Throughput Scaling in Random Wireless Networks

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***Abstract*— We propose and analyze two models of networks in which pairs of nodes communicate over a shared wireless medium. We are interested in the maximum total aggregate traffic flow that is possible through the network. Our first model differs substantially from most existing models in that the channel connections in our network are entirely random: we assume that, rather than being governed by geometry and a decay law, the strength of the connections between nodes is drawn independently from a common distribution. The next model is more general and works at two scales. At a local scale, characterized by nodes being within a distance r from each other, connections are drawn independently from some distribution, but at a global scale, characterized by nodes being further apart from each other than a distance r, channel connections are governed by a Rayleigh distribution, with the power satisfying a distance-based decay law.**

For both models we show that an appropriate distribution for the channel strengths and other parameters can give a throughput that scales almost linearly in the number of nodes of the network. This is a significant improvement over the square-root scaling that has been shown in several previous works.

I. Introduction

Sensor and ad hoc networks have seen much research activity in recent times. An early major result of the field was by Kumar and Gupta [9] where a network of n nodes was studied. Strengths of the connections between two nodes were determined entirely by the distance between them and followed a deterministic power scaling law. With this model, it was shown that a throughput that scaled like $\sqrt{n}$ was the best possible. This implied that the throughput per user fell like $\frac{1}{\sqrt{n}}$ which was a rather discouraging result. Except when nodes were allowed to approach each other [5], similar scaling laws were shown to hold [1], [8], [4], [7], [12], [10] for similar random and deterministic models.

From the study of multi-antenna links [3], [11], it is now generally believed that a rich scattering environment, once thought to be detrimental to wireless communications, may actually be beneficial. We show that a similar effect may hold for the expected aggregate data traffic in a wireless network.

Random models may be preferred over distance-based ones since decay laws of the form $1/r^{\alpha}$ are usually valid in far-field approximations and may not hold for networks of small physical size that are designed with minimum and maximum distances in mind. Additionally, automatic gain control can mitigate many distance effects. Thus, important signal-strength effects are often due to random fluctuations in the medium. For example, [6], [17] start out with very different models and show that in the presence of obstructions and unreliable channels, the probability of good links between nodes that are far apart increases. Therefore we propose a model in which channel connections are independently and identically distributed (i.i.d.) according to some probability distribution function (pdf). This was first proposed in [14], [15].

While the throughput that is possible with this model depends very strongly on the distribution that the channel strengths are drawn from, several distributions, including the Bernoulli and some heavy-tailed distributions led to throughputs that were almost linear in n. Thus the introduction of randomness changes the behavior of the system significantly.

In practice, we expect neither the deterministic model of [9] nor the random model of [15] to hold. A combination of distance-dependent connections and random connections would perhaps make for a better model. Therefore we also propose and analyze such a model. In the two-scale model we assume that nodes are randomly and uniformly distributed on a sphere of radius R. At the local scale, nodes that are within a distance r from each other are connected by channels that are distance-independent. These channel strengths are assumed to be drawn i.i.d. from a distribution, $f(\cdot)$. At a global scale, for nodes that are further apart than r, the channel connections obey a Rayleigh distribution with a mean power that depends on the distance between them and follows a distance-decay law, say $g(\cdot)$.

Such a model incorporates the far field effects at a global level through the decay law, but also recognizes that obstuctions play a role at a local scale. Furthermore, appropriate choices of r and R can help model a full spectrum of networks, from the purely geometric ones of [9] to the purely random ones of [15]. Not surprisingly, a combination of the techniques found in [9] and [15] are employed to study this model.

II. Random Network Model

Consider a network with n nodes labeled $1, \ldots, n$. Every pair of nodes $\{i, j\}$ $(i \neq j)$ is connected by a channel, denoted by the random variable $h_{i,j} = h_{j,i}$. We assume that the channel strengths, $\gamma_{i,j} = |h_{i,j}|^2$ are drawn i.i.d. according to some probability density function (pdf) $f_n(\gamma)$. Once drawn, these channel variables do not change with time.

Node i wishes to transmit signal x_i. We assume that x_i is a complex Gaussian random variable with zero mean and unit variance. Each node is permitted a maximum power of P watts.

We incorporate interference and additive noise in our model as follows. Assume that k nodes $i_1, i_2, \ldots, i_k$ are simultane-

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ously transmitting signals $x_{i_1}, x_{i_2}, \ldots, x_{i_k}$ respectively. Then, the signal received by node $j(\neq i_1, \ldots, i_k)$ is given by

$$y_j = \sum_{t=1}^{k} \sqrt{P} h_{i_t,j} x_{i_t} + w_j \quad (1)$$

where w_j represents additive noise. The additive noise variables $w_1, \ldots, w_n$ are i.i.d., drawn from a complex Gaussian distribution of zero mean and variance σ^2 ($w_j \sim \mathcal{CN}(0, \sigma^2)$). The noise is statistically independent of x_i.

A. Successful communication

In equation (1), suppose that only node i_1 wishes to communicate with node j and the signals $x_{i_2}, \ldots, x_{i_k}$ are interference. Then the signal-to-interference-plus-noise ratio (SINR) for node j is given by

$$\rho_j = \frac{P\gamma_{i_1,j}}{\sigma^2 + P\sum_{l=2}^{k} \gamma_{i_l,j}}$$

We assume that transmission is successful if and only if the SINR exceeds some threshold ρ_0.

III. Network Operation and Objective

We suppose that k nodes, denoted as $s_1, \ldots s_k$, are randomly chosen as sources. For every s_i, a destination node d_i is chosen at random, thus making k source-destination pairs. We assume that these $2k$ nodes are all distinct and therefore $k \leq n/2$. Source s_i wishes to transmit message M_i to destination d_i and has encoded it as signal x_i.

A. Communicating with Hops

In general, we suppose that the source-destination pair (s_i, d_i) communicates using a sequence of relay nodes $r_{i,1}, r_{i,2}, \ldots, r_{i,h-1}$. ($h = 1, 2, \ldots$ represents the number of hops.) Define $r_{i,0} = s_i$ and $r_{i,h} = d_i$. The path from s_i to d_i is then $r_{i,0} = s_i, r_{i,1}, \ldots, r_{i,h-1}, r_{i,h} = d_i$. In time slot $t+1$ we have nodes $r_{1,t}, \ldots, r_{k,t}$ transmitting simultaneously to nodes $r_{1,t+1}, \ldots, r_{k,t+1}$ respectively. We ask that nodes $r_{1,t+1}, \ldots, r_{k,t+1}$ decode their respective signals $x_1, \ldots, x_k$ and transmit them to the next set of relay nodes in the $(t+2)$th time slot, and so on. A natural condition to impose is that the relay nodes that are receiving (or transmitting) messages in any time slot be distinct, i.e., the messages do not collide. In addition, we ask that relay nodes not receive and transmit at the same time. We refer to these conditions together as the property of *no collisions* in the rest of the paper.

B. Throughput

With the above procedure, we have k simultaneous communications occurring in h time slots. Message M_i reaches the intended destination d_i successfully if it can be decoded by each relay $r_{i,t}$. Assume that a fraction $1 - \epsilon$ of messages reach their intended destinations in this way. Then, we define the throughput as

$$T = (1-\epsilon)\frac{k}{h}\log(1+\rho_0) \quad (2)$$

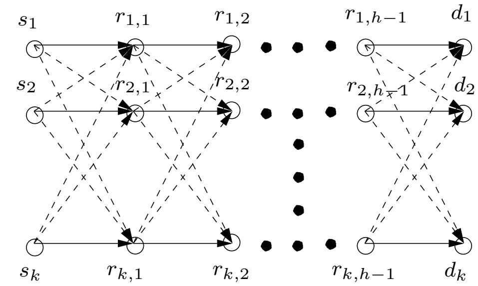

Fig. 1. Schedule of relay nodes: Source s_i communicates with destination d_i using relays $r_{i,1}, \ldots, r_{i,h-1}$. The solid lines indicate intended transmissions and the dashed lines indicate potential interference. The conditions on a schedule are that no node have to receive or transmit more than one message in any time slot and that no node have to perform transmission and reception simultaneously.

The number of source-destination pairs k, the fraction of dropped messages ϵ, the SINR threshold ρ_0 and hence the throughput T depend on n and we sometimes denote them by k_n, ϵ_n, $\rho_{0,n}$ and T_n. Typically, we force ϵ_n to go to zero. We demonstrate a scheme for choosing the relay nodes and analyze the throughput as well as the performance for this scheme. Thus, we give an achievability result for T_n. We begin by stating this result.

IV. Main Result for Random Networks

Theorem 1: Consider a network on n nodes whose edge strengths are drawn i.i.d. from a probability distribution function $f_n(\gamma)$. Let $F_n(\gamma)$ denote the cumulative distribution function corresponding to $f_n(\gamma)$ and define $Q_n(\gamma) = 1 - F_n(\gamma)$. Choose any β_n such that $Q_n(\beta_n) = \frac{\log n + \omega_n}{n}$, where $\omega_n \to \infty$ as $n \to \infty$. Then there exists a positive constant α such that a throughput of

$$T = (1-\epsilon_n)\,\alpha k_n(\beta_n)\frac{\log(nQ_n(\beta_n))}{\log n}\,\log\left(1 + \frac{a_n\beta_n}{\frac{\sigma^2}{P} + (k_n(\beta_n)-1)\mu_\gamma}\right) \quad (3)$$

is achievable for any positive a_n such that $a_n \leq 1$ and any $k_n(\beta_n)$ that satisfy the conditions:

$$k_n(\beta_n) \leq \alpha n \frac{\log(nQ_n(\beta_n))}{\log n} \quad (4)$$

$$\epsilon_n \leq \frac{a_n^2}{\alpha(1-a_n)^2}\frac{(k_n(\beta_n)-1)\sigma_\gamma^2}{(\frac{\sigma^2}{P} + (k_n(\beta_n)-1)\mu_\gamma)^2}\frac{\log n}{\log(nQ_n(\beta_n))} \to 0 \quad (5)$$

where μ_γ and σ_γ^2 are the mean and variance of γ respectively. The SINR threshold ρ_0 is given by $\frac{a_n\beta_n}{\frac{\sigma^2}{P}+(k_n(\beta_n)-1)\mu_\gamma}$.

Channels stronger than the parameter β_n will be called *good* and will allow us to schedule communications. Condition (4) is needed to ensure that we may obtain a non-colliding schedule of relays. (See Section V.) Once the schedule is obtained, we incorporate the effects of interference between non-colliding transmissions and analyze the error, ϵ_n, in Section VI. Condition (5) forces ϵ_n to go to zero. We combine the results of Sections V and VI to prove the theorem. We show how to apply the theorem and choose β_n in Section VII where we give several examples. Following that, the two-scale model is proposed and analyzed in Sections VIII and IX.

V. Scheduling Transmissions

With a view to meeting a minimum SINR of ρ_0 at every relay node at every hop, we impose the condition that each transmitting link be stronger than some threshold β_n. We require that $\gamma_{r_{i,t},r_{i,t+1}} \geq \beta_n$, where β_n is a design parameter. We call links that satisfy $\gamma_{i,j} \geq \beta_n$ as *good*. We require the path from s_i to d_i to use only good links.

Define $p_n = \mathrm{P}(\gamma \geq \beta_n)$ (for convenience, we drop the subscript n in the rest of this section). Using our wireless communication network, we define a graph on n vertices as follows: For (distinct) vertices i and j of the graph, draw an edge (i,j) if and only if $\gamma_{i,j} \geq \beta_n$ in the network. Call the resulting graph $G(n,p)$. The graph $G(n,p)$ then becomes an instance of a model called $\mathcal{G}(n,p)$ on n vertices in which edges are chosen independently and with probability p [2]. This graph shows the possible paths from s_i to d_i using only good links, but does not show the interference between paths. We examine this interference in Section VI.

Graphs taken from the model $\mathcal{G}(n,p)$ have many known properties regarding their connectivity, maximum minimum distance (called diameter) etc. [2]. We invoke a relatively recent result regarding vertex-disjoint paths for this model [13].

A. Scheduling using vertex-disjoint paths in $G(n,p)$

Two paths that do not share a vertex are called vertex-disjoint. Note that any two paths that are vertex-disjoint satisfy our "no-collisions" property; however, the reverse statement is not true. Thus, the vertex-disjoint condition is stronger than our requirement of non-colliding paths. For a set of k (disjoint) pairs of vertices (s_i, d_i), the question of whether there exists a set of vertex-disjoint paths connecting them is addressed in [13]. Their result states that, under certain randomness conditions, with high probability, for every set of k pairs (s_i, d_i) and k not greater than $\alpha n \frac{\log np}{\log n}$, where α is a constant, there exists a set of vertex-disjoint paths. It turns out that the randomness conditions required for their result are easily met in our network setup. Here we state a simplified version of their result that can be directly used for our purposes.

Theorem 2: Suppose that $G = G(n,p)$ and $p \geq \frac{\log n + \omega_n}{n}$, where $\omega_n \to \infty$. Then there exists a constant $\alpha > 0$ such that, with probability approaching 1, there are vertex-disjoint paths connecting s_i to d_i for any set of disjoint, randomly chosen source-destination pairs

$$F = \{(s_i, d_i) | s_i, d_i \in \{1, \ldots, n\}, i = 1, \ldots, k\}$$

provided $k = |F|$ is not greater than $\alpha n \frac{\log np}{\log n}$.
The constant α in this theorem is the same α required in Theorem 1. It is not explicitly specified. It is now easy to reach a conclusion regarding the lengths that these k paths can have. We state it without proof in the following lemma.

Lemma 1: Almost all of the $k = \alpha n \frac{\log np}{\log n}$ vertex-disjoint paths obtainable under Theorem 2 have lengths that grow no faster than $\frac{\log n}{\alpha \log np}$.

Hence the number of hops h is (asymptotically) at most $\frac{\log n}{\alpha \log np}$. We use this fact in the error analysis in the following section.

VI. Probability of Error

Algorithms that choose non-colliding paths without using information regarding the edges between vertices along one path to vertices along another have the property that these edges are i.i.d. Bernoulli distributed with parameter p. An example of a randomized algorithm that does this can be found in [13]. From this we conclude that the channel connections between nodes along different paths in the network are i.i.d. with distribution $f_n(\gamma)$.

We now consider the probability that a particular message fails to reach its intended destination. Destination d_i fails to receive message M_i if the SINR falls below ρ_0 at any of the h relay nodes $r_{i,1}, \ldots, r_{i,h} = d_i$. Denote by E_t the event that relay node $r_{i,t}$ does have an SINR greater than ρ_0. Note that the events $E_1, \ldots, E_h$ are identical. Using a simple union bound, we get that the probability of error in the communication of message M_i, called ϵ_n, is bounded by $hP(\sim E_1)$. We can then bound $P(\sim E_1)$ using a Chebyshev bound and get

$$\epsilon_n \leq h\mathrm{P}(\sim E_1) \leq \frac{\log n}{\alpha \log np} \frac{\sigma_\gamma^2}{(k-1)(\frac{P\beta_n - \rho_0\sigma^2}{(k-1)P\rho_0} - \mu_\gamma)^2}. \quad (6)$$

We force the last expression to go to zero. In order to apply the Chebyshev bound we need the condition $\rho_0 \leq \frac{\beta_n}{\frac{\sigma^2}{P} + (k-1)\mu_\gamma}$ to hold.

We have the condition $k \leq \alpha n \frac{\log np}{\log n}$ from Theorem 2 (this gives us (4)), the condition $\rho_0 \leq \frac{\beta_n}{\frac{\sigma^2}{P} + (k-1)\mu_\gamma}$ and the condition that the upperbound on ϵ_n from (6) go to zero. With these we need to maximize the throughput. Putting these together, we obtain Theorem 1. For details refer to [15].

VII. Examples

Table I lists some distributions of common interest and the throughputs obtained using Theorem 1 on them.

1) We see that the simple Bernoulli distribution, in which nodes are connected by a channel of strength 1 with probability p_n and not connected with probability $(1 - p_n)$ gives an almost linear throughput provided p_n is chosen optimally, namely, $p_n^* = \frac{\log n + \omega_n}{n}$ for any ω_n going to infinity. This optimum connection probability is surprisingly low, in fact, it is just enough to ensure that the network is connected.
2) A network where the channel strengths are drawn from the exponential distribution suffers from being very strongly connected and dominated by interference. It only gives a throughput of $\log n$.
3) Suppose that we are working with a network in which nodes are randomly placed at lattice points with edge distance d in a circular arrangement. Assume that the density of nodes is fixed as Δ. Assume that a power decay law of $1/r^m$, $m > 0$ holds, where r is the distance. When a node at the center of this disk transmits

	Distribution	$f_n(\gamma)$	Throughput
1	Shadow	$(1-p_n^*)\delta(\gamma)+p_n^*\delta(\gamma-1)$	$\frac{1}{w_n}\frac{\log^2(\log n)}{\log^3 n}\ n$
2	Exponential	$e^{-\gamma}$	$\log n$
3	Decay	$\frac{4\pi\Delta}{nm}\frac{1}{\gamma^{1+\frac{2}{m}}}, m>2$	$\frac{1}{w_n}\frac{\log^2(\log n)}{(\log n)^{2+m/2}}\ n$
4	Heavy Tail	$\frac{c}{1+\gamma^4}$	$\frac{\log\log n}{\log^{4/3} n}\ n^{1/3}$

TABLE I

AGGREGATE NETWORK THROUGHPUT FOR VARIOUS CHOICES OF $f_n(\gamma)$.

with power $P=1$, the marginal distribution of the signal powers received by other nodes is given by the Decay pdf mentioned in the table. We see that for the decay exponent being greater than 2, we get almost linear throughput. We see that almost linear throughput can be obtained for $m \geq 2$. This differs substantially from the $O(\sqrt{n})$ results obtained for the structured deterministic model with the same decay law. Our results show that it is not the marginal distribution of the power that impedes the throughput in a geometric power-decay network, but rather the spatial distribution of these powers.

4) The table also lists a heavy tail distribution that gives a throughput polynomial in n.

Thus we see that the throughput varies drastically depending on the precise choice of the pdf. It can go from almost linear to only logarithmic, especially if the interference dominates.

VIII. TWO SCALE NETWORKS

The random model that has been presented and analyzed is at the oppposite end of the spectrum from the deterministic model of [9]. We now propose another model that works like the random model at the local scale and the deterministic model at the global scale. By choosing parameters appropriately in this model we can obtain the random or the deterministic model as special cases. Also, we use a combination of the techniques used in [9] and [15] to analyze the throughput.

Consider a network with N nodes that are uniformly and randomly distributed on the surface of a sphere of radius R. We use a sphere rather than a planar disk to separate edge effects and have symmetry between all nodes. Also, the standard convention of measuring distances along great circles will be followed.

The channel between nodes i and j is denoted by $h_{i,j}=h_{j,i}$ as before. The channel strength is $\gamma_{i,j}=|h_{i,j}|^2$, also as before. The average channel strength is assumed to be distance-dependent for nodes that are more than a certain distance, say r, apart and independent of distance for nodes that are within a distance r.

More precisely, for nodes that are within a distance r, the channel strengths are drawn i.i.d., according to a p.d.f., say $f(\gamma)$. Let the expected value corresponding to this be denoted by μ_γ.

If nodes i and j are at a distance $l(i,j)>r$ from each other, we model $h_{i,j}$ to be a Rayleigh distributed random variable with its power (or second moment), $E|h_{i,j}|^2$, given by $cg(l(i,j))$ where $g(x)$ is used to model the distance-dependence and c is a constant. This gives us that the corresponding $\gamma_{i,j}$ is drawn from an exponential distribution with $cg(x)$ as its mean, i.e., $cg(x)\exp(-\gamma/cg(x))$. Typically, $g(x)$ is a decreasing function such as $\frac{1}{x^m}$ with $m>2$ or $\frac{e^{-\delta x}}{x^m}$ and c is chosen such that $cg(r)$ equals μ_γ. This is done to ensure that the expected value of $\gamma_{i,j}$ does not change abruptly as the distance between i and j changes from being less than r to being greater than r. Therefore, $c=\frac{\mu_\gamma}{g(r)}$. In this paper we use $g(x)=\frac{1}{x^m}$ with $m>2$.

Denote by $p_x(\gamma)$ the distribution from which the channel strength between two nodes with distance x between them is drawn. Then we have

$$p_x(\gamma)=\begin{cases} f(\gamma) & \text{if } x\leq r \\ \frac{\mu_\gamma r^m}{x^m}\exp(-\gamma\frac{x^m}{\mu_\gamma r^m}) & \text{if } x>r \end{cases}.$$

The notion of interference and an SINR threshold of ρ_0 for successful communication are as before. However, we will use K to denote the number of source destination pairs and H to denote the maximum number of time slots required for a message to reach its destination. With this, the througput expression is $(1-\epsilon)\frac{K}{H}\log(1+\rho_0)$ and we seek to maximize this as before. As for the random model we first investigate the scheduling (Section IX) The error analysis follows and leading to a result regarding the achievable throughput in Section X.

IX. SCHEDULING FOR TWO SCALE NETWORKS

Once again, we need to find a set of non-colliding paths connecting the source-destination pairs. In order to accomplish this, we establish a Voronoi tessellation of the surface of our sphere such that each cell of the tessellation can contain a disk of radius $r/12$ and can be contained in a disk of radius $r/6$. From [9], we know that this is always possible. For such a tessellation, we can contain any particular cell, say S_i, and all its neighbors in a disk of diameter r. Therefore all the channel connections in such a group of cells is drawn i.i.d. from $f(\gamma)$. Denote by L_i the line segment connecting them. This segment passes through several cells in order as it traverses from s_i to d_i. Note that the maximum number of cells it can pass through is $M=c_1\frac{R}{r}$ for some constant c_1. Denote these cells, in sequence, by $s_i\in S_{i,0},S_{i,1},S_{i,2},\ldots,S_{i,M}\ni d_i$. (Some sequences may, in actuality, be shorter than M.) We will refer to the set of cells $S_{1,t},S_{2,t},\ldots,S_{K,t}$ as the t-th *layer* of cells. For each occurence of a cell in the same layer or two successive layers, we would like to assign distinct nodes in the cell to do the job of relaying. We can show that this is possible if the condition $K\leq N/(8\cos^2\frac{r}{24R})$ is met. Denote the node that acts as the relay for message i in layer t by $s_{i,t}\in S_{i,t}$. We will call the K sequences $s_i,s_{i,1},\ldots,s_{i,M}$ to be the superschedule.

The relays required for transmitting message M_i from $s_{i,t}$ to $s_{i,t+1}$ are determined through the subschedule. We assume that this takes at most h hops. In order to accomplish this we group the cells of the network into *cell aggregates* such that each aggregate lies entirely in a disk of diameter r. In fact, each aggregate is obtained by considering a cell and some of its neighbors in a specific manner. Time slots are considered in blocks of h time slots each. In any particular block, communications take place only within each aggregate and not across aggregates. Within each aggregate we expect to have atleast $n = c_2 N \sin^2 \frac{r}{24R}$ nodes and atmost $n = c_2 K \sin^2 \frac{r}{12R}$ pairs of relays from successive layers of the superschedule that wish to communicate messages to each other. But this is exactly the scheduling problem that we solved for the random network model in Section V using the notion of good edges, the random graph model $\mathcal{G}(n,p)$ and the result on vertex-disjoint paths from [13]. We use the same procedure here to obtain a vertex-disjoint subschedule that operates over $h = \frac{\log n}{\alpha \log np}$ time slots, provided the condition $K \leq \alpha N \log np / (\log n \cdot \cos^2 \frac{r}{24R})$ holds. Thus, with a particular grouping of cell aggregates, some of the communications from the t-the layer to the $(t+1)$-th layer get scheduled. In order for every communication from the t-the layer to the $(t+1)$-th layer to get scheduled we repeat the process with a different set of randomly chosen cell aggregates. It can be shown that repeating the process $\log N$ times ensures (with probability going to 1) that all the communications from the t-th layer to the $(t+1)$-th layer get accomplished. Details can be found in [16].

To conclude, all K communications can be scheduled using $hM = \frac{\log n}{\alpha \log np} \cdot c_1 \frac{R}{r}$ hops in $hM \log N$ time slots provided the conditions $K \leq N/(8 \cos^2 \frac{r}{24R})$ and $K \leq \alpha N \log np / (\log n \cdot \cos^2 \frac{r}{24R})$ hold.

X. Achievable Throughput and Example

Next we do a probability of error analysis which involves a calculation of $\mathrm{P}(\rho_{i,l} \leq \rho_0)$, where $\rho_{i,l}$ is the SINR at the l-th relay of message i. We do this calculation using a value of $\rho_0 = \frac{P\beta}{\sigma^2 + a'(Pk\mu_\gamma + PK\frac{r^2\mu_\gamma}{2R^2})}$ where $a' \geq 1$. If we wish to force the probability of message M_i getting through to its destination to go to 1, we get the condition $\frac{\log n}{\alpha \log np} c_1 \frac{R}{r} \frac{1}{a'} \to 0$. As for the error analysis of the random network done earlier, this condition comes from using a union bound over the number of hops and bounding the probability $\mathrm{P}(\rho_{i,l} \leq \rho_0)$, this time through a Markov inequality.

Putting together the scheduling and probability of error results, we get the following theorem.

Theorem 3: Consider a network of N nodes, uniformly and randomly distributed over the surface of a sphere of radius R. For two nodes within a distance r, channel strengths are drawn i.i.d. from a pdf $f(\gamma)$ with mean μ_γ. Otherwise they are drawn from an exponential distribution with a mean of $\mu_\gamma r^m / x^m$, where $x > r$ is the distance between them. Let $F(\gamma)$ denote the cumulative distribution function of $f(\gamma)$ and $Q(\gamma) = 1 - F(\gamma)$. Let $n = c_2 N \sin^2 \frac{r}{24R}$ where c_2 is a known constant. Choose any β such that $p = Q(\beta) = \frac{\log n + \omega_n}{n}$, where $\omega_n \to \infty$ as $n \to \infty$. Then a throughput of

$$T = (1-\epsilon) \frac{\alpha K r \log np \cdot \log\left(1 + \frac{P\beta}{\sigma^2 + a'(PK \sin^2 \frac{r}{4R}\mu_\gamma + PK\frac{r^2\mu_\gamma}{2R^2})}\right)}{\log n \cdot c_1 R \cdot \log N}$$

is achievable where α and c_1 are constants and K and $a' \geq 1$ are chosen such that the following conditions are satisfied.

1) $K \leq N/(8 \cos^2 \frac{r}{24R})$.
2) $K \leq \alpha N \log np / (\log n \cdot \cos^2 \frac{r}{24R})$.
3) $\epsilon \leq \frac{\log n}{\alpha \log np} \cdot \frac{R}{r} \cdot \frac{1}{a'} \to 0$

We consider one example to illustrate this theorem. Let $f(\gamma) = \frac{1}{(1+\gamma)^t}$ with $t > 2$ as the distribution from which the channel strengths are drawn i.i.d. for nodes within a distance r from each other. We need $t > 2$ for μ_γ to be finite. We will assume that the other connections are exponential with the mean following a distance decay law of $g(x) = 1/x^m$ for $m > 2$. For this, we can show that a throughput of $T = N^{\frac{1}{t-1}} / \log^2 N$ is achievable. For t just greater than 2, this is almost linear but for $t > 3$, it falls below $\sqrt{N}$. It is interesting to note that m plays no role in the final throughput expression.

XI. Conclusions

We have proposed a random model for a wireless networks and shown that it can give throughputs that are almost linear in the number of nodes. We have also proposed a two-scale model of which the random model is a special case and presented an achievable throughput for this. We see that these models are capable of giving throughputs that scale much better than the $\sqrt{n}$ expected under deterministic distance models.

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