Noncooperative Optimization of Space-Time Signals in Ad hoc Networks

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***Abstract*— Recent results on noncooperative space-time waveform optimization for Ad hoc networks are first reviewed. The optimization criterion is sum power minimization under a QoS constraint (decoupled capacity.) Noncooperative iterative minimum mean-square (IMMSE) algorithms are advocated here as a pragmatic solution to the QoS constrained optimization problem. Connections between IMMSE, time-reversal (TR), taxation concepts in game theory, and Lagrangian optimization are discussed. The ST waveform noncooperative algorithm is then extended to ST eigencoding. The idea of interference taxation leads to a water-filling algorithm based on generalized eigenvalues. A matrix IMMSE-TR update is proposed for ST eigencoding, for which a subset of fixed points correspond to a noncooperative optimum.**

I. Introduction

Ad hoc networks are considered here, where each source node $l(i)$ has a unique destination node i. PHY layer multiaccess is employed allowing simultaneous transmission. Each of N nodes has an M element array and employs space-time transmission with temporal dimension N_s Nyquist samples. We first restrict the waveforms to a single mode with dimension MN_s. The ST received signal from the M elements is then collected over the N_s samples yielding vector $\mathbf{r}_i(m) \in \mathbb{C}^{MN_s}$, defined by $\mathbf{r}_i(m) = [\mathsf{r}_i((m+1)N_s - 1)^T, \ldots \mathsf{r}_i(mN_s)^T]^T$. The array outputs at time k are thus denoted by $\mathsf{r}_i(k) \in \mathbb{C}^M$. This ST vector can be written in vector-matrix form [1] as

$$\mathbf{r}_i(m) = \mathbf{H}^0_{i,l(i)}\tilde{\mathbf{g}}_{l(i)} b_{l(i)}(m) + \sum_{l \neq i,l(i)} \sum_{q=0}^{2} \mathbf{H}^q_{i,l}\tilde{\mathbf{g}}_l b_l(m-q) + \mathbf{n}_i(m). \quad (1)$$

In (1), $b_{l(i)}(m) \in \mathbb{C}$ are the source code symbols and $\tilde{\mathbf{g}}_{l(i)} \in \mathbb{C}^{MN_s}$ is the ST waveform. A unit-norm ST waveform is denoted by $\mathbf{g}_i$ such that $\tilde{\mathbf{g}}_i = \sqrt{P_i}\mathbf{g}_i$, with P_i the transmit power. Destination i thus receives interference from nodes $l \neq i, l(i)$ and white circular Gaussian thermal noise $\mathbf{n}_i(m)$ with covariance $\mathbf{I}$. The channel matrices $\mathbf{H}^q_{i,j} \in \mathbb{C}^{MN_s \times MN_s}$ are block-Toeplitz [1] with index q accounting for intersymbol interference, and sub-blocks $\mathsf{H}_{i,j}(p) \in \mathbb{C}^{M \times M}$, $p = 0, \ldots, L$.

The optimization criterion considered here is minimization of sum power under an SNR (single-mode) or decoupled capacity (ST eigencoding) constraint as used in [2][3]

$$\text{Minimize} \sum_{i=1}^{N} ||\tilde{\mathbf{g}}_i||^2 \quad (2)$$
$$\text{Subject to for all } l(i)$$
$$\Gamma_{l(i)}(\tilde{\mathbf{g}}_i, \tilde{\mathbf{g}}_{-i}) = \tilde{\mathbf{g}}_i^H \mathbf{H}^H_{l(i),i}\mathbf{R}^{-1}_{l(i)}(\tilde{\mathbf{g}}_{-i})\mathbf{H}_{l(i),i}\tilde{\mathbf{g}}_i \geq \gamma_0,$$

where γ_0 is the target SNR. Game-theoretic notation is used in (2) where $\tilde{\mathbf{g}}_{-i}$ corresponds to the set of interferer ST waveforms $\{\tilde{\mathbf{g}}_l : l \neq i, l(i)\}$. The MAI covariance matrix is given by

$$\mathbf{R}_i(\tilde{\mathbf{g}}_{-i}) = \sum_{l \neq i,l(i)} \sum_{q=0}^{2} \mathbf{H}^q_{i,l}\tilde{\mathbf{g}}_l\tilde{\mathbf{g}}_l^H(\mathbf{H}^q_{i,l})^H + \mathbf{I}. \quad (3)$$

The Ad hoc network is readily extended to ST eigencoding by replacing the vectors $\tilde{\mathbf{g}}_l$ in (1) with matrices $\tilde{\mathbf{G}}_l \in \mathbb{C}^{MN_s \times MN_s}$, with individual mode powers $P_{l,k} = ||(\tilde{\mathbf{G}}_l)_k||^2$. Similarly, the code symbols $b_l(m)$ in (1) become vectors $\mathbf{b}_l(m) \in \mathbb{C}^{MN_s}$ when all modes are used. The optimization problem under a decoupled capacity constraint is then following [2][4]

$$\text{minimize} \sum_{i=1}^{N} \text{tr}\{\tilde{\mathbf{G}}_i^H\tilde{\mathbf{G}}_i\} \quad (4)$$
$$\text{subject to for all } l(i)$$
$$\log|\mathbf{I} + \tilde{\mathbf{G}}_i^H \mathbf{H}^H_{l(i),i}\mathbf{R}^{-1}_{l(i)}\mathbf{H}_{l(i),i}\tilde{\mathbf{G}}_i| = r_{l(i)},$$

where $r_{l(i)}$ is the target capacity at destination node $l(i)$.

II. Review of Noncooperative Optimization of ST Waveforms

Before considering ST eigencoding, we first review results on ST waveform (single mode) design. Although cooperative algorithms have been considered for problems similar to (2) [5][6], the required transmission of vectors $\mathbf{r}_i(m)$ to a central processor is not in general practical. The focus here is on IMMSE algorithms which in simplest form only require transmission of a training sequence $\{b_i(m)\}$ to node $l(i)$. The IMMSE concept has been widely employed in cellular beamforming e.g. [7][8][9] and subsequently in TDD networks [2]. The IMMSE time-reversal algorithm in [1] is summarized by Table I

While not converged

For nodes $i = 1, \dots, N$

Initialize $n = 1$, $\mathbf{g}_i^1 = \mathbf{g}_i$

While $||\mathbf{g}_i^{n+1} - \mathbf{g}_i^n|| > \epsilon$

Compute MMSE ST detector using training sequence $\{\mathbf{b}_{l(i)}(m)\}$ and ST waveform $\mathbf{g}_{l(i)}^n$

$$\tilde{\mathbf{w}}_i^{n+1} = \mathbf{R}_i^{-1}\mathbf{H}_{i,l(i)}\mathbf{g}_{l(i)}^n.$$

Set node i ST waveform to *conjugate time-reverse*

$$\mathbf{g}_i^{n+1} = (\mathbf{w}_i^{n+1})^{r,*}$$

Transmit node $l(i)$ SNR $\Gamma_{l(i)}(\tilde{\mathbf{g}}_i^{n+1})$ to i.

Set power $P_i^{n+1} = ||\tilde{\mathbf{g}}_i^{n+1}||^2 = \gamma_0 P_i^n / \Gamma_{l(i)}$

Transmit training sequence $\{b_i(m)\}$ to node $l(i)$

Compute MMSE ST detector $\tilde{\mathbf{w}}_{l(i)}^{n+1}$ at $l(i)$

Set node $l(i)$ ST waveform $\mathbf{g}_{l(i)}^{n+1} = (\mathbf{w}_{l(i)}^{n+1})^{r,*}$

$n \leftarrow n+1$

End While

$\tilde{\mathbf{g}}_i \leftarrow \sqrt{P_i^n}\mathbf{g}_i^n$

Next node i

End While

TABLE I

ST WAVEFORM IMMSE-TR NONCOOPERATIVE ALGORITHM.

Important properties of IMMSE-TR proven in [1][3] that will be useful for designing ST eigencoding algorithms are now summarized.

Proposition 1: IMMSE-TR corresponds to the following noncooperative game where $u_i()$ is the utility function.

$$\begin{aligned} \tilde{\mathbf{g}}_i &\leftarrow \arg\max_{\tilde{\mathbf{g}}_i} u_i(\tilde{\mathbf{g}}_i, \tilde{\mathbf{g}}_{-i}) \qquad (5)\\ u_i(\tilde{\mathbf{g}}_i, \tilde{\mathbf{g}}_{-i}^n) &= \nu(\gamma_0 - \Gamma_{l(i)}(\tilde{\mathbf{g}}_i, \tilde{\mathbf{g}}_{-i}^n)) \\ &+ \ln\left(\mathbf{g}_i^H \mathbf{H}_{l(i),i}^H \mathbf{R}_{l(i)}(\tilde{\mathbf{g}}_{-i}^n)^{-1}\mathbf{H}_{l(i),i}\mathbf{g}_i\right) \\ &- \ln\left(\mathbf{g}_i^H \mathbf{R}_i(\tilde{\mathbf{g}}_{-i}^n)^{r,*}\mathbf{g}_i\right), \end{aligned}$$

where $\nu()$ is an arbitrary smooth concave function. The utility increases with decreasing power P_i as desired. However, the quantity $-\ln(\mathbf{g}_i^H\mathbf{R}_i(\tilde{\mathbf{g}}_{-i}^n)^{r,*}\mathbf{g}_i)$ corresponds to a tax on interference incurred by nodes $l \neq i, l(i)$. Note that $\mathbf{R}^{r,*}$ is the conjugate time-reverse [1] of the MAI covariance matrix.

Remark: The proof of Proposition 1 in [1] makes use of the following property of the transposed channel matrix. Assume the the channel is reciprocal, so the space-time response matrices satisfy $\mathsf{H}_{i,j}^T(p) = \mathsf{H}_{j,i}(p)$ for $p = 0, \dots, L-1$. Let $\mathbf{H}_{i,j} \in \mathbb{C}^{MN_s \times MN_s}$ be the block-Toeplitz space-time channel matrix with sub-blocks $\mathsf{H}_{i,j}(p) \in \mathbb{C}^{M\times M}$. Then $\mathbf{H}_{i,j}^T = \mathbf{H}_{j,i}^r$, where ^{r} is the block time-reversal operator [1]. The relation between the original matrix and time-reverse is specifically (Matlab notation)

$$\begin{aligned} &\mathbf{H}_{i,j}(nM+1:(n+1)M, mM+1:(m+1)M) = \qquad (6)\\ &\quad \mathsf{H}_{i,j}(m-n) \\ &\quad \mathbf{H}_{i,j}^r(nM+1:(n+1)M, mM+1:(m+1)M) = \\ &\quad \mathsf{H}_{i,j}(n-m), \end{aligned}$$

for $n, m = 0, \dots, N_s - 1$.

Proposition 2: An FDMA Ad hoc network corresponds to the constrained ST waveforms

$$\tilde{\mathbf{g}}_i = \sqrt{P_i}\mathbf{c}_{k_i} \otimes \mathbf{a}_i, \qquad (7)$$

where $\mathbf{c}_k \in \mathbb{C}^{N_s}$ is the unit-norm FFT vector with frequency k, $\mathbf{a}_i \in \mathbb{C}^M$ is the spatial signature and $\otimes$ is the Kronecker product. FDMA is the solution to the global optimization (2) when (a) cyclic prefixes are added to the $\tilde{\mathbf{g}}_i$, (b) the sum of propagation delay and multipath spread is less than the time-guard interval and (c) the decoupled *optimum frequencies* k_i are unique with $k_i \neq k_j$ for all $j \neq i, l(i)$. The optimal frequencies k_i and spatial signatures $\mathbf{a}_i$ in the FDMA case are

$$k_i, \mathbf{a}_i = \arg\max_{k,\mathbf{a}} \mathbf{a}^H \hat{\mathbf{H}}_{l(i),i}^H(k)\hat{\mathbf{H}}_{l(i),i}(k)\mathbf{a}. \qquad (8)$$

where $||\mathbf{a}_i||^2 = 1/N_s$ and the frequency response matrix is $\hat{\mathbf{H}}_{i,j}(k_j) = \sum_{p=0}^{N_s-1}\mathsf{H}_{i,j}(p)e^{i2\pi k_j p/N_s}$.

Proposition 3: Let $\{\tilde{\mathbf{g}}_i\}$ be a fixed point of IMMSE-TR in Table I for the ST waveforms and assume zero ISI. Then the $\{\tilde{\mathbf{g}}_i\}$ are a stationary point of the following Lagrangian corresponding to (2), in which each λ_i is minimized and satisfies the Karush-Kuhn-Tucker (KKT) conditions.

$$\begin{aligned} &L(\tilde{\mathbf{g}}, \lambda) = \qquad (9)\\ &\sum_{i=1} ||\tilde{\mathbf{g}}_i||^2 + \sum_{i=1}^{N} \lambda_i \left(\gamma_0 - \tilde{\mathbf{g}}_i^H \mathbf{H}_{l(i),i}^H \mathbf{R}_{l(i)}^{-1}(\tilde{\mathbf{g}}_{-i})\mathbf{H}_{l(i),i}\tilde{\mathbf{g}}_i\right). \end{aligned}$$

This proposition is formalized in [1]. More importantly, a consequence of this result is that a greedy solution to the optimization problem (corresponding to the game (5) without an interference tax), *cannot* be a stationary point of the Lagrangian (9) and hence a greedy algorithm can never yield the global optimum.

Remark: Proposition 3 *does not* imply that IMMSE-TR yields the global optimum. The proposition only means that the noncooperative IMMSE-TR algorithm satisfies the FONCs for a global optimum. For example, there is no guarantee that the global optimum corresponds to the minimum over all feasible multipliers λ_i, whereas IMMSE-TR corresponds to the specific solution minimizing $\{\lambda_i\}$.

III. NONCOOPERATIVE SPACE-TIME EIGENCODING

The above results for ST waveform optimization suggest that a noncooperative algorithm for ST eigencoding should also include an interference tax. The following *local* optimizations extend the IMMSE-TR idea to eigencoding.

$$\begin{aligned}&\text{Minimize } \mathrm{tr}\{\tilde{\mathbf{G}}_i^H \mathbf{R}_i^{*,r} \tilde{\mathbf{G}}_i\} \quad (10)\\ &\quad \text{Subject to}\\ &\quad \log|\mathbf{I} + \tilde{\mathbf{G}}_i^H \mathbf{H}_{l(i),i}^H \mathbf{R}_{l(i)}^{-1}(\tilde{\mathbf{G}}_{-i})\mathbf{H}_{l(i),i}\tilde{\mathbf{G}}_i| = r_{l(i)}\\ &\quad \text{with } \tilde{\mathbf{G}}_{-i} \text{ fixed.}\end{aligned}$$

The local optimizations (10) correspond to the global optimum in the case of zero MAI ($\mathbf{R}_i = \mathbf{I}\ \forall i$). However, in the presence of MAI we see that the term $\mathrm{tr}(\tilde{\mathbf{G}}_i^H \mathbf{R}_i^{*,r} \tilde{\mathbf{G}}_i)$ again plays the role of an interference tax.

It can be shown that the noncooperative optimization (i.e. optimizing w.r.t. $\tilde{\mathbf{G}}_i$ while holding $\tilde{\mathbf{G}}_{-i}$ fixed) is a convex problem. Thus differentiation of a local Lagrangian yields the following solution for the ST eigencode $\mathbf{G}_i$.

$$(\mathbf{R}_i^{*,r})^{-1}\mathbf{H}_{l(i),i}^H \mathbf{R}_{l(i)}^{-1}\mathbf{H}_{l(i),i}\mathbf{G}_i = \mathbf{G}_i \Lambda_i^G, \quad (11)$$

where Λ_i^G is a diagonal matrix of generalized eigenvalues. The generalized eigenmatrix is not orthonormal, although it has the simultaneous diagonalization property

$$\begin{aligned}\mathbf{G}_i^H \mathbf{R}_i^{*,r}\mathbf{G}_i &= \Lambda_i^I \quad (12)\\ \mathbf{G}_i^H \mathbf{H}_{l(i),i}^H \mathbf{R}_{l(i)}^{-1}\mathbf{H}_{l(i),i}\mathbf{G}_i &= \Lambda_i^S,\end{aligned}$$

where Λ_i^I and Λ_i^S are diagonal. It is readily shown that $\Lambda_i^G = \Lambda_i^S(\Lambda_i^I)^{-1}$.

The simultaneous diagonalization property converts the optimization (10) to capacity maximization in parallel channels. The corresponding Lagrangian is

$$\sum_{k=1}^{MN_s}(P_{i,k}\Lambda_{i,k}^I) + \mu_i\left(r_{l(i)} - \sum_{k=1}^{MN_s}\log|1 + P_{i,k}\Lambda_{i,k}^S|\right). \quad (13)$$

The solution to (13) is

$$P_{i,k} = \left(\frac{\mu_i}{\Lambda_{i,k}^I} - \frac{1}{\Lambda_{i,k}^S}\right)^+. \quad (14)$$

where $(x)^+ = x$ for $x > 0$ and zero otherwise. The water-filling level μ_i is set to achieve the capacity target $r_{l(i)}$, and the resulting capacity is

$$r_{l(i)} = \sum_{k:P_{i,k}>0}\log\left(\mu_i \Lambda_{i,k}^G\right). \quad (15)$$

A direct approach to solving the water-filling problem (11), (14) is to estimate the channel $\mathbf{H}_{i,l(i)}$ and interference covariances $\mathbf{R}_i, \mathbf{R}_{l(i)}$. However, this method requires covariance estimation at both nodes $i, l(i)$ and transmission of covariance matrix estimates. The key question is whether a matrix-based IMMSE-TR algorithm can be found that is the eigencoding counterpart of Table I. To develop such a matrix IMMSE-TR update, first consider the well-known MMSE multiuser detector [10]. First, the received ST signal at node i for eigencoding is

$$\mathbf{r}_i = \mathbf{H}_{i,l(i)}\tilde{\mathbf{G}}_{l(i)}\mathbf{b}_{l(i)} + \sum_{k\neq i,l(i)}\mathbf{H}_{i,k}\tilde{\mathbf{G}}_k\mathbf{b}_k + \mathbf{n}_i. \quad (16)$$

For nodes $i = 1, \ldots, N$
  Initialize $n = 1$
  While $||\tilde{\mathbf{G}}_i^{n+1} - \tilde{\mathbf{G}}_i^n||_F > \epsilon$
    Compute $\bar{\text{M}}$MSE TR detector using
    training sequence $\{\mathbf{b}_{l(i)}(m)\}$ and ST eigenmatrix $\mathbf{G}_{l(i)}^n$
      $\tilde{\mathbf{W}}_i^{n+1} = (\mathbf{H}_{i,l(i)}\tilde{\mathbf{G}}_{l(i)}^n(\tilde{\mathbf{G}}_{l(i)}^n)^H\mathbf{H}_{i,l(i)}^H + \mathbf{R}_i)^{-1}\mathbf{H}_{i,l(i)}\tilde{\mathbf{G}}_{l(i)}^n$
    Normalize columns $||\mathbf{W}_i^{n+1}||_k = 1$
    Set node i ST waveform to *conjugate time-reverse*
      $\mathbf{G}_i^{n+1} = (\mathbf{W}_i^{n+1})^{r,*}$
    Transmit node $l(i)$ SNR $\Gamma_{l(i)}(\tilde{\mathbf{G}}_i^{n+1})$ to i.
    If $||\mathbf{W}_i^{n+1} - \mathbf{W}_i^n||_F \leq \epsilon$
      Compute the normalized SNR for each eigenmode
        $\gamma_i(k) = P_{l(i)}^n(k)/\Gamma_{l(i)}^k(\tilde{\mathbf{G}}_i^{n+1})$
      Performing waterfilling algorithm
        $P_{new}(k) = \left[\frac{\mu_i}{\Lambda_{i,k}^I} - \frac{1}{\gamma_i(k)}\right]^{++}$
      Set new power values
        $P_i^{n+1}(k) = aP_{new}(k) + (1-a)P_i^n(k), \quad a \in [0,1]$
    End If
    Transmit training sequence $\{b_i(m)\}$ to node $l(i)$
    Compute MMSE TR detector $\tilde{\mathbf{W}}_{l(i)}^{n+1}$ at $l(i)$
    Set node $l(i)$ TR matrix $\mathbf{G}_{l(i)}^{n+1} = (\mathbf{W}_{l(i)}^{n+1})^{r,*}$
    $n \leftarrow n+1$
  End While
Next node i

TABLE II

THE IMMSE-TR ALGORITHM FOR EIGENCODING

The MMSE detector is then given by

$$\tilde{\mathbf{W}}_{MMSE,i} = \arg\min_{\mathbf{W}} E\{||\mathbf{b}_{l(i)} - \mathbf{W}_i^H\mathbf{r}_i||^2\}. \quad (17)$$

The unnormalized MMSE solution has two alternative representations

$$\begin{aligned}&\tilde{\mathbf{W}}_{MMSE,i} = \quad (18)\\ &[\mathbf{H}_{i,l(i)}\tilde{\mathbf{G}}_{l(i)}\tilde{\mathbf{G}}_{l(i)}^H\mathbf{H}_{i,l(i)}^H + \mathbf{R}_i]^{-1}\mathbf{H}_{i,l(i)}\tilde{\mathbf{G}}_{l(i)} =\\ &\mathbf{R}_i^{-1}\mathbf{H}_{i,l(i)}\tilde{\mathbf{G}}_{l(i)})\left[\mathbf{I} + \tilde{\mathbf{G}}_{l(i)}^H\mathbf{H}_{i,l(i)}^H\mathbf{R}_i^{-1}\mathbf{H}_{i,l(i)}\tilde{\mathbf{G}}_{l(i)}\right]^{-1}\\ &= \mathbf{R}_i^{-1}\mathbf{H}_{i,l(i)}\tilde{\mathbf{G}}_{l(i)}\mathbf{M}_i.\end{aligned}$$

The matrix $\mathbf{M}_i$ is the mean-square error covariance

$$\begin{aligned}&\mathbf{M}_i = \quad (19)\\ &E\{[\mathbf{b}_i - \tilde{\mathbf{W}}_{MMSE,i}^H\mathbf{r}_i][\ldots]^H\} =\\ &\left[\mathbf{I} + \tilde{\mathbf{G}}_{l(i)}^H\mathbf{H}_{i,l(i)}^H\mathbf{R}_i^{-1}\mathbf{H}_{i,l(i)}\tilde{\mathbf{G}}_{l(i)}\right]^{-1}.\end{aligned}$$

The proposed IMMSE-TR algorithm for eigencoding is given in Table II. The eigenvalues for waterfilling are determined using the estimated SNRs per mode k, $\gamma_i(k)$. For stability, we determined that the powers should then be updated only after the MMSE detector $\mathbf{W}_i^n$ has converged. Furthermore, to avoid ill-convergence, the powers on all modes are lower bounded by P_{min}, as represented by $[x]^{++} = \min(x, P_{min})$. To increase algorithm stability, it was also found that P_i^{n+1} should be updated slowly using the averaging of $P_{new}(k)$ and $P_i^n(k)$. The key step in matrix-based IMMSE-TR is setting $\mathbf{G}_i \leftarrow \mathbf{W}_i^{r,*}$ on each iteration, where $\mathbf{A}^r$ implies time-reversal of each column of matrix $\mathbf{A}$. However, the power algorithm interpretation of IMMSE-TR for the vector case [1] only applies to the dominant eigenmode of $\mathbf{G}_i$, and hence convergence of IMMSE-TR to the generalized eigenmatrix solution (11) cannot be proven.

Although convergence of IMMSE-TR to (11) is not guaranteed, the following Theorem gives a connection between IMMSE-TR and the noncooperative optimum (11).

Theorem 1: Consider the IMMSE-TR algorithm where the transmit matrices satisfy $\mathbf{G}_i = \mathbf{W}_i^{r,*}$ for all nodes i. Then at least one fixed point of IMMSE-TR corresponds to the generalized eigenmatrix solution (11) rewritten as

$$\mathbf{G}_i = (\mathbf{R}_i^{r,*})^{-1}\mathbf{H}_{l(i),i}^H\mathbf{R}_{l(i)}^{-1}\mathbf{H}_{l(i),i}\mathbf{G}_i(\Lambda_i^g)^{-1}. \quad (20)$$

Proof: At a fixed point of IMMSE-TR the normalized MMSE receiver is

$$\mathbf{W}_i = \mathbf{R}_i^{-1}\mathbf{H}_{i,l(i)}\mathbf{G}_{l(i)}\mathbf{P}_{l(i)}^{1/2}\mathbf{M}_i\mathbf{D}_i, \quad (21)$$

where $\mathbf{D}_i$ is a diagonal matrix with k-th entry $1/||(\mathbf{W}_i)_k||$, and $\mathbf{P}_{l(i)}$ is diagonal with k-th entry $||(\mathbf{G}_{l(i)})_k||^2$. The MMSE error covariance is

$$\mathbf{M}_i = [\mathbf{I} + \tilde{\mathbf{G}}_{l(i)}^H\mathbf{H}_{i,l(i)}^H\mathbf{R}_i^{-1}\mathbf{H}_{i,l(i)}\tilde{\mathbf{G}}_{l(i)}]^{-1}. \quad (22)$$

At an IMMSE-TR fixed point, $\mathbf{W}_i = \mathbf{G}_i^{r,*}$, and $\mathbf{G}_{l(i)} = \mathbf{W}_{l(i)}^{r,*}$. Substituting these relationships into (21)) and using time-reversal relationships for the channel matrices yields the following fixed point equation for $\mathbf{G}_i$.

$$\mathbf{G}_i = (\mathbf{R}_i^{r,*})^{-1}\mathbf{H}_{l(i),i}^H\mathbf{R}_{l(i)}^{-1}\times \quad (23)$$
$$\mathbf{H}_{l(i),i}\mathbf{G}_i\mathbf{P}_i^{1/2}\mathbf{M}_{l(i)}\mathbf{D}_{l(i)}\mathbf{P}_{l(i)}^{1/2}\mathbf{M}_i^{r,*}\mathbf{D}_i^{r,*}.$$

The solutions $\mathbf{G}_i$ in (23) do not correspond to the noncooperative optimum (11) unless the MMSE covariances $\mathbf{M}_i, \mathbf{M}_{l(i)}$ are diagonal. However, if $\mathbf{G}_i$ satisfies (11), then it diagonalizes $\mathbf{M}_i$ from (12) and (19), yielding $\mathbf{M}_i = [\mathbf{I}+\mathbf{P}_{l(i)}\Lambda_i^S]^{-1}$. Hence a solution $\mathbf{G}_i$ in (11) is one of the fixed points of IMMSE-TR.

IV. Results

The first simulated network consists of $N = 12$ nodes in a 1 km square region as in Figure 3. Each node use $M = 8$ transmit/receive elements. The pathloss coefficient is 4. The channel is assumed to have full rank with direction of arrival uniformly distributed in $[-\pi/6, \pi/6]$. The number of Nyquist samples/symbol is $N_s = 16$. The targeted capacity is $c = 64$ bits/transmission, or 4 b/s/Hz assuming zero excess bandwidth.

To evaluate performance of the algorithm we compare IMMSE-TR with a greedy algorithm for the given network. The greedy algorithm corresponds to solving (11) with $\mathbf{I}$ replacing $\mathbf{R}_i^{r,*}$, i.e. each node minimizes its power without an interference tax. The performance benchmark is power efficiency η. Let $P_{sum} = \sum_{i=1}^N tr\{\tilde{\mathbf{G}}_i^H\tilde{\mathbf{G}}_i\}$ represent the sum power. The minimum possible sum power for N nodes is obtained when MAI is zero, i.e. each $\mathbf{R}_i = \mathbf{I}$. In this case, the Table 1 IMMSE-TR algorithm can be shown to yield the optimum minimum sum power by computing the eigenvectors of $\mathbf{H}_{l(i),i}^H\mathbf{H}_{l(i),i}$ and waterfilling. Denote this minimum possible sum power as P_{sum}^* when MAI is absent. Let P_{sum} be the actual sum power when MAI is included and IMMSE-TR or greedy optimization is employed. Then $\eta = P_{sum}^*/P_{sum}$, with $0 \leq \eta \leq 1$ and $\eta = 1$ being the maximum efficiency.

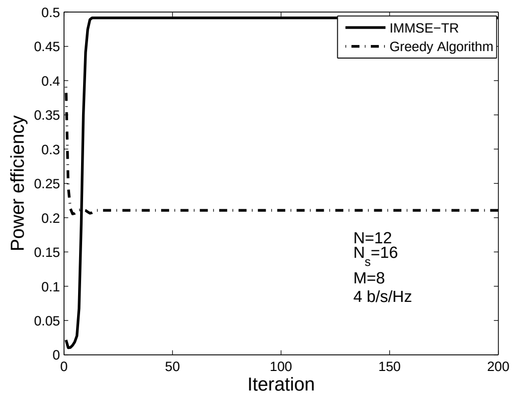

Fig. 1. Power efficiencies of IMMSE-TR and greedy algorithm

Fig. 1 shows that IMMSE-TR yields significantly higher power efficiency in comparison with the greedy algorithm. Figure 2 is the power efficiency of a network of 32 nodes. For 32 nodes, IMMSE-TR in Table II could not be simulated due to the computational load of multiple IMMSE subiterations. In this case, the generalized eigenmatrix computation (11) and waterfilling (14) was performed directly. Each node has the same $N_s = 16, M = 8$ parameters as in the previous case. The targeted spectral efficiency is 2 b/s/Hz. Results demonstrate that the generalized eigencoding algorithm required much lower transmit power than the greedy version corresponding to $\mathbf{R}_i = \mathbf{I}$ in (11).

For illustrative purposes, a rank-1 approximation to the spatial and temporal signatures of each node corresponding to the largest eigenmode are shown in Fig. 3 and Fig. 4. That is, at node i column $(\mathbf{G}_i)_1$ corresponding to the largest generalized eigenvalue is rearranged into an $N_s \times M$ matrix, and the rank-1 SVD approximation then computed. The rank-1 matrix then yields the spatial and temporal signatures in Figs. 3 and 4.

V. Conclusion

Acknowledgment

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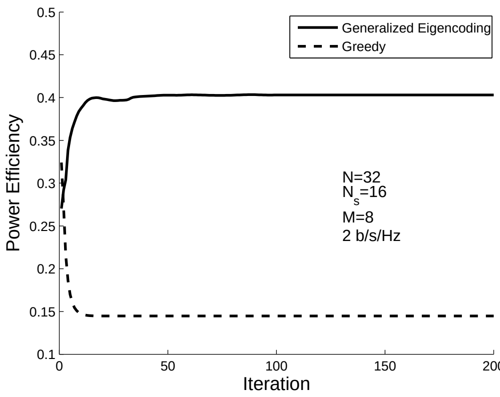

Fig. 2. Power efficiencies of Generalized Egiencoding and greedy algorithm for a network of 32 nodes

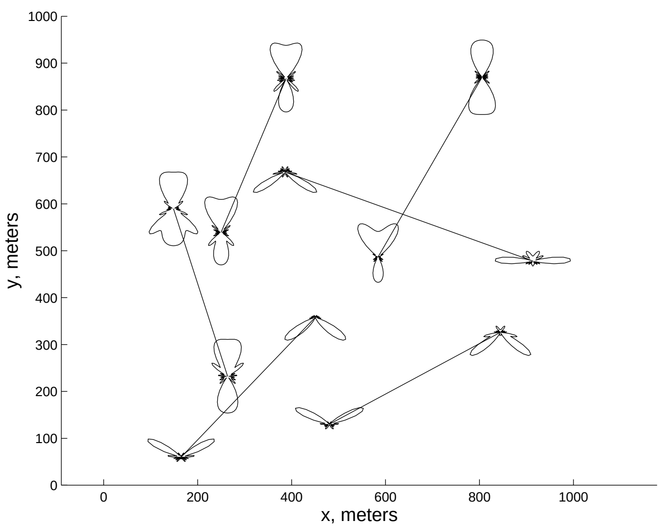

Fig. 3. Beampatterns of the largest eigenmode of nodes in the network for a network of 12 nodes

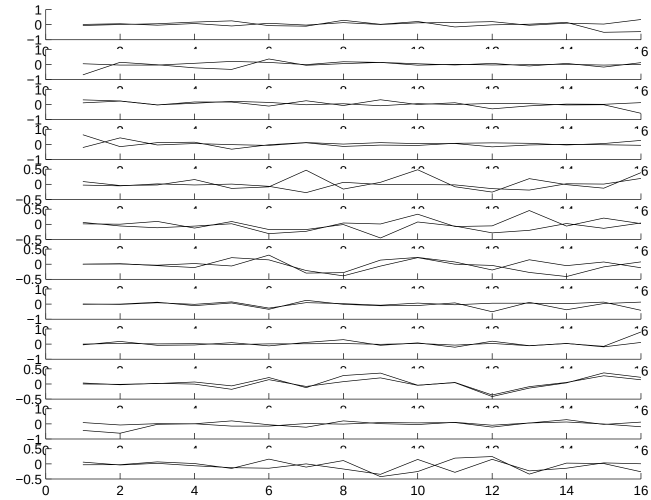

Fig. 4. Waveform of the largest mode of nodes in the network

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