, we consider a cooperative transmission strategy based on a technique called *time-reversal communication* (TRC), which can be regarded as generalization of beamforming. Various signal processing techniques based on time-reversal (TR) processing have been proposed and studied previously for many applications, including wireless communication channels [7, 8, 14].

In two recent publications [1, 2], we have studied some aspects of the application of TRC to cooperative communication in wireless networks. In this paper, we investigate the utility of cooperative TRC for improving the achievable data rate in the data aggregation problem. We demonstrate that a rate of $\ (\log n/n)$ is achievable using cooperative TRC and that this rate is in fact order optimal for the data aggregation problem with a single antenna sink in a fading environment.

II. Cooperative Time-Reversal Communication

Cooperative TRC operates as follows. We assume that a cluster of network nodes will cooperate to transmit a common data stream to the sink node. During a training phase, the sink node transmits a short sequence of wide-band training pulses which are received at all of the nodes in the cluster. After transmission of the training sequence, the receiving nodes in the cluster independently perform pulse estimation in order to estimate the exact arrival time, duration, and shape of the received pulse.

After the completion of the training phase, information is transmitted from an arbitrary node in the cluster to the sink using the following cooperative TRC scheme. Whenever a node in the cluster has data to transmit to the sink node, the source node first disseminates the information to all of the other nodes in the cluster. The ensemble of nodes in the cluster then cooperate to transmit the information to the sink node by synchronously transmitting a stream of identical data symbols modulated onto the time-reverse of their respective estimated received waveforms.

The motivation for utilizing TR for communication is based primarily on the optimality property presented in the following lemma, which is a simple consequence of the Cauchy-Schwarz inequality.

Lemma. Let $h(t)$ represent the impulse response of the channel between an arbitrary source node and the sink node as measured at the sink node relative to time $t=0$ at the source node. Let $\tilde{h}(t)=h(\ t)$ represent the time reverse of the impulse response. Then, the instantaneous power output from the sink node at an arbitrary time $t=t_0$ relative to time $t=0$ at the source node subject to an energy constraint is maximized by transmitting an appropriate multiple of the signal $\tilde{h}(t\ \ t_0)$. Furthermore, the received signal $r(t)$ corresponding to the transmission of the signal $s(t)=\tilde{h}(t)$ is given by the autocorrelation function $R_h(t)$ of the channel impulse response; that is,

$$r(t)=R_h(t)=\quad h(\ )h(\ +t)d\ .$$

It follows from this lemma and the principle of superposition that all of the transmitted waveforms from the cooperating cluster will converge at the sink node to produce an impulsive waveform that maximizes the peak power output from the channel at the desired time. Note the following:

1. TRC can be regarded as “matched signaling” for a channel rather than matched filtering.
2. The lemma implies that cooperative TRC can be regarded as a generalization of cooperative or distributed beamforming
3. A side effect of the optimality property presented in the lemma is that the output from a TRC wireless channel tends to be concentrated in both space and time at the receiver.
4. In practice, neither the training pulse shape, the pulse estimation, nor the transmission synchronization will be perfect, and system performance will suffer as a result. The degradation in the performance of cooperative TRC due to pulse estimation and timing errors has been studied previously in [1, 2].

III. Physical-Layer and Network-Layer Models

We consider a wireless communication network $\mathcal{G}_n$ consisting of a group of n nodes, $N=\{1,2,\ldots,n\}$ located on the plane at constant density $\ $. Without loss of generality, we let $\ =1$. Nodes are power limited, i.e., their transmission power cannot exceed $P_{\max}$. For ease of analysis, we assume nodes follow a regular layout on a grid of size $\sqrt{n}\ \ \sqrt{n}$. Though random placement has been considered in the literature in the analysis of transport capacity [5, 17], it has often been demonstrated that random geometric graphs and regular grids exhibit similar asymptotic properties in capacity and energy consumption when transmission power is allowed to vary from node to node [3].

At the physical layer, all wireless links are assumed to be baseband channels corrupted by circularly symmetric, complex-valued additive white Gaussian noise (AWGN) with power spectral density N_0 as well as additive interference from other transmitting nodes in the network, which is also assumed to be Gaussian in the aggregate. However, two different channel propagation models are assumed in our analysis depending on the nature of the communication link being analyzed.

For links in a multihop relay, we adopt a simple propagation model often used for network throughput analysis [12]. In particular, the power on the channel is assumed to decay with distance deterministically at an exponential rate with path-loss exponent $\ > 2$. Hence, the maximum achievable rate (in bits/s) for communication from node i to node j in a multihop relay is given by

$$r_{ij} = B \log\left(1 + SINR_{ij}\right),$$

where B represents the common bandwidth for all links on the network, P_i is the power transmitted by node i, $\ _{ij}$ is the distance between nodes i and j, I is the set of interfering users, and

$$SINR_{ij} = \frac{P_i \ _{ij}}{BN_0 + \ _{k \ I} P_k \ _{ik}}.$$

Similarly, when common information is broadcast from node i to a set of nodes $\mathcal{R}$ in a multihop relay, the maximum achievable rate for the broadcast is given by

$$r_i = \min_{j \ \mathcal{R}} \left\{ B \log\left(1 + SINR_{ij}\right) \right\}.$$

For analysis of cooperative TRC links, all channels are modeled as compound channels consisting of a deterministic path-loss channel with exponent $\ > 2$ cascaded with a Raleigh fading channel. The fading channels are assumed to be independent and identically distributed (i.i.d.) for every distinct pair of nodes (i, j), and the frequency response $H_{ij}(f)$ of the channel between any two such nodes is modeled as a stationary, circularly symmetric, complex-valued Gaussian random process in the frequency domain that remains fixed during each network realization. The fading processes are normalized to have unit power and coherence bandwidth $\ << B$, so that the autocorrelation function $\ _{ij}(f) \ \ (f)$ of the process $H_{ij}(f)$ satisfies $\ (0) = 1$ and $\ (f) = 0$ for all $|f| > \ /2$.

To determine the capacity for an arbitrary cooperative TRC link in our network, we assume that the cooperating cluster consists of $m << n$ nodes contained within a circular region of radius R and that the distance d from the center of the circle to the sink satisfies $d/R >> 1$. Under this assumption, the distance $\ _i$ from an arbitrary node i in the cluster to the sink is approximately the same for all nodes in the cluster, and we can write $\ _i \ d$ for all $i = 1, 2, \ldots, m$. Finally, we assume that all cooperating nodes in a cluster transmit with common power $P \ P_{\max}$ and that while the sink is receiving a cooperative TRC transmission from an arbitrary cluster, the interference at the sink can be modeled as radiating uniformly at power P from all nodes in the network at distance greater than $\ _0$ from the sink, where $\ _0$ is an arbitrary constant.

IV. Results

Lemma 1. Under the physical-layer model discussed above for cooperative TRC, it can be shown that the maximum achievable rate for an arbitrary cooperative TRC link (assuming the information has been previously broadcast to all cooperating nodes) is well modeled as

$$r_{TRC} = B \log \ 1 + \frac{PX(\ \ 2) \ _0^{\ 2} \ (}{BN_0(\ \ 2) \ _0^{\ 2} + 2 \ PK_0)},$$

where

$$K_0 = \frac{1}{B} \ ^{\ /2}_{\ /2} \mathrm{sinc}\left(\tfrac{f}{B}\right) \ ^2(f) df \ \frac{}{B},$$

X is a Gaussian random variable with mean $\ _X$ and variance $\ ^2_X$ given by

$$\ _X = d^{\ } m^2,$$
$$\ ^2_X = K_X d^{\ 2 \ } m^3,$$

and

$$K_X = \frac{8}{B} \ _0^{B/2} \ 1 \ \frac{f}{B} \ 1 \ (\ ^2(f) \ _2F_1(\ 1, \ 1; 1; (\ ^2(f)) df \ 1.$$

Corollary 1. It follows immediately from Lemma 1 that as $m \ $ we have

$$r_{TRC} \ B \log\left(1 + \frac{m^2 P(\ \ 2) \ _0^{\ 2} d^{\ }}{BN_0(\ \ 2) \ _0^{\ 2} + 2 \ PK_0}\right).$$

Lemma 2. Under the physical-layer model discussed above for multihop relay, for any integer $k \ 0$, there exists a time-division-multiple-access (TDMA) scheduling scheme such that one node per square of edge length l can transmit concurrently to nodes located within a radius of k squares (in Manhattan distance) with fixed rate $R(k)$ given by

$$R(k) \ \frac{B}{4(k+1)^2} \log\left(1 + \frac{P_{\max}}{BN_0 \ l(k+1)^{\ } + K_1 P_{\max}}\right),$$

where K_1 is a constant independent of k and l.

Using these lemmas, we establish our main result in the following manner. To maximize the rate of data fusion at the sink, we divide the $\sqrt{n}\quad\sqrt{n}$ grid into three areas as shown in Figure 1. In Areas I and III, data are aggregated using naive multihop relay on balanced trees. In Area II, nodes are organized into $R\quad R$ square clusters. Data are first broadcast among all nodes inside the cluster (*intra-cluster communication*), then all nodes in a cluster cooperatively perform time reversal communication towards the sink (*inter-cluster communication*). Communication in each distinct area is carried out in non-overlapping time slots independently. This allows different communication strategies to be used without interfering with one another. The rate sustainable for each sensor in the network is determined by the minimum of the achievable rates in each area.

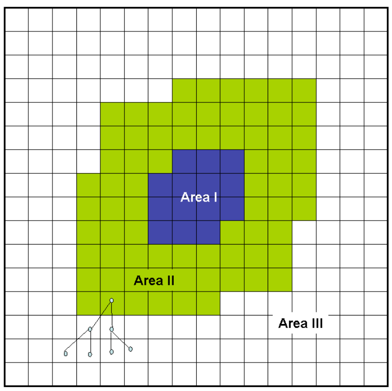

Figure 1. Partition of the network into three distinct Areas

In the following, we let be the rate achievable for each sensor, and we consider the collection of clusters comprising Area II with centers at distance no greater than $\sqrt{2}d$ from the sink. Each cluster is a square with sides of length *R*. The number of clusters in Area II is roughly $M = (2d+R)/R$, and the number of nodes in each cluster is $m = R^2$. Similarly, the number of nodes in Area I is approximately $n_1 = (2d \quad R)^2$ and the number of nodes in Area III is approximately $n_3 = n \quad (2d+R)^2$. Since we are interested in the rate of growth in achievable rate as m, n , we use the result of Corollary 1 to characterize the asymptotic capacity of the TRC links, and we rewrite it using more compact notation as

$$r_{TRC} \quad B\log\left(1+\frac{m^2 d}{BN_0K}\right) = B\log\left(1+\frac{R^4 d}{BN_0K}\right),$$

where K is appropriately defined. Similarly, we rewrite the result of Lemma 2 as

$$R(k) \quad \frac{B}{4(k+1)^2}\log\left(1+\frac{1}{BN_0K}\right),$$

where K is appropriately defined.

Achievable Rate in Areas I and III: In Area III, we construct trees rooted at a border node of a cluster. The trees are well balanced such that the size of each tree is approximately $n \quad (2d+R)^2 / M$. By Lemma 2 with $k = l = 1$, each root node can receive information from its closest neighbor at rate $R(1) = \frac{B}{16}\log\left(1+\frac{1}{BN_0K}\right)$. Therefore, the sustainable rate in Area III satisfies

$$\frac{n \quad (2d+R)^2}{M} \qquad \frac{B}{48}\log\left(1+\frac{1}{BN_0K''}\right). \tag{1}$$

Similarly, in Area I, using naïve multihop data fusion [4, 9], we have

$$(2d \quad R)^2 \qquad \frac{B}{16}\log\left(1+\frac{1}{BN_0K''}\right). \tag{2}$$

Achievable Rate In Area II: From Corollary 1, we know that each cluster of size R^2 can transmit at rate $B\log\left(1+\frac{R^4 d}{BN_0K}\right)$ to a node at distance *d*. Using TDMA to separate the cluster transmissions, the *effective rate* of each cluster is then $r_{inter} = \frac{B}{M}\log\left(1+\frac{R^4 d}{BN_0K'}\right)$. On the other hand, assuming that we separate intra-cluster and inter-cluster communication into different time slots and allow all clusters to perform intra-cluster communication concurrently, the intra-cluster broadcast rate is given by $r_{intra} = \frac{B}{16}\log\left(1+\frac{1}{BN_0K}\right)$, as in Area I. If M , it follows that the total effective achievable rate for each cluster is given by

$$\frac{r_{intra}r_{inter}}{r_{intra}+r_{inter}} \quad r_{inter} = \frac{B}{M}\log\left(1+\frac{R^4 d}{BN_0K'}\right).$$

Now, since the nodes in Area II must transport all of the traffic from Area III to the sink, the amount of traffic that each cluster must carry is given by $\frac{}{M}\left(n \quad (2d \quad R)^2\right)$. If we let $d = n$ and $R = n$ with $0 < \quad < \quad < \frac{1}{2}$, then the sustainable rate in Area II satisfies

$$\frac{}{M}\; n \;\; \left(2d \quad R\right)^2 (\;)\; \frac{}{M}\left(n \quad 4d^2\right))\; \frac{n}{M} \qquad \frac{B}{M}\log\;\; 1+\frac{R^4 d^{\;\;+}}{BN_0K'}\Big(,$$

or equivalently that

$$\frac{B}{n}\log\;\; 1+\frac{R^4 d}{BN_0K'}\Big(\;. \qquad (3)$$

Achievable Rate for the Network: Comparing Equations (1)-(3), we see that the entire network can sustain a rate of $\frac{B}{n}\log\;\; 1+\frac{R^4 d}{BN_0K'}\Big)\Big($. Furthermore, if <4 and we choose $0< \;\; < \;\; < \frac{4}{}$ with $< \frac{1}{2}$, then we can achieve a sustainable rate of $\frac{\log n}{n}$ bit/s per node, as claimed.

V. Conclusion

We have established that a sustainable data aggregation rate of $\left(\log n/n\right)$ can be achieved using cooperative TRC in a wireless sensor network in situations where the path loss exponent satisfies $2< \;\; <4$. It is not as yet clear whether this rate can also be achieved for >4 using a different protocol. What is clear is that the rate $\left(\log n/n\right)$ is in fact order-optimal for data aggregation. This follows immediately from the "genie-aided" upper bound that can be derived from results in [15] where it is shown that the capacity of a multiple-input, single-output fading channel with *n* transmitting antennas, approaches $B\log\left(1+nP/BN_0\right)$ for large *n*. Hence, if we could somehow achieve an infinite broadcast rate for exchange of information between all nodes in the network excluding the sink, and if all nodes remained at constant distance from the sink as the number of nodes in the network grew to infinity, the achievable data aggregation rate for the network would still be $O\left(\log n/n\right)$ bits/s per node. Since we have demonstrated that $\left(\log n/n\right)$ is achievable, the data aggregation rate is thus $\left(\log n/n\right)$.

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