Multiuser detection in a dynamic environment

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***Abstract*— In mobile multiple-access communications, not only the location of active users, but also their number varies with time. In typical analyses, multiuser detection theory is developed under the assumption that the number of active users is constant and known at the receiver, and coincides with the maximum number of users entitled to access the system. This assumption is often overly pessimistic, since many users might be inactive at any given time, and detection under the assumption of a number of users larger than the real one may impair performance.**

This paper undertakes a different, more general approach to the problem of identifying active users and estimating their parameters and data in a dynamic environment where users are continuously entering and leaving the system. Using a mathematical tool known as Random Set Theory, we derive Bayesian-filter equations which describe the evolution with time of the a posteriori probability density of the unknown user parameters, and use this density to derive optimum detectors.

I. Introduction

In a typical mobile multiple-access communications scenario, the number of active users, their location, as well as the parameters that characterize their channel state, vary with time. Thus, techniques aimed at identifying not only the transmitted data, but also the users parameters, play a central role in analysis and design of wireless transmission systems. Identification of active users, and estimation of their parameters, can be done by properly training a receiver, which may use a proper known sequence transmitted within a frame of useful data. Now, this training phase may be made more efficient if one can account for dynamic models of the number of active users and of their parameters. In fact, accounting for the past history of the parameters may bring a considerable amount of extra information if their changes are not overly abrupt (for example, if the number of active users does not change considerably from frame to frame). This paper deals with this situation: focusing for simplicity on MUD, we lay the foundation of a theory in which the number and the parameters of active users, assumed to be unknown at the receiver, may change from one observation time to the next following a known dynamic model.

Here we derive Bayesian-filter equations which describe the evolution with time of the a posteriori probability density of the unknown user parameters and data. The mathematical tool we apply is *Random Set Theory* (RST: see the Appendix and the references therein). This tool, often applied in the context of multitarget tracking and identification (see, e.g., [3], [5]–[7], [11], [12]) is based on a probability theory of finite sets that exhibit randomness not only in each element, but also in the number of elements. Thus, since the active users and their parameters can be thought of as elements of a finite random set, RST provides a fairly natural approach to multiuser detection in a dynamic environment, as it unifies in a single step two steps that would be taken separately without it: viz., detection of active users and estimation of their parameters. A motivation for the development presented in this paper can be obtained by glancing over Fig. 1. The results in this figure show how, whenever the probability that a given user is active is significantly lower than 1, the relaxation of the assumption that the number of active users is known can provide considerable performance improvement. System performance can be further improved if the receiver is able to exploit additional information about the behavior of the users, i.e., a model for the appearance, disappearance, and movement of the users.

In this paper we restrict ourselves to interferer identification and data detection, while in a paper in preparation [2] we examine the problem of estimating users' parameters. This paper is organized as follows. Section II describes the channel model, while Section III states the detection problem in the context of RST. Section IV describes an application of the theory to CDMA, while Section V provides some numerical results illustrating the theory.

II. Channel model and statement of the problem

We assume $K+1$ users transmitting digital data over a common channel. Let $s(\mathbf{x}_t^{(0)})$ denote the signal transmitted by the reference user at discrete time t, $t = 1, 2, \ldots$, and $s(\mathbf{x}_t^{(i)})$, $i = 1, \ldots, K$, the signals that may be transmitted at the same time by K interferers. Each signal has in it a number of known parameters, reflected by the deterministic function $s(\,\cdot\,)$, and a number of random parameters, summarized by $\mathbf{x}_t^{(i)}$. The index i reflects the identity of the user, and is typically associated with its signature. The observed signal at time t is a sum of $s(\mathbf{x}_t^{(0)})$, of the signals generated by the users active at time t, which are in a random number, and of stationary random noise $\mathbf{z}_t$. We write

$$\mathbf{y}_t = s(\mathbf{x}_t^{(0)}) + \sum_{\mathbf{x}_t^{(i)} \in \mathbf{X}_t} s(\mathbf{x}_t^{(i)}) + \mathbf{z}_t \tag{1}$$

where $\mathbf{X}_t$ is random set, encapsulating what is unknown about the active users. With this notation we implicitly assume that user 0 is active with probability 1 and its parameters (but not its data) are known (this restriction can be easily removed).

Using RST, the whole set of interferers is modeled as a single entity. Roughly speaking, a random set is a map $\mathbf{X}$ between a sample space and a family of subsets of a space $\mathbb{S}$. This is the space of the unknown data and parameters of the active interferers. For example, we have $\mathbb{S} = \{1, \dots, K\}$ if all parameters of the interferers are known, except their number and their identity. Or, we have $\mathbb{S} = \{1, \dots, K\} \times \{\pm 1\}$ if the users' (binary antipodal) data are unknown, and we want to detect them. We may also have $\mathbb{S} = \mathbb{R} \times \{1, \dots, K\}$, $\mathbb{R}$ the set of real numbers, if one parameter (e.g., the interferer power) is also unknown in addition to the interferers' number and identities, while the transmitted data are known (for example, in a training phase). In mathematical terms, $\mathbb{S}$ is generally a *hybrid space* $\mathbb{S} \triangleq \mathbb{R}^d \times U$, with U a finite discrete set, and d the number of parameters to be estimated for each user. In the remainder of this paper we shall mainly restrict ourselves to the case $d = 0$, and leave to a companion paper [2] the discussion of the case $d \neq 0$.

With our channel model, the receiver detects only a superposition of interfering signals. Thus, the random set describing the receiver, denoted $\mathbf{Y}_t$, is the singleton $\{\mathbf{y}_t\}$, where $\mathbf{y}_t$ has conditional probability density function

$$f_{\mathbf{Y}_t|\mathbf{X}_t}(\mathbf{y}_t \mid \mathbf{B}) = f_{\mathbf{z}}(\mathbf{y}_t - \sigma(\mathbf{B})) \tag{2}$$

where $\mathbf{B} = \{\mathbf{b}_1, \dots, \mathbf{b}_k\}$ is a realization of $\mathbf{X}_t$, that is, a realization of a random set of users and their parameters/data. Moreover, $f_{\mathbf{z}}(\,\cdot\,)$ is probability density function (pdf) of the additive noise, and

$$\sigma(\mathbf{B}) \triangleq \sum_{\mathbf{b}_i \in \mathbf{B}} s(\mathbf{b}_i) \tag{3}$$

A. Defining estimators

Development of estimators with our model must take into account the peculiarities of RST. In particular, expectations cannot be defined, because there is no notion of set addition, and hence estimators based on a posteriori expectations do not exist (this point is discussed thoroughly and eloquently in [5]). A possible estimator maximizes the a posteriori probability (APP) of $\mathbf{X}_t$ given $\mathbf{y}_{1:T}$, the latter denoting the whole set of observations corresponding to a data frame transmitted from $t = 1$ to $t = T$. Another possibility is to restrict oneself to a *causal* estimator, which searches for the maximum probability of $\mathbf{X}_t$ given $\mathbf{y}_{1:t}$. In a delay-constrained system, one may estimate $\mathbf{X}_t$ on the basis of the observations $\mathbf{y}_{t-\Delta:t+\Delta}$, with Δ a fixed interval duration (sliding-window estimator).

B. Consideration of a dynamic environment

Since $\{\mathbf{X}_t\}_{t=1}^{\infty}$ forms a random set sequence, the calculation of $\widehat{\mathbf{X}}_t$ is needed for all discrete time instants t. If a dynamic model of the transmission system is available (which is what we assume in this paper), then the APPs can be updated recursively, thus allowing one to take advantage of the information gathered from the past evolution of the system. We observe here that the concept of an adaptive receiver was examined previously by several authors (see, e.g., [9] and references therein), while the effects on analysis of a dynamic model were touched upon, among others, by the authors of [4], [8].

We make the assumption that $\{\mathbf{X}_t\}_{t=1}^{\infty}$ forms a random set sequence with the Markov property, i.e., such that $\mathbf{X}_t$ depends on its past only through $\mathbf{X}_{t-1}$. This allows us to use *Bayesian-filter* recursions for $\widehat{\mathbf{X}}_t$ [6]:

$$\begin{aligned} f_{\mathbf{X}_{t+1}|\mathbf{Y}_{1:t}}&(\mathbf{B} \mid \mathbf{y}_{1:t}) = \\ &= \int f_{\mathbf{X}_{t+1}|\mathbf{X}_t}(\mathbf{B} \mid \mathbf{C}) f_{\mathbf{X}_t|\mathbf{Y}_{1:t}}(\mathbf{C} \mid \mathbf{y}_{1:t})\, \delta\mathbf{C} \end{aligned} \tag{4}$$

$$\begin{aligned} f_{\mathbf{X}_{t+1}|\mathbf{Y}_{1:t+1}}&(\mathbf{B} \mid \mathbf{y}_{1:t+1}) \\ &\propto f_{\mathbf{Y}_{t+1}|\mathbf{X}_{t+1}}(\mathbf{y}_{t+1} \mid \mathbf{B}) f_{\mathbf{X}_{t+1}|\mathbf{Y}_{1:t}}(\mathbf{B} \mid \mathbf{y}_{1:t}) \end{aligned} \tag{5}$$

The integrals appearing in the equations are *set integrals*, defined in the sense of RST (see the Appendix). The notation δ for the differential reflects this definition.

Assuming from now on real signals, and the noise to be Gaussian with mean 0 and known variance $N_0/2$, we have

$$f_{\mathbf{z}}(\mathbf{y}_t \mid \mathbf{X}_t) \propto \exp\{-(\mathbf{y}_t - \sigma(\mathbf{X}_t))^2/N_0\}$$

Thus, the causal maximum-a-posteriori estimate of $\mathbf{X}_t$ is obtained by maximizing, over $\mathbf{B}$, the APP $f_{\mathbf{X}_t|\mathbf{Y}_{1:t}}(\mathbf{B} \mid \mathbf{y}_{1:t})$, which is tantamount to minimizing

$$m(\mathbf{B}) \triangleq (\mathbf{y}_t - \sigma(\mathbf{B}))^2 - \varepsilon(\mathbf{B})$$

where $\varepsilon(\mathbf{B}) \triangleq N_0 \ln f_{\mathbf{X}_t|\mathbf{Y}_{1:t-1}}(\mathbf{B} \mid \mathbf{y}_{1:t-1})$. The first term in the RHS of definition above is the Euclidean distance between the observation and the sum of the interfering signals at time t. Its minimization yields the maximum-likelihood (ML) estimate of $\mathbf{X}_t$. The second term in the RHS, generated by the uppermost step of iterations, reflects the influence on $\mathbf{X}_t$ of its past history, and its consideration yields maximum-a-posteriori (MAP) estimation. Notice how this term becomes less and less relevant as $N_0 \to 0$.

Application of the Bayesian-filter recursions requires the determination of the dynamic model of the process $\mathbf{X}_t$, described by the function $f_{\mathbf{X}_{t+1}|\mathbf{X}_t}(\,\cdot \mid \cdot\,)$ that models the time evolution of data and parameters of the system. Examples of this modeling procedure are available for the problem of tracking multiple targets [6], [11].

From now on we restrict ourselves to the detection of the number and identity of active interferers, and of the data they carry, under the assumption that the remaining parameters, which were previously estimated by the receiver in a training phase, do not change in any appreciable way during the tracking phase. Estimation of these parameters using RST is described in the companion paper [2].

III. Detection

A. Active users

We assume first that we are only interested in detecting which interferers, out of a universe of K potential system users, are present at time t. This information may be used for example to do decorrelation detection, under the assumption that the signatures of all users are known at the receiver. In our theory, this situation corresponds to choosing as $\mathbb{S}$ the set $\mathbb{K} \triangleq \{1, \ldots, K\}$. Thus, $\mathbf{X}_t$ takes values in the power set of $\mathbb{K}$, denoted $2^{\mathbb{K}}$. Since this is finite, a probability measure for $\mathbf{X}_t$ can be defined by assigning probabilities $\mathbb{P}(\mathbf{A})$, $\mathbf{A} \in 2^{\mathbb{K}}$.

1) Static model: At any fixed time t, suppose that the probability of interferer $\mathbf{x}_t^{(i)}$ to be active is α, independent of t and i. In this case the probability of the interferer set $\mathbf{X}_t$ depends only on its cardinality $|\mathbf{X}_t|$, and we can write

$$f_{\mathbf{X}_t}(\mathbf{B}) = \alpha^{|\mathbf{B}|}(1-\alpha)^{K-|\mathbf{B}|} \tag{6}$$

2) Dynamic model: Consider now the evolution of $\mathbf{X}_t$. We assume that from $t-1$ to t some new users become active, while some old users become inactive. We write

$$\mathbf{X}_t = \mathbf{S}_t \cup \mathbf{N}_t \tag{7}$$

where $\mathbf{S}_t$ is the set of *surviving* users still active from $t-1$, and $\mathbf{N}_t$ is the set of *new* users becoming active at t. The condition $\mathbf{N}_t \cap \mathbf{X}_{t-1} = \emptyset$ is forced, because a user ceasing transmission at time $t-1$ cannot reënter the set of active users at time t. We proceed by constructing separate dynamic models for $\mathbf{S}_t$ and $\mathbf{N}_t$, which will be eventually combined to yield a model for $\mathbf{X}_t$.

Consider first $\mathbf{S}_t$. Suppose that there are k active users at $t-1$, the elements of the random set $\mathbf{X}_{t-1} = \{\mathbf{x}_{t-1}^{(1)}, \ldots, \mathbf{x}_{t-1}^{(k)}\}$. Then we may write, for the set of surviving users,

$$\mathbf{S}_t = \bigcup_{i=1}^{k} \mathbf{X}_t^{(i)} \tag{8}$$

where $\mathbf{X}_t^{(i)}$ denotes either an empty set (if user i has become inactive) or the singleton $\{\mathbf{x}_t^{(i)}\}$ (user i is still active). Let μ denote the "persistence" probability, i.e., the probability that a user survives from $t-1$ to t. We obtain, for the conditional probability of $\mathbf{S}_t$ given that $\mathbf{X}_{t-1} = \mathbf{B}$:

$$f_{\mathbf{S}_t|\mathbf{X}_{t-1}}(\mathbf{C} \mid \mathbf{B}) = \begin{cases} \mu^{|\mathbf{C}|}(1-\mu)^{|\mathbf{B}|-|\mathbf{C}|}, & \mathbf{C} \subseteq \mathbf{B} \\ 0, & \mathbf{C} \nsubseteq \mathbf{B} \end{cases} \tag{9}$$

For new users, a reasonable model is

$$f_{\mathbf{N}_t|\mathbf{X}_{t-1}}(\mathbf{C} \mid \mathbf{B}) = \begin{cases} \alpha^{|\mathbf{C}|}(1-\alpha)^{K-|\mathbf{B}|-|\mathbf{C}|}, & \mathbf{C} \cap \mathbf{B} = \emptyset \\ 0, & \mathbf{C} \cap \mathbf{B} \neq \emptyset \end{cases} \tag{10}$$

Finally, by assuming that births and deaths of users are conditionally independent given $\mathbf{X}_{t-1} = \mathbf{B}$, the pdf of the union of the independent random sets $\mathbf{S}_t$ and $\mathbf{N}_t$ is obtained from the *generalized convolution* [3]

$$\begin{aligned} & f_{\mathbf{X}_t|\mathbf{X}_{t-1}}(\mathbf{C} \mid \mathbf{B}) \\ &= \sum_{\mathbf{W} \subseteq \mathbf{C}} f_{\mathbf{S}_t|\mathbf{X}_{t-1}}(\mathbf{W} \mid \mathbf{B})\, f_{\mathbf{N}_t|\mathbf{X}_{t-1}}(\mathbf{C} \setminus \mathbf{W} \mid \mathbf{B}) \\ &= f_{\mathbf{S}_t|\mathbf{X}_{t-1}}(\mathbf{C} \cap \mathbf{B}) f_{\mathbf{N}_t|\mathbf{X}_{t-1}}(\mathbf{C} \setminus (\mathbf{C} \cap \mathbf{B})) \end{aligned}$$

3) Bayes' recursions: In our context, Bayes' recursions (4)-(5) go as follows:

$$\begin{aligned} f(\mathbf{X}_{t+1} \mid \mathbf{y}_{1:t}) &= \sum_{\mathbf{X}_t \in 2^{\mathbb{K}}} f(\mathbf{X}_{t+1} \mid \mathbf{X}_t)\, f(\mathbf{X}_t \mid \mathbf{y}_{1:t}) \\ f(\mathbf{X}_{t+1} \mid \mathbf{y}_{1:t+1}) &\propto f_{\mathbf{z}}(\mathbf{y}_{t+1} - \sigma(\mathbf{X}_{t+1}))\, f(\mathbf{X}_{t+1} \mid \mathbf{y}_{1:t}) \end{aligned}$$

B. Active users and their data

Under the assumption of binary antipodal data, independent from time to time and across users, this case corresponds to having $\mathbb{S} = \mathbb{K} \times \{\pm 1\}^N$, where N denotes the length of the data frame transmitted by each user in a discrete-time unit. In this case (6) becomes

$$f_{\mathbf{X}_t}(\mathbf{B}) = 2^{-N|\mathbf{B}|}\alpha^{|\mathbf{B}|}(1-\alpha)^{K-|\mathbf{B}|} \tag{11}$$

where the new factor $2^{-N|\mathbf{B}|}$ accounts for the fact that there are $N|\mathbf{B}|$ equally likely binary symbols transmitted at time t by $|\mathbf{B}|$ interferers.

Similarly, (9) is transformed into

$$f_{\mathbf{S}_t|\mathbf{X}_{t-1}}(\mathbf{C} \mid \mathbf{B}) = \begin{cases} 2^{-N|\mathbf{C}|}\mu^{|\mathbf{C}|}(1-\mu)^{|\mathbf{B}|-|\mathbf{C}|}, & \mathbf{C} \subseteq \mathbf{B} \\ 0, & \mathbf{C} \nsubseteq \mathbf{B} \end{cases} \tag{12}$$

and (10) into

$$f_{\mathbf{N}_t|\mathbf{X}_{t-1}}(\mathbf{C} \mid \mathbf{B}) = \begin{cases} 2^{-N|\mathbf{C}|}\alpha^{|\mathbf{C}|}(1-\alpha)^{K-|\mathbf{B}|-|\mathbf{C}|}, & \mathbf{C} \cap \mathbf{B} = \emptyset \\ 0, & \mathbf{C} \cap \mathbf{B} \neq \emptyset \end{cases} \tag{13}$$

C. Possible scenarios

We recall that throughout this paper we assume that the only unknown signal quantities may be the identities of the users and their data. Specifically, we may distinguish four cases in our context:

① *Static channel, unknown identities, known data.* This corresponds to a training phase intended at identifying users, and assumes that the user identities do not change during transmission. In this case we write $\mathbf{X}$ in lieu of $\mathbf{X}_t$.

② *Static channel, unknown identities, unknown data.* This may correspond to a tracking phase following ① above. We write again $\mathbf{X}$ in lieu of $\mathbf{X}_t$, and assume that $\mathbf{X}$ contains the whole transmitted data sequence.

③ *Dynamic channel, unknown identities, known data.* This corresponds to identification of users preliminary to data detection (which, for example, may be based on decorrelation).

④ *Dynamic channel, unknown identities, unknown data.* This corresponds to simultaneous user identification and data detection in a time-varying environment.

IV. An Example of Application

Assume now the specific situation of a DS-CDMA system with signature sequences of length L and additive white Gaussian noise. At discrete time t, we may write, for the sufficient statistics of the received signal,

$$\mathbf{y}_t = \mathbf{R}\mathbf{A}\mathbf{b}_t(\mathbf{X}_t) + \mathbf{z}_t, \qquad t = 1, \ldots, T \tag{14}$$

where $\mathbf{X}_t$ is now the random set of all active users, $\mathbf{R}$ is the $L \times L$ correlation matrix of the signature sequences (assumed to have unit norm), $\mathbf{A}$ is the diagonal matrix of the users' signal amplitudes, the vector $\mathbf{b}_t(\mathbf{X}_t)$ has nonzero entries in the locations corresponding to the active-user identities described by the components of $\{\mathbf{X}_t\}$, and $\mathbf{z}_t \sim \mathcal{N}(0, (N_0/2)\mathbf{R})$ is the noise vector, with $N_0/2$ the power spectral density of the received noise. We further assume that, at every discrete time instant, only one binary antipodal symbol is transmitted.

A. Static channel

The a posteriori probability of $\mathbf{X}$, given the whole received sequence, is

$$f(\mathbf{X} \mid \mathbf{y}_1, \ldots, \mathbf{y}_T) \propto f_{\mathbf{X}}(\mathbf{X})\, f(\mathbf{y}_1, \ldots, \mathbf{y}_T \mid \mathbf{X}) \qquad (15)$$

Thus, the MAP estimator of users' identities is

$$\widehat{\mathbf{X}} = \arg \max_{\mathbf{X} \in 2^{\mathbb{K}}} f(\mathbf{X} \mid \mathbf{y}_{1:T}) \qquad (16)$$

where, as usual, $\mathbf{y}_{1:T} \triangleq \mathbf{y}_1, \ldots, \mathbf{y}_T$. The MAP estimator of users' identities and data has the same form of (16), where now $\mathbf{X}$ takes values in a set including all possible combinations of users and their data.

The expression above can be rewritten in such a way that the presence of the sequence of transmitted data is made more explicit. Specifically, we write, in lieu of $\mathbf{X}$, the sequence $(\mathbf{X}, \mathbf{b}_1(\mathbf{X}, \ldots, \mathbf{b}_T(\mathbf{X}))$. Doing so, we may express the MAP estimator of users' identities and data in the more explicit form

$$\begin{aligned}&(\widehat{\mathbf{X}}, \widehat{\mathbf{b}}_1(\mathbf{X}), \ldots, \widehat{\mathbf{b}}_T(\mathbf{X}))\\ &\quad = \quad \arg\max f(\mathbf{X}, \mathbf{b}_1(\mathbf{X}), \ldots, \mathbf{b}_T(\mathbf{X}) \mid \mathbf{y}_{1:T})\end{aligned}$$

where the maximum has to be taken with respect to $\mathbf{X} \in 2^{\mathbb{K}}$ and $\mathbf{b}_t(\mathbf{X}) \in \{\pm 1\}^{|\mathbf{X}|} \times \{0\}^{K-|\mathbf{X}|}$. The introduction of this "fine-grain" notation for the random set $\mathbf{X}$ suggests that the MAP detector may be implemented in the form of a sequential detector, thus simplifying its operation (more on this *infra*).

B. Dynamic channel

Consider now a dynamic channel, and examine first the case of known data. We have, accounting for the Markov property of our channel model,

$$\begin{aligned}&f(\mathbf{X}_1, \ldots, \mathbf{X}_T \mid \mathbf{y}_{1:T}) \propto f(\mathbf{y}_{1:P} \mid \mathbf{X}_1, \ldots, \mathbf{X}_T)\\ \times &\prod_{t=1}^{T} f(\mathbf{X}_t \mid \mathbf{X}_{t-1}) \times f(\mathbf{y}_{1:T} \mid \mathbf{X}_1, \ldots, \mathbf{X}_T)\end{aligned}$$

The MAP estimator here maximizes the RHS of the above (or its logarithm) with respect to the values taken on by the sequence $(\mathbf{X}_1, \ldots, \mathbf{X}_T)$. Even in this case we may think of a sequential detector, which searches for the maximum-APP path traversing a trellis with T stages and 2^K states, each of the latter being associated to a realization of the random set $\mathbf{X}_i$, $i = 1, \ldots, T$.

The case of unknown data can be dealt with likewise, resulting in sequential detector operating on a trellis with an augmented number of states.

a) Implementing a sequential detector.: Implementation of the sequential detector through a version of Viterbi algorithm leads to the following consequences:

① The decision on the whole sequence of users' identities and their data should be taken only after the whole sequence of observations $\mathbf{y}_1, \ldots, \mathbf{y}_T$ has been recorded.
② The decision on the users' identities and their data at time t depends not only on the past observations, but also on observations that have not been recorded yet at time t.
③ A suboptimum version of the optimum sequential algorithm, the *sliding-window Viterbi* algorithm (see, e.g., [1, p. 133 ff.]) can be implemented. This consists of forcing a decision on $\mathbf{X}_t$, $\mathbf{b}_t(\mathbf{X}_t)$ based on a sliding window of observations that includes $\mathbf{y}_t$, but whose length is $D < T$.

V. Numerical results

In this section we show some numerical examples that illustrate the theory developed in the previous sections.

Fig. 1 compares, by computer simulation, "classic" ML multiuser detection [10], which assumes that all users are simultaneously active, and ML detection based on RST, which detects simultaneously the number of active users and the data of the reference user. The ordinate shows the bit error probability of the reference user in a multiuser system with 3 users transmitting binary antipodal signals, different active-user probabilities ($\alpha = 0.1$, 0.5, and 1), spreading-sequences consisting of Kasami sequences with length 15, and perfect power control (and hence a scalar matrix $\mathbf{A}$). The Gaussian channel is static. The single-user bound is also shown as a reference. It is seen that RST provides a detector much more

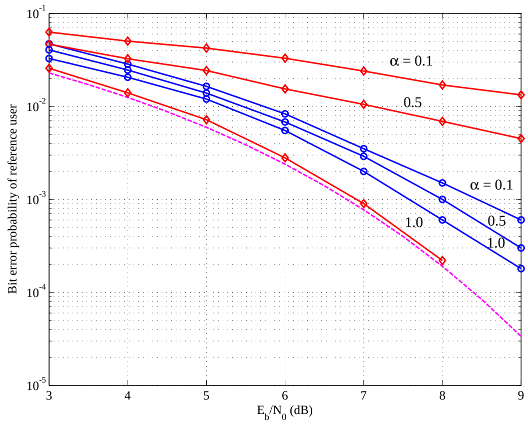

Fig. 1. *Bit error probability of the reference user in a multiuser system with 3 users, independently active with probability α. Lines with diamond markers: Classic multiuser ML detection, assuming that all users are active. Line with circle markers: ML detection using RST. Dashed curve: Single-user bound.*

robust that classic MUD to variations in the users activity factor. We also observe that classic MUD can outperform RST for hich values of α, as this situation corresponds to having side information about the number of active users.

Fig. 2 refers to a dynamic channel. Now $K = 3$ and $L = 7$; the data-frame length is $T = 10$. Here we compare two receivers, one based on a Viterbi algorithm and one based on Bayes recursions for estimating the set of interferers and the transmitted bits. The ordinate shows the bit-sequence error probability at time $t = 1$ and at time $t = T$. This is defined as the probability that the estimated and the true set do not coincide: notice that a wrong estimate of the identities of the active users implies a wrong estimate of the data stream, while the converse is not true.

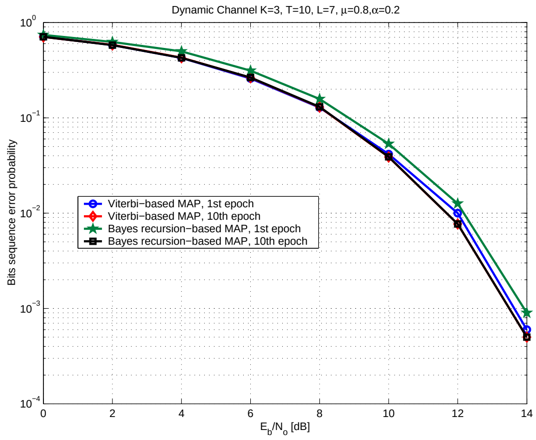

Fig. 2. Bit-sequence error probability in a dynamic environment with $K = 3$, $T = 10$, $\mu = .8$, and $\alpha = .2$.

VI. Conclusions

We have described a technique for estimating the received-signal parameters and data in a multiuser transmission system. Since the number of active interferers is itself a random variable, the set of parameters to be estimated has a random number of random elements. A dynamic model for the evolution of this random set accounts with new interferers appearing and old interferers disappearing in each measurement interval. A multiuser detection scheme in this context can be developed using random-set theory. This allows us to develop Bayes-filtering equations describing the evolution of the MAP multiuser detector in a dynamic environment.

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Appendix

This appendix describes, in a rather qualitative fashion, the fundamentals of Random-Set Theory. For a rigorous approach and additional details, see [3], [11], [12] and the references therein.

Given a sample space Ω (the space of all the outcomes of a random experiment), a probability measure can be defined on it. If a random variable (i.e., a mapping from Ω to another space $\mathbb{S}$) is defined, it is convenient to generate a probability measure directly on $\mathbb{S}$. This can be given in terms of a density function, once certain mathematical operations, such as integration, are defined on $\mathbb{S}$. Random sets can be viewed as a generalization of the concept of a random variable. A *finite random set* is a mapping $\mathbf{X} : \Omega \to \mathcal{F}(\mathbb{S})$ from the sample space Ω to the collection of closed sets of the space $\mathbb{S}$, with $|\mathbf{X}(\omega)| < \infty$ for all $\omega \in \Omega$. For our purposes, the space $\mathbb{S}$ of finite random sets is assumed to be the *hybrid space* $\mathbb{S} = \mathbb{R}^d \times U$, the direct product of the d-dimensional Euclidean space $\mathbb{R}^d$ and a finite discrete space U. The elements of $\mathbb{S}$ characterize the users' parameters, some of which continuous (d real numbers) and some discrete (for example, the users' signatures and their transmitted data).

A. Belief mass functions

A natural probability law for $\mathbf{X}$ is the probability distribution $P_{\mathbf{X}}$, defined for any (Borel) subset $\mathcal{T}$ of $\mathcal{F}(\mathbb{S})$ by $P_{\mathbf{X}}(\mathcal{T}) \triangleq \mathbb{P}(\mathbf{X} \in \mathcal{T})$. However, RST is based on a probability law given differently. Specifically, the *belief mass function* of a finite random set $\mathbf{X}$ is defined as $\beta_{\mathbf{X}}(\mathbf{C}) \triangleq \mathbb{P}(\mathbf{X} \subseteq \mathbf{C})$, where $\mathbf{C}$ is a closed subset of $\mathbb{S}$. The belief function characterizes the probability distribution of a random finite set $\mathbf{X}$, and allows the construction of a density function of $\mathbf{X}$ through the definition of a *set integral* and a *set derivative*, which, through a *generalized fundamental theorem of calculus*, turn out to be the inverse of each other. Thus, belief functions and belief densities can be derived from one another.