On network coding and routing in dynamic wireless multicast networks

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***Abstract*— We compare multicast network coding and routing for a time-varying wireless network model with interference-determined link capacities. We use dynamic back pressure algorithms that are optimal for intra-session network coding and routing respectively. Our results suggest that under such conditions, the gap in multicast capacity between network coding and routing can decrease relative to a collision-based wireless model with fixed link capacities.**

I. INTRODUCTION

In this paper we consider dynamic multicasting in time-varying wireless networks. We investigate the capacity benefit of network coding relative to multicast routing, i.e. forwarding and replication over one or more multicast trees. Network coding has been shown to be necessary for achieving multicast capacity in some wired networks such as the multicast network of Figure 1 [1]; many known examples of such networks are extensions or generalizations of this basic example. Reference [16] gives an analogous example, shown in Figure 2, of a static wireless network with fixed link rates and a half-duplex constraint, i.e. each node can either send or receive a single transmission at any one time. In this example, the optimal network coding solution, shown in Figure 3a, achieves 4/3 times the multicast throughput of the optimal routing solution, shown in Figure 3b.

As we consider more realistic wireless network models, we move progressively further from the static wireless model that is closest to the original wired model. For instance, if we consider the effect of interference on link capacities, it is not clear to what extent the network coding and routing solutions of Figure 3 are affected, since their transmit scenarios involve different sets of interfering links. Also, channel conditions often vary and traffic is usually bursty because either the sources generate traffic in bursts or the network nodes employ queuing and scheduling across multiple sessions. In such scenarios, optimal scheduling, routing and coding should vary dynamically depending on the current state of the network – channel conditions and buffer occupancy.

Routing, scheduling, rate, and power control in networks with bursty traffic has recently received significant attention in the context of wireless networks [2], [10], [12], [13], [14], [15], [19], [21], [22]. Much of the recent work in this area builds on the ideas of [3], [18] that describe algorithms for routing and scheduling flows using queue sizes, or differences

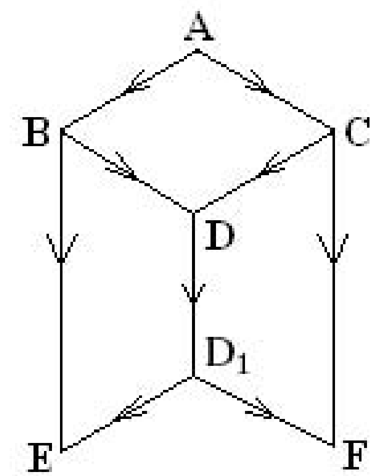

Fig. 1. Wired multicast network example, with a single node A multicasting information to two sink nodes E and F.

in queue size between the queues at the source and the destination of a link, as the metric to select between different flows. Such an approach is usually said to be *back-pressure* based since heavily loaded nodes downstream slow down the flow coming down from nodes upstream. Such approaches are optimal in the sense of allowing transmission at the maximum possible arrival rates into the network for which the queues at the various network nodes can be stabilized.

While the back-pressure approach has mostly been applied in the context of unicast transmissions, it has also been extended to the case of multicast transmissions [2], [17]. However, in the multicast case without network coding the algorithms are significantly more complex, even for wired networks.

We have recently extended the above back-pressure based dynamic routing and scheduling algorithms to include network coding and correlated sources [8]. Random network coding [6], introduced for the flow model, extends naturally to a time-varying network with bursty traffic and provides a distributed implementation. For networks with one or more multicast sessions, each consisting of a set of sources and sinks such that data from all the sources is intended for all the sinks,

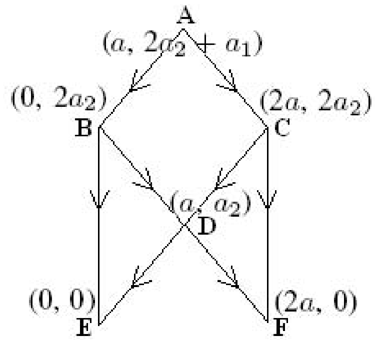

Fig. 2. Wireless multicast network example, with a single node A multicasting information to two sink nodes E and F. The coordinates represent physical distances which are varied in our simulation experiments.

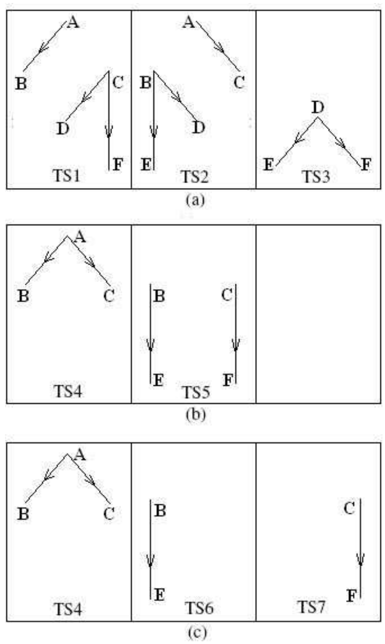

Fig. 3. Illustration of optimal schedules for coding (a) and routing (b). An alternative schedule for routing is given in (c). Each box shows a set of wireless links that can be simultaneously activated, termed a transmit scenario (TS).

our algorithm for routing, network coding, scheduling and rate control achieves stability for all input rates that can be stabilized with intra-session network coding.

Apart from the potential capacity gain of allowing network coding within multicast sessions, we also obtain much reduced complexity compared to the existing algorithms of [17], which involves enumeration of all multicast trees used, and [2], which involves maintaining a virtual queue for every subset of sinks for every session. In our approach, each node has just one virtual queue for each sink of each session (for independent sources) or for each source-sink pair of each session (for correlated sources). Routing, network coding and scheduling decisions are made locally by comparing, for each link, the difference in length of corresponding virtual queues, summed over each session's queues. For correlated sources, the sinks locally determine and control transmission rates across the sources. This gives a completely distributed algorithm for wired networks; in the wireless case, scheduling and power control among interfering transmitters is done centrally.

This paper is organized as follows. We present the system and network coding model in Section II. Optimal back pressure algorithms for multicast with and without network coding are described in Section III. In Section IV, we compare the two algorithms on the example of Figure 2. We conclude with a summary in Section V.

II. Model

A. Wireless network model

We use a model similar to that in [15]. We consider a network composed of a set $\mathcal{N}$ of $N = |\mathcal{N}|$ nodes with communication links between them that are fixed or time-varying according to some specified ergodic processes, and transmission of a set of multicast sessions $\mathcal{C}$ through the network. Each session $c \in \mathcal{C}$ is associated with a set $\mathcal{S}_c \subset \mathcal{N}$ of sources, and an exogenous process of data arrivals at each of these sources which must be transmitted over the network to each of a set $\mathcal{T}_c \subset \mathcal{N}$ of sinks. Transmissions are assumed to occur in slotted time, with time slots of length T. Decisions on routing, scheduling, etc. are made at most once a slot. For simplicity, we assume fixed length packets and link transmission rates that are restricted to integer multiples of the packet-length/time-slot quotient. That is, an integer number of packets can be transmitted in each slot.

The link transmission rate μ_{ij} from node i to node j, with other nodes $n \in \mathcal{N}$ transmitting independent information simultaneously, is given by the Shannon formula [5]

$$\mu_{ij}(\underline{P}, \underline{S}) = \log\left(1 + \frac{P_i S_{ij}}{N_0 + \sum_{n \in \mathcal{N}} P_n S_{nj}}\right)$$

where P_l is the power transmitted by node l, S_{lj} is the channel gain from node l to node j and N_0 is additive white Gaussian noise power over the signaling bandwidth. We assume that the channel conditions are fixed over the duration of a slot, and known at the beginning of the slot. For simplicity of exposition we assume that the channel and arrival processes are independent and identically distributed across slots; a straightforward generalization to ergodic processes is possible using a similar approach as that in [15].

We denote by (a, Z) a wireless broadcast link where a is the originating node and Z is the set of receiving nodes. Link rates $\underline{\mu}(\underline{P}, \underline{S}) = (\mu_{aZ}(\underline{P}, \underline{S}))$ are determined by the vector of transmit powers $\underline{P}(t) = (P_{aZ}(t))$ and a channel state vector $\underline{S}(t)$. $\underline{S}(t)$ is assumed to be constant over each time slot, i.e., state transitions occur only on slot boundaries $t = kT$, k integer. We also assume that $\underline{S}(t)$ takes values from a finite set and is ergodic; we denote by $\pi_{\underline{S}}$ the time average probability of state $\underline{S}$. $\underline{P}(t)$ is also held constant over each time slot, and is chosen from a compact set Π of power allocations representing limits on transmit power per node and/or across nodes.

B. Network coding

We use the approach of distributed random linear network coding [4], [6], [7], in which network nodes form output data by taking random linear combinations of input data. The contents of each packet, as a linear combination of the input packets, are specified by a *coefficient vector* in the packet header, updated by applying to the coefficient vectors the same linear transformations as to the data. The coefficient vector is thus a function of the random code coefficients specifying the linear combinations at intermediate nodes. A sink is able to decode when it receives a full set of packets with linearly independent coefficient vectors.

For simplicity, we consider the case where no restrictions are placed on coding among packets from the same multicast session. This asymptotically achieves optimal capacity, but in the worst-case decoding may not be possible until the end of the entire session.

III. Dynamic back-pressure algorithms

A. Multicast with intra-session network coding

In [8] we give a dynamic algorithm that uses queue state information to make network coding and transmit scenario decisions, without requiring any knowledge of the input or channel statistics.

Back-pressure policy

In each time slot $[t, t+T)$, the following are carried out:

Scheduling: For each link (a, Z), one session

$$c^*_{aZ} = \arg\max_c \left\{ \sum_{\beta\in\mathcal{T}_c} \max\left(\max_{b\in Z} \left(U_a^{c\beta} - U_b^{c\beta} \right), 0 \right) \right\}$$

is chosen. Let

$$w^*_{aZ} = \sum_{\beta\in\mathcal{T}_{c^*_{aZ}}} \max\left(\max_{b\in Z} \left(U_a^{c^*_{aZ}\beta} - U_b^{c^*_{aZ}\beta} \right),\ 0 \right). \quad (1)$$

Power control: The state $\underline{S}(t)$ is observed, and a power allocation

$$\underline{P}(t) = \arg\max_{\underline{P}\in\Pi} \sum_{a,Z} \mu_{aZ}(\underline{P}, \underline{S}(t)) w^*_{aZ} \quad (2)$$

is chosen.

Network coding: For each link (a, Z), a random linear combination of data corresponding to each (session, sink) pair $(c^*_{aZ}, \beta \in \mathcal{T}_{c^*_{aZ}})$ for which $\max_{b\in Z} \left(U_a^{c^*_{aZ}\beta} - U_b^{c^*_{aZ}\beta} \right) > 0$ is sent at the rate offered by the power allocation. Each destination node $d \in Z$ associates the received information with the virtual buffers corresponding to sinks $\beta \in \mathcal{T}_{c^*_{aZ}}$ for which $d = \arg\max_{b\in Z} \left(U_a^{c^*_{aZ}\beta} - U_b^{c^*_{aZ}\beta} \right)$. If the originating queues are empty within the time slot, no data is sent.

In a network where simultaneous transmissions interfere, optimizing (2) requires a centralized solution. In practice, the optimization (2) can be done heuristically using a greedy approach similar to that in [11], [20] but with the added guidance of weights w^*_{aZ} for prioritization among candidate links (a, Z). If there are enough channels for independent transmissions, the optimization can be done independently for each transmitter.

This algorithm stabilizes any set of input rates stabilizable with intra-session network coding [8]:

Theorem 1: If input rates (λ_i^c) satisfy $(\lambda_i^c + \epsilon) \in \Lambda$, the back-pressure policy stabilizes the system and guarantees an average total buffer occupancy upper bounded by $\frac{TBN}{\epsilon}$, where

$$B = \frac{\tau_{max}}{2} \left(\frac{1}{N} \sum_{i,c} E\left\{ \left(\frac{A_i^c}{T} \right)^2 \right\} + (\mu_{max}^{out} + \mu_{max}^{in})^2 \right)$$

B. Back pressure algorithm for multicast routing

We compare our back pressure multicast network coding algorithm with a back pressure algorithm for optimal multicast routing, for the case of a single multicast session with two sinks. The routing algorithm is similar to that in [2] in that each node maintains a queue for every subset of sinks. The algorithms differ in their policies for updating queues, as the algorithm of [2] is for adversarial wired networks, whereas our algorithm is for non-adversarial wireless networks.

In general, the number of queues is exponential in the number of sinks. We describe the algorithm for the case of two sinks, where each node maintains three queues: two individual queues containing data that is to be transmitted to each of the sinks and a common queue containing data that is to be transmitted to both sinks. We denote by $U_i^{(k)}, k = 1, 2, 3,$ respectively the lengths of these three queues at node i, and refer to data in the respective queues as commodity $k = 1, 2, 3$ data. Back pressure is used to control the branch points of the multicast distribution trees used.

In each time slot $[t, t+T)$, the following are carried out:

Balancing: At the start of each timeslot, for each non-sink node i, if $U_i^{(3)} - U_i^{(1)} - U_i^{(2)} > 0$, then an amount $\left(U_i^{(3)} - U_i^{(1)} - U_i^{(2)} \right)/3$ of data is removed from the common queue and the same amount of data is added to each of the individual queues at i.

Scheduling: For each link (a, Z),

$$k^*_{aZ} = \arg\max_k \max_{b\in Z} \left(U_a^{(k)} - U_b^{(k)} \right).$$

If

$$\max_{b\in Z} \left(U_a^{(k^*_{aZ})} - U_b^{(k^*_{aZ})} \right) > U_a^{(3)} - \sum_{j=1}^{2} \min_{b\in Z} U_b^{(j)},$$

then we set

$$w^*_{aZ} = \max_{b\in Z} \left(U_a^{(k^*_{aZ})} - U_b^{(k^*_{aZ})} \right).$$

Otherwise, we set $k^*_{aZ} = 0$ and

$$w^*_{aZ} = U_a^{(3)} - \sum_{j=1}^{2} \min_{b\in Z} U_b^{(j)}.$$

Power control: The state $\underline{S}(t)$ is observed, and a power allocation

$$\underline{P}(t) = \arg\max_{\underline{P}\in\Pi} \sum_{a,Z} \mu_{aZ}(\underline{P}, \underline{S}(t)) w^*_{aZ}$$

is chosen.

Routing: For each link (a, Z), if $k^*_{aZ} \neq 0$ and $\max_{b\in Z}\left(U_a^{(k^*_{aZ})} - U_b^{(k^*_{aZ})}\right) > 0$, commodity k^*_{aZ} data is sent from a to $\arg\max_{b\in Z}\left(U_a^{(k^*_{aZ})} - U_b^{(k^*_{aZ})}\right)$ at the rate offered by the power allocation. Otherwise, if $k^*_{aZ} = 0$ and $U_a^{(3)} - \sum_{j=1}^{2} \min_{b\in Z} U_b^{(j)} > 0$, data from the common queue at a is broadcast on (a, Z) at the offered rate, and a corresponding amount is added to $\min_{b\in Z} U_b^{(j)}$, $j = 1, 2$. If the originating queues are empty within the time slot, no data is sent.

This algorithm asymptotically achieves the optimal multicast throughput for routing. The proof is omitted for brevity.

IV. Simulation experiments

A. Experimental setup

We run the two multicast algorithms of the previous section on the network of Fig 2, with a single node A multicasting information to two sink nodes E and F. We assume a common transmission power and a half-duplex constraint at each network node. The channel gain for from node i to node j is modeled as:

$$S_{ij} = \frac{|f_{ij}|^2}{d_{ij}^k} \tag{3}$$

where $|f_{ij}|$ is the Rayleigh fading state and d_{ij} is the distance between i and j, and k is the propagation power loss exponent. We assume $\mu = E[|f|^2]$ is the same for all (i, j).

The two algorithms asymptotically achieve, for linear coding and routing respectively, the optimal multicast throughput achievable with the following seven transmit scenarios, chosen so as to cover the network coding and routing schedules of Figure 3:

- transmit scenario 1, where A transmits to B, and C broadcasts to D and F.
- transmit scenario 2, where A transmits to C, and B broadcasts to D and E.
- transmit scenario 3, where D broadcasts to E and F.
- transmit scenario 4, where A broadcasts to B and C.
- transmit scenario 5, where B transmits to E, and C transmits to F, respectively.
- transmit scenario 6, where B transmits to E.
- transmit scenario 7, where C transmits to F.

To investigate the effects of network geometry and SNR, the two algorithms were run for a number of different values for parameter a_1, defined in Figure 2, and SNR. For each choice of parameter values, a series of simulation runs was carried out to find the maximum stable multicast rate. For each run, the throughput was increased by 0.1, until the source queue became unstable, giving an approximate maximum transmission rate of R, defined as the maximum value for which the network is stable at rate R and unstable at rate $R + 0.1$. The network was considered stable for a given transmission rate and policy if the maximum backlog in all queues of all nodes was bounded by 200 when run for 20,000 timeslots.

B. Results

The simulation results are shown in Table 1 and Figures 4-6. From Table 1, we see that under this time-varying wireless network model with interference, the capacity gain of the network coding strategy of [16] over routing is lower than the 4/3 factor obtained in the fixed-rate collision model. Figures 4-6 show the proportion of different transmit scenarios used by the two policies, which varies depending on the experimental parameters. In the network coding case, the proportion of transmit scenarios gives a rough indication of the proportion of time network coding is used.

TABLE I

MAXIMUM TRANSMISSION RATES (R) FOR CODING AND ROUTING

snr	a_2	R for coding	R for routing
1000	$\sqrt{2}/4$	3.7	3.6
1000	$\sqrt{2}/2$	3.3	3.2
100	$\sqrt{2}/2$	2.3	2.2
10	$\sqrt{2}/2$	1.4	1.3

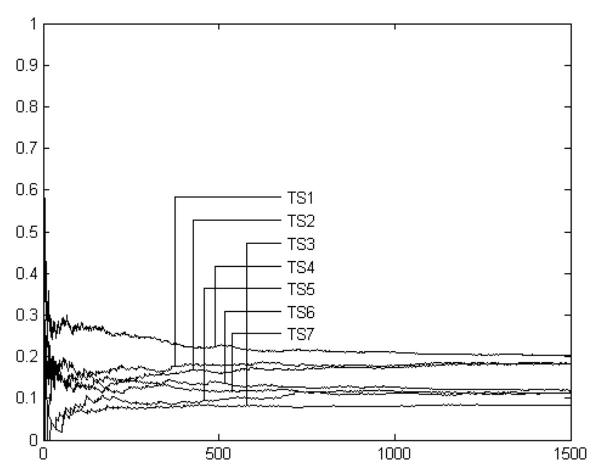

Fig. 4. The proportion of different transmit scenarios used by the network coding algorithm for snr $= 10$ (10dB), $a_1 = a_2 = a = \sqrt{2}/2, k = 2, \mu = 1.5$, tranmssion rate $= 1.4$.

V. Conclusion

We have compared multicast network coding and routing for a time-varying wireless network model with interference. Our results suggest that when link capacities are affected by interference, and power control, scheduling, network coding and routing are dynamically controlled in response to network conditions, the gap in multicast capacity between network

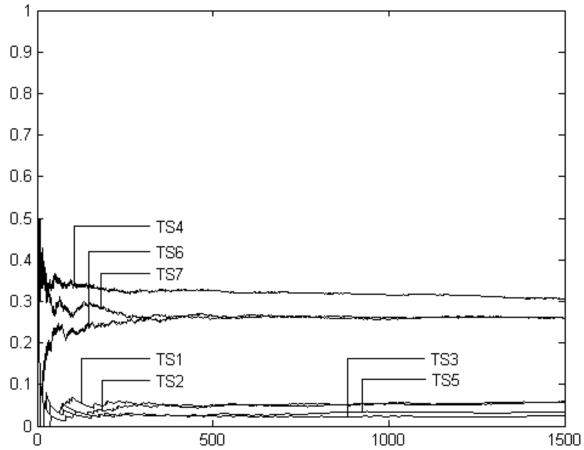

Fig. 5. The proportion of different transmit scenarios used by the network coding algorithm for snr $= 100$ (20dB), $a_1 = a_2 = a = \sqrt{2}/2, k = 2, \mu = 1.5$, transmission rate $= 2.3$.

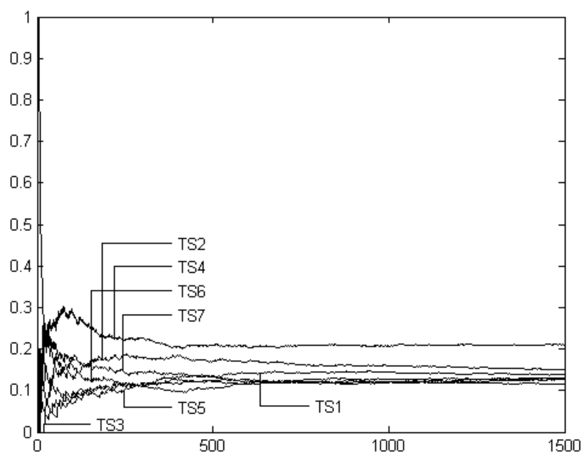

Fig. 6. The proportion of different transmit scenarios used by the routing algorithm for snr $= 10$ (10dB), $a = a_1 = a_2 = \sqrt{2}/2, k = 2, \mu = 1.5$, tranmssion rate $= 1.3$.

coding and routing can decrease relative to a collision-based wireless model with fixed link capacities. In such cases, the main advantage of network coding may be the reduction in complexity of optimization and operation. The coding advantage may also increase in situations where routing, power control and scheduling are not done optimally, as has been shown for the multiple unicasts case [9].

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