Joint Source Channel Coding with Side Information Using Hybrid Digital Analog Codes

Makesh Pravin Wilson
Dept. of Electrical Engineering
Texas A&M University
College Station, TX 77843
makesh@ece.tamu.edu

Krishna Narayanan
Dept. of Electrical Engineering
Texas A&M University
College Station, TX 77843
krn@ece.tamu.edu

Giuseppe Caire
Dept. of Electrical Engineering
University of Southern California
Los Angeles, CA 90089
caire@usc.edu

***Abstract*— We study the joint source channel coding problem of transmitting an analog source over a Gaussian channel in two cases - (i) the presence of interference known only to the transmitter and (ii) in the presence of side information known only to the receiver. We introduce hybrid digital analog forms of the Costa and Wyner-Ziv coding schemes. Our schemes are based on random coding arguments and are different from the nested lattice schemes by Kochman and Zamir that uses dithered quantization. We also discuss superimposed digital and analog schemes for the above problems which show that there are infinitely many schemes for achieving the optimal distortion for these problems. This provides an extension of the schemes by Bross et al to the interference/side information case.**

I. Introduction and Problem Statement

For the classical problem of transmitting K samples of an i.i.d Gaussian source in K uses of an additive white Gaussian noise channel (AWGN), there are many approaches to obtain the optimal performance. Recently, it was shown by Bross, Lapidoth and Tinguely [4] that there are infinitely many schemes that contain pure separation based scheme and uncoded transmission as special cases.

Here, we first consider the problem of transmitting K samples of the source through an AWGN channel in the presence of an interferer known only to the transmitter. We introduce a hybrid digital analog (HDA) Costa coding scheme where the source is not explicitly quantized, which is optimal. We also show that there infinitely many schemes similar to that for the case without the interference. We discuss applications of this HDA Costa coding scheme for transmitting a Gaussian source in the presence of a channel signal to noise mismatch and in broadcasting a Gaussian source to two users with different SNRs. We show that the HDA Costa coding has advantages over digital Costa coding schemes in these applications. The HDA Costa coding scheme is related to the scheme considered by Kochman and Zamir in [5], although this was developed independently. The proposed scheme is based only on random coding arguments and does not use nested lattices like in [5]. As a result, the relationship between the auxiliary random variable and the source is made more explicit. We also consider the non-asymptotic SNR case unlike in [5]. Further, the performance of this scheme in the presence of SNR mismatch is analyzed.

For completeness, we also consider the dual problem of transmitting a Gaussian source with side information available only to the receiver, where we show an HDA version of Wyner-Ziv coding. This is similar to the scheme in [2] and again uses random coding arguments instead of nested lattices.

The paper is organized as follows. In Section II, we first discuss the problem of transmitting an i.i.d Gaussian source in the presence of a Gaussian interference known only to the transmitter. We discuss the analog Costa coding scheme and also the hybrid form of costa coding in this section. In Section III, we discuss similar schemes for the Wyner-Ziv problem and in Section IV, briefly consider the combination of Costa and Wyner-Ziv coding. In Section V we study the performance of these schemes when the SNR of the channel is different from the designed SNR. In Section VI, we consider the problem of transmitting a Gaussian source in the absence of an interference, but when the channel bandwidth is smaller than the source bandwidth and show how the HDA Costa coding scheme is useful. Finally, in Section VII, we consider the problem of broadcasting a Gaussian source to two users through AWGN channels and propose a joint source channel coding scheme based on HDA Costa coding.

We use the following notation in this paper. Vectors are denoted by bold face letters such as $\mathbf{x}$. Upper case letters are used to denote scalar random variables. When considering a sequence of i.i.d random variables, a single upper case letter is used to denote each component of the random vector.

II. Transmission of a Gaussian source over a Gaussian channel with interference known only at the transmitter

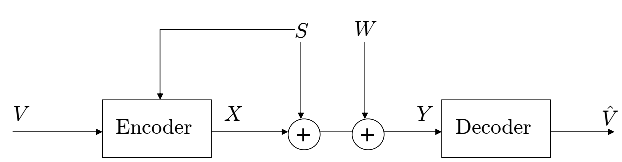

Fig. 1. Block diagram of the joint source channel coding problem with interference known only at the transmitter.

We first consider the problem of transmitting N samples of a real analog source $\mathbf{v} \in \mathbb{R}^N$, where v_i's are independent Gaussian random variables with $v_i \sim \mathcal{N}(0, \sigma_v^2)$ in N uses of an AWGN channel with noise variance σ^2 in the presence of an interference $\mathbf{s} \in \mathbb{R}^N$ which is known to the transmitter but unknown to the receiver. Further, let us assume that s_i's are a sequence of real i.i.d Gaussian random variables with zero mean and variance Q and let the input power to the channel $\mathbb{E}[x_i^2]$ be constrained to be P. The problem setup is shown schematically in Fig. 1. The received signal $\mathbf{y}$ is given by

$$\mathbf{y} = \mathbf{x} + \mathbf{s} + \mathbf{w} \tag{1}$$

where $\mathbf{s}$ is the interference and $\mathbf{w}$ is the AWGN.

The optimal distortion of $\frac{\sigma_v^2}{(1+\frac{P}{\sigma^2})}$ can be obtained even in the presence of the interference by using the following (obvious) separate source and channel coding scheme.

A. Separation based scheme with Costa coding (Digital Costa Coding)

We first quantize the source using an optimal quantizer to produce an index $m \in \{1, 2, \ldots, 2^{NR}\}$, where $R = \frac{1}{2}\log\left(1 + \frac{P}{\sigma^2}\right) - \epsilon$. Then, the index is transmitted using Costa's writing on dirty paper coding scheme [7]. Since the quantizer output is digital information, we refer to this scheme as digital Costa coding. We briefly review this here to make it easier to describe our proposed techniques later on.

Let U be an auxiliary random variable given by

$$U = X + \alpha S \tag{2}$$

where $X \sim \mathcal{N}(0, P)$ is independent of S and $\alpha = \frac{P}{P+\sigma^2}$.

We first create an N-length i.i.d Gaussian code book $\mathcal{U}$ with $2^{N(I(U;Y)-\delta)}$ codewords, where each component of the codeword is Gaussian with zero mean and variance $P + \alpha^2 Q$. Then evenly (randomly) distribute these over 2^{NR} bins. For each $\mathbf{u}$, let $i(\mathbf{u})$ be the index of the bin containing $\mathbf{u}$. For a given m, we look for an $\mathbf{u}$ such that $i(\mathbf{u}) = m$ and $(\mathbf{u}, \mathbf{s})$ are jointly typical. Then, we transmit $\mathbf{x} = \mathbf{u} - \alpha\mathbf{s}$. Note that since $(\mathbf{u}, \mathbf{s})$ are jointly typical, from (2), we can see that $\mathbf{x} \perp \mathbf{s}$ and satisfies the power constraint.

The received sequence $\mathbf{y}$ is given by

$$\mathbf{y} = \mathbf{x} + \mathbf{s} + \mathbf{w} \tag{3}$$

At the decoder, we look for a $\mathbf{u}$ that is jointly typical with $\mathbf{y}$ and declare $i(\mathbf{u})$ to be the decoded message. Since $R = \frac{1}{2}\log\left(1 + \frac{P}{\sigma^2}\right) - \epsilon$, the distortion in $\mathbf{v}$ given by $D(R)$, where D is the distortion rate function. For a Gaussian source and mean squared error distortion $D(R) = \sigma_v^2 2^{-2R}$ and, hence, the overall distortion can be made to be arbitrarily close to $\frac{\sigma_v^2}{(1+\frac{P}{\sigma^2})}$ by a proper choice of ϵ and δ.

While the above scheme is straightforward, in the following three sections we show that there are a few other joint source channel coding schemes, which are also optimal. In fact, there are infinitely many schemes which are optimal. Although, these schemes are all optimal when the channel SNR is known at the transmitter, their performance is in general different when there is an SNR mismatch. The joint source channel coding schemes to be discussed in the next sections have advantages over the separation based scheme discussed in such a situation.

B. Hybrid Digital Analog Costa Coding

Let us now describe a joint source-channel coding scheme where the source $\mathbf{v}$ is not explicitly quantized. We refer to this scheme as hybrid digital analog (HDA) Costa coding for which the code construction, encoding and decoding procedures are as follows.

We first define an auxiliary random variable U given by

$$U = X + \alpha S + \kappa V \tag{4}$$

where $X \sim \mathcal{N}(0, P)$ and X, S and V are pairwise independent.

1) Codebook generation: Generate a random i.i.d code book $\mathcal{U}$ with 2^{NR_1} sequences, where each component of each codeword is Gaussian with zero mean and variance $P + \alpha^2 Q + \kappa^2 \sigma_v^2$.
2) Encoding: Given an $\mathbf{s}$ and $\mathbf{v}$, find a $\mathbf{u}$ such that $(\mathbf{u}, \mathbf{s}, \mathbf{v})$ are jointly typical with respect to the distribution obtained from the model in (4) and transmit $\mathbf{x} = \mathbf{u} - \alpha\mathbf{s} - \kappa\mathbf{v}$. If such an $\mathbf{u}$ cannot be found, we declare an encoder failure. Let P_{e_1} be the probability of an encoder failure. From standard arguments on typicality and its extensions to the infinite alphabet case [11], it follows that $P_{e_1} \to 0$ as $N \to \infty$ provided

$$\begin{aligned} R_1 &> I(U; S, V) && (5)\\ &= h(U) - h(U|S, V) && (6)\\ &= h(U) - h(X|S, V) && (7)\\ &= h(U) - h(X) && (8)\\ &= \frac{1}{2}\log\frac{P + \alpha^2 Q + \kappa^2\sigma_v^2}{P} && (9)\end{aligned}$$

where the results follow because $X = U - \alpha S - \kappa V$ and $X \perp S, V$. Notice that when a $\mathbf{u}$ that is jointly typical with $\mathbf{s}$ and $\mathbf{v}$ is found, $\mathbf{x}$ satisfies the power constraint.
3) Decoding : The received signal is $\mathbf{y} = \mathbf{x} + \mathbf{s} + \mathbf{w}$. At the decoder, we look for an $\mathbf{u}$ that is jointly typical with $\mathbf{y}$. If such a unique $\mathbf{u}$ can be found, we declare $\mathbf{u}$ as the decoder output or, else, we declare a decoder failure. Let P_{e_2} be the probability of the event that the decoder output is not equal to the encoded $\mathbf{u}$ (this includes the probability of decoder failure as well as the probability of a decoder error).

   In order to analyze P_{e_2}, consider the equivalent communication channel between U and Y. Notice that we have in effect transmitted a codeword $\mathbf{u}$ from a random i.i.d codebook for U with 2^{NR_1} codewords through the equivalent channel whose output is $\mathbf{y}$. Again, from the extension of joint typicality to the infinite alphabet case,

$P_{e_2} \to 0$ as $N \to \infty$ provided that

$$\begin{aligned} I(U;Y) &> R_1 \\ I(U;Y) &= h(U) - h(U|Y) \\ &= h(U) - h(U - \alpha Y|Y) \\ &= h(U) - h(X + \alpha S + \kappa V - \alpha X - \alpha S - \alpha W|Y) \\ &= h(U) - h(\kappa V + (1-\alpha)X - \alpha W|Y) \end{aligned} \quad (10)$$

Now, let us choose

$$\alpha = \frac{P}{P+\sigma^2} \quad (11)$$

$$\kappa^2 = \frac{P^2}{(P+\sigma^2)\sigma_v^2} - \frac{\epsilon}{\sigma_v^2} \quad (12)$$

For the above choice of α, it can be seen that

$$\mathbb{E}[(\kappa V + (1-\alpha)X - \alpha W)Y] = 0$$

and, hence, (10) reduces to

$$\begin{aligned} I(U;Y) &= h(U) - h(\kappa V + (1-\alpha)X - \alpha W) \\ &= \frac{1}{2}\log\frac{P + \alpha^2 Q + \kappa^2\sigma^2}{P-\epsilon} \end{aligned} \quad (13)$$

Hence, P_{e_2} can be made arbitrarily small as long as

$$R_1 < \frac{1}{2}\log\frac{P+\alpha^2 Q + \kappa^2\sigma^2}{P-\epsilon} \quad (14)$$

Combining this with the condition for encoder failure, P_{e_1} and P_{e_2} can both be made arbitrarily small provided

$$\frac{1}{2}\log\frac{P+\alpha^2 Q+\kappa^2\sigma_v^2}{P} < R_1 < \frac{1}{2}\log\frac{P+\alpha^2 Q + \kappa^2\sigma^2}{P-\epsilon} \quad (15)$$

Therefore, by choosing an ϵ_1, $0 < \epsilon_1 < \epsilon$ and $R_1 = \frac{1}{2}\log\frac{P+\alpha^2 Q+\kappa^2\sigma^2}{P-\epsilon_1}$ we can satisfy (15) and make $P_{e_1} \to 0$ and $P_{e_2} \to 0$ as $N \to \infty$.

4) Estimation: If there is no decoding failure, we form the final estimate of $\mathbf{v}$ as an MMSE estimate of $\mathbf{v}$ from $[\mathbf{y}\ \mathbf{u}]$. This is given by,

$$\hat{\mathbf{v}} = \frac{\kappa\sigma_v^2}{P-\epsilon}(\mathbf{u} - \alpha\mathbf{y}) \quad (16)$$

The distortion is then given by,

$$E[(V-\hat{V})^2] = \frac{\sigma_v^2}{1+\frac{P}{\sigma^2}}\frac{P}{P-\epsilon} \le \frac{\sigma_v^2}{1+\frac{P}{\sigma^2}} + \delta(\epsilon) \quad (17)$$

with $\delta(\epsilon)$ is vanishing for arbitrarily small ϵ.
If an encoder or decoder failure was declared, we set the estimate of $\mathbf{v}$ to be the zero vector. However, as shown above the probability of these events can be made arbitrarily small and, hence, they do not contribute to the overall distortion, which can be seen to be arbitrarily close to the optimal distortion achievable in the absence of the interference.

We have presented a joint source channel coding scheme in the presence of an interference known only to the transmitter. The use of the term hybrid digital analog Costa coding needs some explanation. The scheme is not entirely analog in that the auxiliary random variable is from a discrete codebook. However, in contrast to digital Costa coding, the source is not explicitly quantized and appears in an analog fashion in the transmitted signal $\mathbf{x}$. This is the reason for calling this as HDA Costa coding and this has some interesting consequences which are discussed in the following section.

Another feature of the HDA Costa coding scheme is that it does not make use of binning, rather it needs a single quantizer codebook that is also a good channel code. In practice, this may have some impact on the design since there are ensembles of codes that are provably good quantizers and channel codes. In the Gaussian case, good lattices that are both good for coding and for quantization are known. The binning approach however, requires a nesting condition. That is, the big code much be a good channel code, but it much contain a subcode (and its cosets) that must be good quantizers. This may be a more difficult condition to obtain in practice.

C. *Generalized Hybrid Costa coding*

In the previous section, we described a technique where the source was not explicitly quantized. Here, we show that the source can be quantized to any rate $R < C$, the capacity of the channel and yet optimal performance can be obtained.

The main idea is to initially quantize the source $\mathbf{v}$ to $\mathbf{v}^*$ at a rate R,that is strictly lesser than the channel capacity, using an optimal vector quantizer. Let $\mathbf{e} = \mathbf{v} - \mathbf{v}^*$ be the quantization error vector.

We next define an auxiliary random variable U given by

$$U = X + \alpha S + \kappa_1 E \quad (18)$$

where $X \sim \mathcal{N}(0,P), E \sim \mathcal{N}(0, \sigma_v^2 2^{-2R})$ and $\kappa_1^2 = \frac{P}{P+\sigma^2}\frac{(P+\sigma^2)-\sigma^2 2^{2R}}{\sigma_v^2} - \frac{\epsilon((P+\sigma^2)-\sigma^2 2^{2R})}{P\sigma_v^2}$. Also X, S and E are independent.

1) Codebook generation: Generate a random i.i.d code book $\mathcal{U}$ with $2^{NI(U;Y)}$ sequences, where each component of each codeword is Gaussian with zero mean and variance $P + \alpha^2 Q + \kappa_1^2\sigma_v^2 2^{-2R}$. These codewords are uniformly distributed in 2^{NR} bins and this is shared between the encoder and the decoder.
2) Encoding: Let m be the quantization index corresponding to the quantized source $\mathbf{v}^*$. Let $i(\mathbf{u})$ represent the index of a bin that contains $\mathbf{u}$. For a given m find an $\mathbf{u}$ such that $i(\mathbf{u}) = m$ and $(\mathbf{u}, \mathbf{s}, \mathbf{e})$ are jointly typical with respect to the distribution in model (18). We next transmit the vector $\mathbf{x} = \mathbf{u} - \alpha\mathbf{s} - \kappa_1\mathbf{e}$. Note that since $(\mathbf{u}, \mathbf{s}, \mathbf{e})$ are jointly typical, from (18), we can see that $\mathbf{x} \perp \mathbf{s}, \mathbf{e}$ and satisfies the power constraint.
3) Decoding : The received signal is $\mathbf{y} = \mathbf{x} + \mathbf{s} + \mathbf{w}$. At the decoder, we look for an $\mathbf{u}$ that is jointly typical with $\mathbf{y}$. If such a unique $\mathbf{u}$ can be found, we declare $\mathbf{u}$ as the decoder output or, else, we declare a decoder failure. Next we make an estimate of $\mathbf{e}$ from $\mathbf{u}$ and $\mathbf{y}$.
We can see by similar Gelfand-Pinsker coding arguments that $R < I(U;Y) - I(U;S,E)$. Note

$$
\begin{aligned}
I(U;Y) &- I(U;S,E) \\
&= h(U|S,E) - h(U|Y) \\
&= h(X) - h(U-\alpha Y|Y) \\
&= h(X) - h(\kappa_1 E + (1-\alpha)X - \alpha W|Y) \\
&= h(X) - h(\kappa_1 E + (1-\alpha)X - \alpha W) \\
&= \frac{1}{2}\log\left(\frac{P}{\kappa_1^2\sigma_v^2 2^{-2R} + (1-\alpha)^2 P + \alpha^2\sigma^2}\right) \\
&> R
\end{aligned} \tag{19}
$$

This shows that we can decode the codeword $\mathbf{u}$ with a very high probability and we can decode the message $m = i(\mathbf{u})$ and v^*.

4) Estimation: If there is no decoding failure, we form the final estimate of $\mathbf{v}$ as an MMSE estimate of $\mathbf{v}$ from $[\mathbf{y}\ \mathbf{u}\ \mathbf{v}^*]$. The estimator is linear and the estimation error in $\mathbf{e}$ is $\left(\frac{\sigma_v^2}{1+\frac{P}{\sigma^2}}\right) + \delta(\epsilon)$. Since $\mathbf{v} = \mathbf{v}^* + \mathbf{e}$ and $\mathbf{v}^*$ is decoded correctly with a high probability the error in estimating $\mathbf{v}$, $D = \left(\frac{\sigma_v^2}{1+\frac{P}{\sigma^2}}\right) + \delta(\epsilon)$. Choosing ϵ arbitrarily small $\delta(\epsilon) \to 0$ and $D = \left(\frac{\sigma_v^2}{1+\frac{P}{\sigma^2}}\right)$.

It must be noted that this scheme is an intermediate between digital costa coding scheme with the maximum possible bins equal to the capacity of the channel and the analog Costa coding scheme with no bins. Thus we can get a family of schemes with varying bins for the Gaussian channel.

D. *Superimposed digital and analog Costa coding scheme*

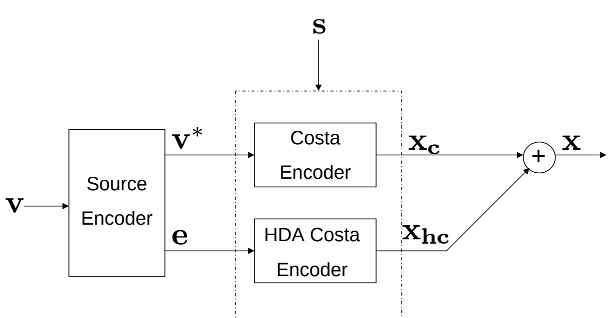

Fig. 2. Encoder model for superimposed coding

Recently in [4], Bross, Lapidoth and Tinguely considered the problem of transmitting N samples of a Gaussian source in N uses of an AWGN channel, in the absence of the interferer. They showed that there are infinitely many superposition based schemes, which contain pure separation based scheme and uncoded transmission as special cases. In this section, we show that the same is true in the presence of an interference also and show the corresponding superposition scheme, which is given in Fig. 2.

Here we first quantize the source at a rate of $R < C$ and let the quantization error be $\mathbf{e} = \mathbf{v} - \mathbf{v}^*$, where $\mathbf{v}^*$ is the reconstruction. The first stream in Fig. 2 is a digital Costa encoder that encodes the quantization index by treating $\mathbf{s}$ as interference and the channel noise and the second stream $\mathbf{x_{hc}}$ as independent Gaussian noise. A power of $(P+\sigma^2)(1-2^{-2R})$ is used in the first stream.

In the second stream, the quantization error $\mathbf{e}$ is encoded using an HDA Costa coding scheme and a power of $(P+\sigma^2)2^{-2R} - \sigma^2$ is used. The auxiliary random variable is chosen as $\mathbf{u} = \mathbf{x_{hc}} + \mathbf{x_c} + \mathbf{s} + \kappa\mathbf{e}$, where $\mathbf{x_{hc}}$ is chosen to be independent to $\mathbf{x_c}$, $\mathbf{s}$ and $\mathbf{e}$. $\mathbf{x_c} + \mathbf{s}$ is the net interference. This gives the optimal $\kappa = \sqrt{\frac{P}{P+\sigma^2}\frac{(P+\sigma^2)2^{-2R}-\sigma^2}{\sigma_v^2}}$. The optimal distortion can then be obtained as $D = \frac{\sigma_e^2}{1+\frac{(P+\sigma^2)2^{-2R}-\sigma^2}{\sigma^2}} = \frac{\sigma_v^2}{1+\frac{P}{\sigma^2}}$.

III. Transmission of a Gaussian source through a channel with side information available only at the receiver

In this section we consider the problem of sending an analog source over a Gaussian noise channel when the receiver has some side information about the source. This problem is a dual of the problem considered in the previous section and is considered here for the sake of completeness. The following schemes can be shown to be optimal for this case.

A. *Separation Based Scheme with Wyner Ziv Coding (Digital Wyner Ziv Coding)*

One strategy is using a separation scheme of Wyner-Ziv coding followed by a channel code. This coding scheme cannot improve the distortion performance when the actual SNR of the channel is better than the designed SNR. We briefly explain the digital Wyner-Ziv scheme and then establish our information theoretic model for the analog Wyner-Ziv coding using random coding arguments.

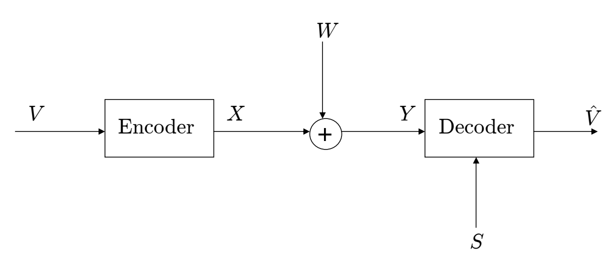

Fig. 3. Block diagram of the joint source channel coding problem with side information known only at the receiver.

Let S be the side information known at the receiver given by

$$V = S + Z \tag{20}$$

where $Z \sim \mathcal{N}(0, \sigma_z^2)$ and let $Y = X + W$, where $W \sim \mathcal{N}(0, \sigma^2)$. When the side information is available at the encoder as well as the receiver, the best possible distortion is $D=\frac{\sigma_z^2}{1+\frac{P}{\sigma^2}}$. The same distortion can be achieved using the following scheme and is a direct consequence of Wyner and Ziv's result [12]. This can be achieved as follows

Let U be an auxiliary random variable given by

$$U=\sqrt{\alpha}V+B \tag{21}$$

where $\alpha=1-\frac{D}{\sigma_z^2}=\frac{P}{P+\sigma^2}$ and $B\sim\mathcal{N}(0,D)$. We create an N-length i.i.d Gaussian code book $\mathcal{U}$ with $2^{NI(U;V)}$ codewords, where each component of the codeword is Gaussian with zero mean and variance $\alpha\sigma_v^2+D$ and evenly distribute them over 2^{NR} bins. Let $i(\mathbf{u})$ be the index of the bin containing $\mathbf{u}$. For each $\mathbf{v}$, find an $\mathbf{u}$ such that $(\mathbf{u},\mathbf{v})$ are jointly typical. The index $m=i(\mathbf{u})$ is transmitted over the Gaussian channel using a channel code. At the receiver decoding of m is possible with high probability as $R=\frac{1}{2}\log(1+\frac{P}{\sigma^2})$ which is the capacity of the channel. Next for the decoded m we look for an $\mathbf{u}$ such that $i(\mathbf{u})=m$ and $(\mathbf{u},\mathbf{s})$ are jointly typical. From $\mathbf{s}$ and the decoded $\mathbf{u}$ we make an estimate of the source $\mathbf{v}$ as follows.

$$\hat{\mathbf{v}}=\mathbf{s}+\sqrt{\alpha}(\mathbf{u}-\sqrt{\alpha}\mathbf{s}) \tag{22}$$

This yields the optimal distortion D.

B. Hybrid Digital Analog Wyner Ziv Coding

In this section, we discuss a different joint source channel coding that does not involve quantizing the source explicitly. This scheme is quite similar to the modulo lattice modulation scheme in [2]; the difference being that a nested lattice is not used. The auxiliary random variable U is generated as follows.

$$U=\kappa V+X \tag{23}$$

where κ is defined as before as $\kappa=\sqrt{\frac{P^2}{(P+\sigma^2)\sigma_z^2}-\frac{\epsilon}{\sigma_z^2}}$ and $X\sim\mathcal{N}(0,P)$.

1) Codebook generation: Generate a random i.i.d code book $\mathcal{U}$ with $2^{NI(U;V)}$ sequences, where each component of each codeword is Gaussian with zero mean and variance $P+\kappa^2\sigma_v^2$. This codebook is shared between the encoder and the decoder.
2) Encoding: For a given $\mathbf{v}$ find an $\mathbf{u}$ such that $(\mathbf{u},\mathbf{v})$ are jointly typical and transmit $\mathbf{x}=\mathbf{u}-\kappa\mathbf{v}$.
3) Decoding: The received signal is $\mathbf{y}=\mathbf{x}+\mathbf{w}$. From $\mathbf{y}$ find an $\mathbf{u}$ such that $(\mathbf{s},\mathbf{y},\mathbf{u})$ are jointly typical. This is possible because $I(U;V)>I(U;S,Y)$ since

$$\begin{aligned}
I(U;S,Y) &= h(U)-h(U|S,Y)\\
&= h(U)-h(U-\kappa S-\alpha Y|S,Y)\\
&= h(U)-h(\kappa Z+(1-\alpha)X-\alpha W|Y,S)\\
&= h(U)-h(\kappa Z+(1-\alpha)X-\alpha W)\\
&= \frac{1}{2}\log\left(\frac{P+\kappa^2\sigma_v^2}{\kappa^2\sigma_z^2+(1-\alpha)^2P+\alpha^2\sigma^2}\right)\\
&= \frac{1}{2}\log\left(\frac{P+\kappa^2\sigma_v^2}{P}\right)-\delta(\epsilon)\\
&= h(U)-h(U|V)-\delta(\epsilon)\\
&= I(U;V)-\delta(\epsilon)
\end{aligned} \tag{24}$$

Hence from knowing $\mathbf{u}$ and $\mathbf{s}$ we can make an estimate of $\mathbf{v}$ as

$$\hat{\mathbf{v}}=\mathbf{s}+\frac{\kappa\sigma_z^2}{P}(\mathbf{u}-\kappa\mathbf{s}-\alpha\mathbf{y}) \tag{25}$$

We once again obtain the optimal distortion D by making ϵ arbitrarily small and $\delta(\epsilon)\to 0$.

C. Superimposed digital and HDA Wyner-Ziv scheme

The above results could also be extended to a form of superimposed digital and analog coding. This is similar to the Costa coding case. We once again have two streams. The first stream uses a rate R Wyner Ziv code to quantize the source assuming the side information $\mathbf{s}$ is known at the receiver. Next a channel code is used to transmit the Wyner Ziv coded bits. The channel code is a superposition code that treats the second stream also as independent noise. At the receiver this channel code can be decoded and which gives the Wyner Ziv bits. The Wyner Ziv bits along with the side information bits $\mathbf{s}$ can be used to make an estimate of the source $\mathbf{v}$. This new estimate acts as the new side information $\tilde{\mathbf{s}}$.

Now the second stream employs the HDA Wyner Ziv scheme that is designed for the new side information $\tilde{\mathbf{s}}$ known only at the receiver. The two streams are superimposed and sent into the channel. At the decoder the first stream is decoded and cancelled from the received signal. The decoded first stream is used to make an estimate of the source to get the new side information $\tilde{\mathbf{s}}$. The final estimate of the source is obtained by the HDA decoding of the second stream.

The auxiliary random variable U is given by

$$U=X+\kappa_1 V \tag{26}$$

$X\sim\mathcal{N}(0,P_{aw})$ with $P_{aw}=(P+\sigma^2)2^{-2R}-\sigma^2$ and X and V are pairwise independent. Choose $\kappa_1\approx\sqrt{\frac{P_{aw}^2}{(P_{aw}+\sigma^2)\sigma_e^2}}$ where $\sigma_e^2=\sigma_z^2 2^{-2R}$. The digital part is first decoded and canceled from the received signal to get an equivalent channel with power constrain P_{aw} and channel noise σ^2. Next we use HDA Wyner Ziv coding in this equivalent channel. At the receiver the side information and the digital part can be used to construct an equivalent model relating the side information and source given by $V=\tilde{S}+\tilde{Z}$ where $\tilde{Z}$ has a variance $\sigma_z^2 2^{-2R}$. Then,

$$
\begin{aligned}
D &= \sigma_e^2 \frac{\sigma^2}{P_{aw}+\sigma^2} \\
&= \sigma_z^2 2^{-2R} \frac{\sigma^2}{P_{aw}+\sigma^2} \\
&= \sigma_z^2 \frac{P_{aw}+\sigma^2}{P+\sigma^2} \frac{\sigma^2}{P_{aw}+\sigma^2} \\
&= \frac{\sigma_z^2}{1+\frac{P}{\sigma^2}}
\end{aligned}
\qquad (27)
$$

IV. Transmission of a Gaussian Source with Interference at the Transmitter and Side Information at the Receiver

We can consider the transmission of the Gaussian source $\mathbf{v}$ through an AWGN channel with channel variance σ^2 in the presence of an interference $\mathbf{s}_c$ known to the transmitter and side information $\mathbf{s}_w$ known only at the receiver, where the side information is given by $\mathbf{v} = \mathbf{s}_w + \mathbf{z}$. The channel output $\mathbf{y}$ is given by

$$\mathbf{y} = \mathbf{x} + \mathbf{s}_c + \mathbf{w}$$

We can combine the results from the previous two sections as follows. Choose $U = X + \alpha S_c + \kappa V$ with $\kappa \approx \sqrt{\frac{P^2}{(P+\sigma^2)\sigma_z^2}}$. Now X is transmitted over the channel and from Y and S_w, U is decoded using typical set decoding. From the knowledge of U, S_w and Y, a linear estimate of V is formed to get the required optimal distortion.

V. Analysis of the Schemes for SNR Mismatch

In this section, we consider the performance of the above JSCC schemes for the case of SNR mismatch where we design the scheme to be optimal for a channel noise variance of σ^2, but the actual noise variance is $\sigma_a^2 < \sigma^2$.

Separation based digital schemes suffer from a pronounced threshold effect. When the channel SNR is worse than the designed SNR, the index cannot be decoded and when the channel SNR is better than the designed SNR, the distortion is limited by the quantization and does not improve. However, the hybrid digital analog schemes considered offer better performance in this situation.

Let us consider the joint source channel coding setup with side information at both the transmitter and receiver. We can decode $\mathbf{u}$ at the receiver when the SNR is better than the designed SNR and make an estimate of the source from the various observations at the receiver as shown below.

$$U = X + \alpha S_c + \kappa_w V \qquad (28)$$

$$V = S_w + Z \qquad (29)$$

$$Y = X + S_c + W_a \qquad (30)$$

where $\kappa_w = \sqrt{\frac{P^2}{(P+\sigma^2)\sigma_z^2}}$,$S_c \sim \mathcal{N}(0,Q)$ and $Z \sim \mathcal{N}(0,\sigma_z^2)$

From these observations an estimate of V is made by optimal linear estimation as all the variables are Gaussian.

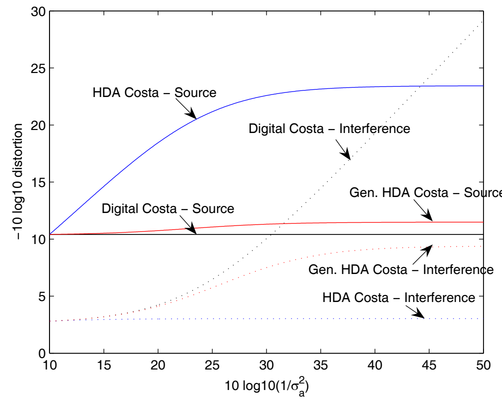

Fig. 4. Performance of the different Costa coding schemes for the joint source channel coding problem.

Define $\mathbf{R_m} = \begin{pmatrix} \sigma_v^2 - \sigma_z^2 & \kappa(\sigma_v^2-\sigma_z^2) & 0 \\ \kappa(\sigma_v^2-\sigma_z^2) & P+\alpha^2 Q + \kappa_w^2\sigma_v^2 & P+\alpha Q \\ 0 & P+\alpha Q & P+Q+\sigma_a^2 \end{pmatrix}$ and $\mathbf{P_m} = \begin{pmatrix} \sigma_v^2-\sigma_z^2 & \kappa\sigma_v^2 & 0 \end{pmatrix}$

Then the Distortion achievable for the channel SNR is given by

$$D_a = \sigma_v^2 - \mathbf{P_m}{\mathbf{R_m}}^{-1}{\mathbf{P_m}}^T \qquad (31)$$

This on further simplification yields

$$
\begin{aligned}
&D_a = \left[(Q\sigma^4 + (P(P+Q) + 2P\sigma^2 + \sigma^4)\sigma_a^2)\sigma_z^2\right] \times \\
&\left[P^2(P+Q) + P(P+Q)\sigma^2 + Q\sigma^4 + \right. \\
&\left.(P(2P+Q) + 3P\sigma^2 + \sigma^4)\sigma_a^2\right]^{-1}
\end{aligned}
\qquad (32)
$$

Let us now look at a few special cases

A. Hybrid Digital Analog Costa Coding

In this setup there is side information only at the transmitter. The distortion achievable for the user under SNR mismatch with the actual SNR greater than the designed SNR is given below and is obtained by setting $\sigma_v = \sigma_z$ in (32).

$$
\begin{aligned}
&D_{va} = \left[(Q\sigma^4 + (P(P+Q) + 2P\sigma^2 + \sigma^4)\sigma_a^2)\sigma_v^2\right] \times \\
&\left[P^2(P+Q) + P(P+Q)\sigma^2 + Q\sigma^4 + \right. \\
&\left.(P(2P+Q) + 3P\sigma^2 + \sigma^4)\sigma_a^2\right]^{-1}
\end{aligned}
\qquad (33)
$$

The distortion in the source $\mathbf{s}$ is shown in Fig.4 for a designed SNR of 10 dB as the actual channel SNR ($10\log 1/\sigma_a^2$) varies when the source and interference both have unit variance. It can be seen that the distortion in the source is smaller with the HDA Costa scheme than with the digital Costa scheme.

In some case, the distortion in estimating the interference at the receiver may also be of interest and can be obtained by estimating S from (28) and (30). The distortion is given below,

$$\begin{aligned} D_{sa} = &\left[Q(P+\sigma^2)(P^2+(2P+\sigma^2)\sigma_a^2)\right]\times \\ &\left[P^2(P+Q)+P(P+Q)\sigma^2+Q\sigma^4+\right. \\ &\left.(P(2P+Q)+3P\sigma^2+\sigma^4)\sigma_a^2\right]^{-1} \end{aligned} \tag{34}$$

It can be seen from Fig. 4 that the distortion in estimating the interference is better for the digital scheme than for the HDA Costa scheme.

B. Generalized HDA Costa Coding under channel mismatch

Next we analyze the performance of the generalized HDA costa coding under channel mismatch. This case leads to a few interesting analysis. By changing the source coding rate of the digital part R, we can tradeoff the distortion between the source and the interference in the presence of mismatch.

The different random variables and their relations are given below.

$$U = X + \alpha S + \kappa_1 E \tag{35}$$

$$Y = X + S + W_a \tag{36}$$

$$V = V^* + E \tag{37}$$

In the above equation $\kappa_1 = \sqrt{\frac{P}{P+\sigma^2}\frac{(P+\sigma^2)-\sigma^2 2^{2R}}{\sigma_v^2}}$

From the above equations an estimate of S as well as V is made. The resulting expressions of estimation error $D_{sa}(R)$ and $D_{va}(R)$ are given by

$$\begin{aligned} D_{va}(R) = &\left[(\sigma_a^2(\sigma^2+P)^2+(\sigma^4+\sigma_a^2 P)Q)\sigma_v^2\right]\times \\ &\left[(\sigma^2+P)^2(\sigma_a^2+P+Q)-2^{2R}(\sigma^2-\sigma_a^2)P(\sigma^2+P+Q)\right]^{-1} \end{aligned} \tag{38}$$

$$\begin{aligned} D_{sa}(R) = &\left[(\sigma^2+P)(2^{2R}(\sigma^2-\sigma_a^2)P-(\sigma^2+P)(\sigma_a^2+P))Q\right] \\ &\times\left[2^{2R}(\sigma^2-\sigma_a^2)P(\sigma^2+P+Q)-(\sigma^2+P)^2(\sigma_a^2+P+Q)\right]^{-1} \end{aligned} \tag{39}$$

The performance of the hybrid Costa scheme in relation to digital and analog schemes is shown in fig. 4. We can see that the hybrid scheme performs in between the digital and analog schemes.

C. Hybrid Digital Analog Wyner Ziv

In this case the distortion could be obtained by setting $Q = 0$ in (32). The actual distortion is given by

$$D_a = \frac{(P+\sigma^2)\sigma_a^2\sigma_z^2}{P^2+(2P+\sigma^2)\sigma_a^2} \tag{40}$$

This is clearly better than $\frac{\sigma_z^2\sigma^2}{P+\sigma^2}$ which is what is achievable with a separation based approach. However, we don't know if this is the optimal distortion that is achievable in the presence of channel mismatch. A simple lower bound on the achievable distortion in the presence of mismatch is to assume that the

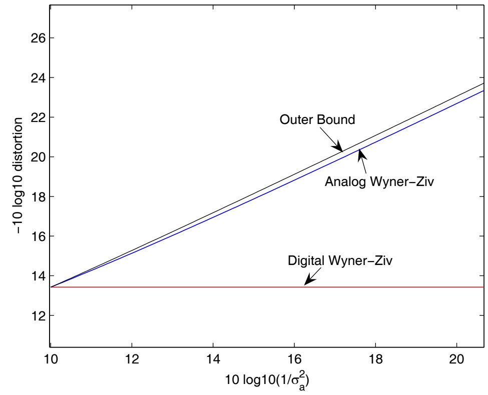

Fig. 5. Performance of the different Wyner-Ziv schemes for the joint source channel coding problem.

transmitter knows the channel SNR. Based on this we can make an analysis of the gap in db between the performance of HDA Wyner Ziv and the lower bound as follows.

The lower bound is given by

$$D_{bd} = \frac{\sigma_z^2}{1+P/\sigma_a^2} \tag{41}$$

Now the gap between the analog Wyner-Ziv and the bound at high SNR can be easily calculated as $\lim_{\sigma_a\to 0}\frac{D_{bd}}{D_a}$. The gap in db, G_{db} is hence given by

$$G_{db} = 10\log\left(\frac{P}{P+\sigma^2}\right) \tag{42}$$

This result is interesting since this says that if our designed SNR is say 10 db, for high SNRs, we loose at most $G_{db} = -0.41db$ which is numerically very close to the outer bound as shown in Fig. 5.

VI. Applications to Transmitting a Gaussian Source with Bandwidth Compression

We now consider the problem of transmitting K samples of the i.i.d Gaussian source to a single user in $N = K/\lambda$ uses of an AWGN channel with noise variance σ^2, where $\lambda < 1$. There is no interference in the channel, but since $\lambda < 1$, we will see that the techniques described in the previous sections are useful for this problem.

There at least three ways to achieve the optimal distortion in this case. One is to use a conventional separation based approach. The second one is to use superposition coding and the third one is to use Costa coding. Although, they are all optimal for the single user case, they perform differently when there is a mismatch in the channel SNR and, hence, the last two approaches are briefly described here.

a) Superposition Coding: Here we split the source in two parts and take N samples of the source $\mathbf{v}$, namely v_1^N and scale it by $\sqrt{a}$ creating the systematic signal $\mathbf{x}_1 = \sqrt{a}v_1^N$. We take the other $K-N$ source samples v_{N+1}^K and use a conventional source encoder followed by a capacity achieving channel code resulting in the N dimensional vector $\mathbf{x}_c = \mathcal{C}(\mathcal{Q}(v_{N+1}^K))$, where $\mathcal{C}$ denotes a channel encoding operation and $\mathcal{Q}$ denotes a source encoding operation. Then $\mathbf{x}_c$ is normalized so that the average power is $\sqrt{1-a}$. The overall transmitted signal is $\mathbf{y} = \mathbf{x}_s + \mathbf{x}_c$ and the received signal is $\mathbf{z} = \mathbf{y} + \mathbf{n}$. At the receiver, the digital part is first decoded assuming the systematic (analog) part is noise and then $\mathbf{x}_c$ is subtracted from $\mathbf{z}$. Then an MMSE estimate of v_1^N is formed. For the optimal choice of a, the optimal overall distortion can be obtained given by

$$a_{sup}^* = \sigma^2 \left[\left(1 + \frac{1}{\sigma^2}\right)^{\lambda} - 1 \right] \text{ and } D_{sup}^* = \frac{1}{\left(1+\frac{1}{\sigma^2}\right)^{\lambda}} \quad (43)$$

.

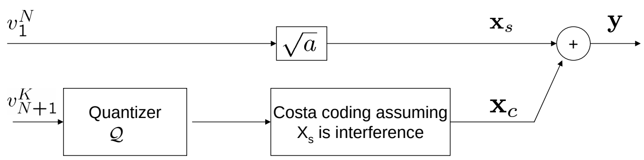

Fig. 6. Encoder model using Costa coding for single user

b) Digital Costa Coding: We split the source exactly as in the previous case and one stream is formed as $\mathbf{x}_s = \sqrt{a} v_1^N$. However, here the digital part assumes that $\mathbf{x}_s$ is interference and uses Costa coding to produce $\mathbf{x}_c$ with power $1-a$ as shown in Fig. 6. In Costa coding, we define an auxiliary random variable $\mathbf{u} = \mathbf{x}_c + \alpha_1 \mathbf{x}_s$ where $\alpha_1 = \frac{1-a}{1-a+\sigma^2}$ is the optimum scaling coefficient. At the receiver, the digital part is decoded which means that $\mathbf{u}$ can be obtained. In spite of knowing $\mathbf{u}$ exactly, the optimal estimate of v_1^N is obtained by simply treating $\mathbf{x}_c$ as noise since for the optimal choice of α_1, $\mathbf{x}_c = \mathbf{u} - \alpha_1 \mathbf{x}_s$ and v_1^N are uncorrelated. Therefore, an MMSE estimate of v_1^N is formed assuming $\mathbf{x}_c$ were noise. Hence, the overall distortion becomes

$$D = \frac{\lambda}{1 + \frac{a}{1-a+\sigma^2}} + \frac{1-\lambda}{\left(1+\frac{1-a}{\sigma^2}\right)^{\lambda/1-\lambda}} \quad (44)$$

Again, minimizing D w.r.t. a gives

$$a_{costa}^* = (1+\sigma^2)\left[1 - \frac{1}{\left(1+\frac{1}{\sigma^2}\right)^{\lambda}}\right] \text{and } D_{costa}^* = \frac{1}{\left(1+\frac{1}{\sigma^2}\right)^{\lambda}} \quad (45)$$

which is the best possible distortion.

c) Hybrid Digital Analog Costa Coding: For the case of $\lambda = 0.5$, the digital Costa coding part can be replaced by a hybrid digital analog (HDA) Costa coding. We refer to such a scheme as HDA Costa coding. The same power allocation however, remains the same and hence, we can simply use a_{Costa}^* without the need to differentiate the digital and HDA costa coding. It is quite straightforward to show that $a_{Costa}^* > a_{sup}^*$ for $\lambda < 1$. Hence, the Costa coding approach allocates higher power to the systematic part than the superposition approach, since the systematic part is treated as interference.

A. Performance in the presence of SNR mismatch

Now, we consider the same set up as above, but when the actual channel noise variance is σ_a^2, whereas the designed noise variance is σ^2.

Case 1: $\sigma_a^2 > \sigma^2$

The distortion for the superposition code can be computed to be the sum of the distortions in the systematic part and the digital part. When $\sigma_a^2 > \sigma^2$, the digital part cannot be decoded and, hence, we assume that the distortion in the digital part is the variance of the source, 1.

$$D_{sup} = \frac{\lambda}{1 + \frac{a_{sup}^*}{1-a_{sup}^*+\sigma_a^2}} + (1-\lambda)\cdot 1 \quad (46)$$

Both the digital and analog Costa coding schemes perform identically when $\sigma_a^2 \geq \sigma^2$ and the distortion for the Costa code can be computed to be

$$D_{digCosta} = D_{analogCosta} = \frac{\lambda}{1 + \frac{a_{costa}^*}{1-a_{costa}^*+\sigma_a^2}} + (1-\lambda)\cdot 1 \quad (47)$$

Case 2: $\sigma_a^2 < \sigma^2$ In this case, the digital part can be decoded exactly and, hence, the distortion for superposition coding is

$$D_{sup} = \lambda \, \frac{1}{1 + \frac{a_{sup}^*}{\sigma_a^2}} + (1-\lambda) \, \frac{1}{\left(1 + \frac{1-a_{sup}^*}{a_{sup}^*+\sigma^2}\right)^{\lambda/(1-\lambda)}} \quad (48)$$

For digital Costa coding, the decoder first decodes the digital part when the auxiliary random variable $\mathbf{u}$ is perfectly known. In the case when $\sigma_a^2 \neq \sigma^2$, the receiver must form the MMSE estimate of v_1^N from the channel observation $\mathbf{y}$ and $\mathbf{u}$. Therefore, the overall distortion is

$$D_{digCosta} = \lambda \Bigg(1 - [\sqrt{a_{Costa}^*} \;\; \alpha\sqrt{a_{Costa}^*}] \times \begin{bmatrix} 1+\sigma_a^2 & 1-a_{Costa}^*+\alpha a_{Costa}^* \\ 1-a_{Costa}^*+\alpha a_{Costa}^* & 1-a_{Costa}^*+\alpha^2 a_{Costa}^* \end{bmatrix}^{-1} \times \begin{bmatrix} \sqrt{a_{Costa}^*} \\ \alpha\sqrt{a_{Costa}^*} \end{bmatrix}\Bigg) + (1-\lambda)\frac{1}{\left(1+\frac{1-a_{costa}^*}{a_{costa}^*+\sigma^2}\right)^{\lambda/(1-\lambda)}} \quad (49)$$

For the hybrid digital analog Costa coding, we can decode $\mathbf{u}$ and form MMSE estimates of v_1^N and v_{N+1}^K separately and, hence, the overall distortion is given by

$$D_{HDACosta} = \lambda \Bigg(1 - [\sqrt{a^*_{Costa}} \ \alpha\sqrt{a^*_{Costa}}] \times \begin{bmatrix} 1+\sigma_a^2 & 1-a^*_{Costa}+\alpha a^*_{Costa} \\ 1-a^*_{Costa}+\alpha a^*_{Costa} & 1-a^*_{Costa}+\alpha^2 a^*_{Costa}+\kappa^2 \end{bmatrix}^{-1} \times \begin{bmatrix} \sqrt{a^*_{Costa}} \\ \alpha\sqrt{a^*_{Costa}} \end{bmatrix} \Bigg) + (1-\lambda)\,(1-[0 \ \kappa] \times \begin{bmatrix} 1+\sigma_a^2 & 1-a^*_{Costa}+\alpha a^*_{Costa} \\ 1-a^*_{Costa}+\alpha a^*_{Costa} & 1-a^*_{Costa}+\alpha^2 a^*_{Costa}+\kappa^2 \end{bmatrix}^{-1} \times \begin{bmatrix} 0 \\ \kappa \end{bmatrix} \Bigg) \quad (50)$$

The performance of the superposition scheme, digital Costa and HDA costa scheme are shown for an example with $\lambda = 0.5$ in Fig. 7. The designed SNR is defined as $10\log_{10}\frac{1}{\sigma^2}$ whereas the actual SNR is defined as $10\log_{10}\frac{1}{\sigma_a^2}$. In the example, the designed SNR is fixed at 10dB and the actual SNR is varied from 0 dB to 20 dB. It can be seen that the Costa coding approach is better than superposition coding when $\sigma_a^2 < \sigma^2$ and worse for the other case. The HDA Costa coding scheme performs the best over the entire range of SNRs.

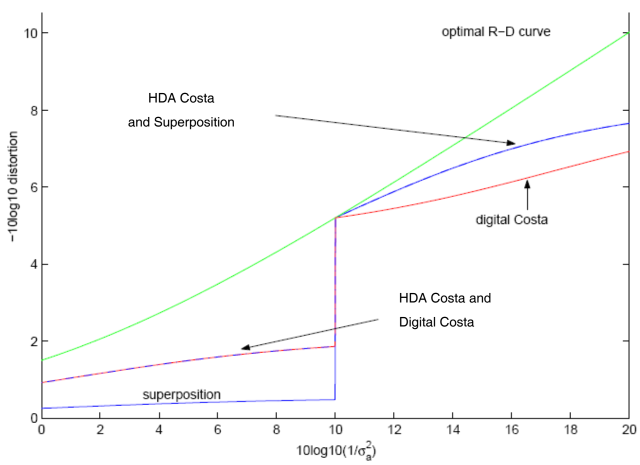

Fig. 7. Performance of different schemes for the source splitting approach for the bandwidth compression problem with SNR mismatch.

VII. Applications to Broadcasting with Bandwidth Compression

We now consider the problem of transmitting $K = 2N$ samples of a unit variance Gaussian source $\mathbf{v}$ in N uses of the channel to two users through AWGN channels with noise variances σ_1^2 (weak user) and σ_2^2 (strong user) with $\sigma_1 > \sigma_2$. The channel has the power constraint $P = 1$. We are interested in joint source channel coding schemes that provide a good region of pairs of distortion that are simultaneously achievable at the two users. This problem was considered in [1, 3, 6]. The best known region to date is given by the schemes therein.

Notice that when we design a source channel coding scheme to be optimal for the weak user, the strong user operates under the situation of SNR mismatch explained in Section VI-A with $\sigma_a^2 < \sigma^2$. Similarly, when the system is designed to be optimal for the strong user, for the weak user $\sigma_a^2 > \sigma^2$. Motivated by the fact that for $\lambda = 0.5$, the HDA costa coding scheme performs the best, we propose a scheme which is shown in Fig. 8.

There are three layers in the proposed coding scheme. The first layer is the systematic part where N out of the K samples of the source are scaled by $\sqrt{a}$. Let us call this as $\mathbf{x}_s = \sqrt{a}v_1^N$. The other $K - N$ samples of the Gaussian source are hybrid digital analog Costa coding, treating $\mathbf{x}_s$ as the interference and transmits the signal $\mathbf{x}_1$ with power b in the second layer. So $\mathbf{x}_1 = \mathbf{u}_1 - \alpha_1\mathbf{x}_s - \kappa_c v_{N+1}^K$, where α_1 and κ_c are the optimal scaling coefficient to be used in the hybrid digital analog Costa coding process and $\mathbf{u}_1$ is the auxiliary variable. This layer is meant to be decoded by the weak user and, hence, the scaling factor α_1 is set to be $b/(b + c + \sigma_1^2)$. That is, this layer sees the third layer also as *independent* noise.

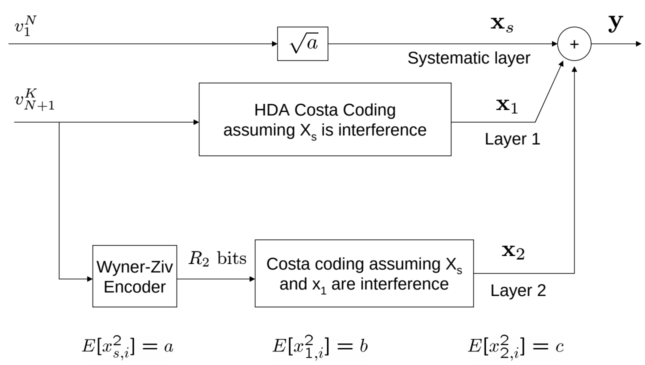

Fig. 8. Encoder model using Costa coding

The third layer is first Wyner Ziv coded at a rate R_2 assuming the estimate of v_{N+1}^K at the receiver as side information. The Wyner-Ziv index is then encoded using digital Costa coding assuming $\mathbf{x}_s$ and $\mathbf{x}_1$ are interference and uses power $c = 1 - a - b$. Therefore, $\mathbf{x}_2 = \mathbf{u}_2 - \alpha_2(\mathbf{x}_s + \mathbf{x}_1)$. This layer is meant for the strong user and, hence, the scaling factor $\alpha_2 = c/(c + \sigma_2^2)$. We then transmit $\mathbf{y} = \mathbf{x}_s + \mathbf{x}_1 + \mathbf{x}_2$.

At the receiver, from the second layer an estimate of v_{N+1}^K is obtained. This estimate acts as side information that can be used in refining the estimate of v_{N+1}^K for the strong user using the decoded Wyner-Ziv bits. The Wyner-Ziv bits are decoded from the third layer by Costa decoding procedure.

The users estimate the systematic part v_1^N and non-systematic part v_{N+1}^K by MMSE estimation from the received $\mathbf{y}$, the decoded $\mathbf{u}_1$ and $\mathbf{u}_2$. So the overall distortion seen at the weak user is

$$D_1 = \frac{1}{2}\frac{1}{1+\frac{a}{b+c+\sigma_1^2}} + \frac{1}{2}\frac{1}{1+\frac{b}{c+\sigma_1^2}}$$

The distortion for the strong user is given by

$$D_2 = \frac{1}{2}\left(1 - [\sqrt{a}\ \alpha_1\sqrt{a}]\begin{bmatrix} 1+\sigma_2^2 & b+\alpha_1 a \\ b+\alpha_1 a & b+\alpha_1^2 a+\kappa^2 \end{bmatrix}^{-1} \times \begin{bmatrix} \sqrt{a} \\ \alpha_1\sqrt{a} \end{bmatrix}\right) + \frac{1/2}{1+\frac{c}{\sigma_2^2}}\left(1 - [0\ \kappa] \times \begin{bmatrix} 1+\sigma_2^2 & b+\alpha_1 a \\ b+\alpha_1 a & b+\alpha_1^2 a+\kappa^2 \end{bmatrix}^{-1}\begin{bmatrix} 0 \\ \kappa \end{bmatrix}\right) \tag{51}$$

The corner points of the distortion region corresponding to being optimal for the strong and weak user respectively, can be obtained by setting $c = 0$ and $b = 0$, respectively.

The distortion region for this scheme for the case of $\sigma_1^2 = 0$ dB and $\sigma_2^2 = 5$ dB is shown in Fig. 9. The distortion region for three other schemes are also shown. They are the scheme proposed by Mittal and Phamdo in [1], a different broadcasting scheme which uses digital Costa coding in both the layers proposed in [6] (details can be found there) and the broadcast scheme with one layer of superposition coding and one layer of digital Costa coding considered in [3, 6]. This scheme, referred to as superposition+Costa currently appears to be the best known scheme.

The proposed broadcast scheme in Fig. 8 significantly outperforms the scheme in Mittal and Phamdo and the digital Costa based broadcast scheme for this example. The corner points of this scheme also coincide with that of the superposition+Costa scheme, which are the best known corner points.

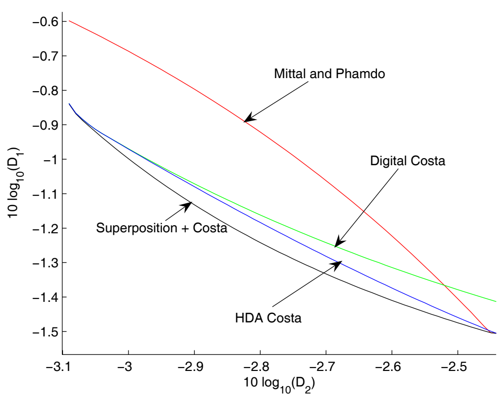

Fig. 9. Distortion regions of the different schemes for broadcasting with bandwidth compression.

VIII. Conclusion and Future Work

We discussed hybrid digital analog version of Costa coding and Wyner-Ziv coding for transmitting an analog Gaussian source through an AWGN channel in the presence of an interferer known only to the transmitter and side information available only to the receiver respectively. These schemes are closely related to the schemes by Reznic and Zamir [2] and [5], but make the auxiliary random variable model more explicit. We also showed that there are infinitely many schemes that are optimal for this problem, extending the work of Bross, Lapidoth and Tinguely [4] to the side information case. The HDA coding schemes have advantages over strictly digital schemes when there is a mismatch in the channel SNR. This makes them also useful for broadcasting a Gaussian source to two users with different SNRs.

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