Neighbor Discovery in Wireless Networks: A Multiuser-Detection Approach

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Abstract— **We examine the problem of determining which nodes are neighbors of a given one in a wireless network. We consider an unsupervised network operating on a frequency-flat Gaussian channel, where $K+1$ nodes associate their identities to nonorthogonal signatures, transmitted at random times, synchronously, and independently. A number of neighbor-discovery algorithms, based on different optimization criteria, are introduced and analyzed. Numerical results show how reduced-complexity algorithms can achieve a satisfactory performance.**

I. Introduction

Of late, wireless networks, and in particular sensor networks, have been the object of a good deal of interest, also spurred by the manifold applications they can be associated with (see, for example, their applications to classification and tracking [1] and to monitoring [2]). A characteristic requirement of several wireless networks, which enables them to adapt themselves to a changing environment, is that they be "self-configuring," i.e., that a large number of wireless nodes organize themselves to perform the tasks required by the application they have been deployed for: examples of self-configuration include construction of routing paths, clustering, and formation of minimum-weight trees. In this paper, we consider an aspect of self-configuration in wireless networks referred to as *neighbor discovery* (ND). Neighbor discovery is the determination of all nodes in the network a given node may directly communicate with. Knowledge of neighbors is essential for all routing protocols, medium-access control protocols, and several other topology-control algorithms. Ideally, nodes should discover their neighbors as quickly as possible, which will allow nodes to save energy in their discovery phase. Also, rapid discovery allows for other protocols (such as routing protocols) to quickly start their execution. In addition, ND may also be the solution for "partner selection" in cooperative wireless networks. In fact, cooperation among users may carry advantages only if the partners are chosen in a proper way: for example, "decode-and-forward" (DAF) protocols may suffer from cooperation with weak users, thus failing in the goal of increasing the diversity order [3].

Recently, a number studies on ND algorithms have appeared (see, e.g., [4], [5] and the references therein). Most of these approach ND at a protocol level, defining node *A* to be a neighbor of node *B* if *A* can exceed *B*'s signal to noise-ratio requirement: as a consequence, *A* is inserted in the neighbor list of *B* based solely upon successful reception, at node *B*, of a packet sent by node *A*. Moreover, the Internet Engineering Task Force proposes to perform Neighbor Discovery "at IP Layer" [6]. The corresponding protocol assumes a broadcast capability at physical layer, and a MAC which handles contention. Now, ND algorithms for wireless networks may not be contention-based when energy constraints are tight: retransmission in the case of a collision costs energy, which might be a resource at a premium. In this context, we consider a transmission scheme which avoids collisions at modulation level and is based on simultaneous transmission of signatures. In principle, if the nodes' waveforms were orthogonal, no collision would occur. In practice, these waveforms have a small correlation, causing an interference whose amount may be controlled by multiuser-detection algorithms.

ND can be performed in a supervised or unsupervised manner. In supervised methods, there is a central controller (e.g., a leader node) which processes the signal received from all nodes, determines the network configuration, and communicates to all nodes their neighbor lists. Supervised ND algorithms are expected to cost a large amount of energy, and hence they should be discarded for energy-limited networks. Unsupervised ND algorithms have no central controller: there, each node discovers its own neighbors. Another important issue in ND problems is the timing aspect. In [7], the frame-synchronous assumption is justified by the presence in each node of Global Positioning System (GPS) devices. In [5], asynchronous algorithms are addressed, assuming that nodes can synchronize at bit level (which is the assumption we make in the following).

The goal of this work is to provide the foundations of signal processing for ND in wireless networks. We consider an unsupervised wireless network in a frequency-flat Gaussian multiple-access channel, shared by $K+1$ nodes which transmit, synchronously and independently, a set of known signatures according to the scheme advocated in [5]. Each node is identified by its own unique signature, and every node keeps a list of all the signatures of the network. A node is called a *neighbor* of the reference node if its amplitude, received by the latter, exceeds a preassigned *activity threshold*, say τ_A. Moreover, nodes cannot transmit and receive simultaneously on the same channel,[1] and the maximum number of active nodes is fixed and finite. We clarify that a neighbor relation between two nodes need not be bidirectional, since each node discovers those nodes it can receive from.

The organization of this paper is the following. In Section II we provide a model for the physical aspects of the networks, and we formulate our problem. ND algorithms are introduced in Section III, and analyzed in Section IV. Section V shows some numerical results, while Section VI concludes the paper.

II. Signal model and problem formulation

Our scenario is based on the transmission scheme illustrated in Fig. 1, which corresponds to node 0 searching its own neighbors among four other nodes. [2] In every time interval ("slot"), each node i, $i = 1, \ldots, K$, transmits its own signature, independently of the other nodes, with probability ε_i, while otherwise (and hence with probability $1 - \varepsilon_i$) it senses the channel.

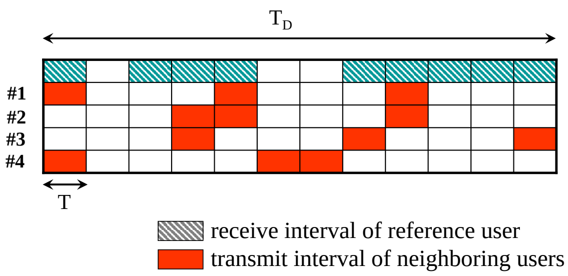

Fig. 1. *A scheme for synchronous neighbor discovery.*

The ND algorithm runs in a finite period, called a *discovery session*, whose duration is denoted T_D. During T_D, every active node transmits a number of signals containing one or more copies of its signature. Each signal has duration $T = T_D/N$, with N the number of slots in the discovery session. The network is assumed to be unsupervised, which implies that all nodes are independent and at the same hierarchical level: as a consequence, the ND algorithm is run in parallel by all nodes. Under the assumptions made in Section I, the baseband representation of the signal received by node 0 in the time interval $\big[(n-1)T, nT\big)$, $n \in \{1, 2, \ldots, N\}$, is

$$y(t) = \begin{cases} \sum_{k=1}^{K} \psi_{k,n} \alpha_k s_k\big(t-(n-1)T\big) + z(t) & \text{if } \psi_{0,n} = 0 \\ 0 & \text{if } \psi_{0,n} = 1 \end{cases} \tag{1}$$

where α_k denotes the channel gain, i.e., the complex amplitude of the signal received from node k and assumed to be constant during all the discovery session, $s_k(\cdot)$ is the kth node signature, $\psi_{k,n}$ is a random variable taking value 1 if node k is transmitting at time n, and value 0 otherwise (so that $\mathbb{P}(\psi_{k,n} = 1) = \varepsilon_k$), and $z(t)$ is additive white complex Gaussian noise having spectral density $2N_0$. We assume α_k to be modeled by a complex circularly symmetric Gaussian random variable with variance $2\sigma_k^2$. The signatures can be expressed as

$$s_k(t) = \sum_{l=1}^{L} s_{l,k} \phi\big(t-(l-1)T_c\big)/\sqrt{L} \tag{2}$$

where $s_{l,k} \in \{-1, +1\}$ is the lth chip of the kth signature, L is the processing gain, $T_c = T/L$ is the chip duration, and $\phi(\cdot)$ is the (unit-energy) chip waveform.[3] The slots devoted to channel sensing need not be adjacent: however, due to our flat-fading assumption, we may assume, without any loss of generality, a sensing phase of

$$M = \sum_{n=1}^{N} (1 - \psi_{0,n}) = N - \nu_0 \tag{3}$$

consecutive slots with intermittent other-users activity, with ν_0 the number of slots where node "0" is transmitting. Notice that M is random (N is assumed fixed and node 0 has its own activity factor ε_0), but the value it takes is known to node 0. Hence, in all subsequent derivations we refer to a given value of M. Of course, we may adopt the silent phases of node 0 as a time scale, recasting (1), with a slight notational abuse, in the form:[4]

$$y(t) = \sum_{k=1}^{K} \psi_{k,p} \alpha_k s_k\big(t-(p-1)T\big) + z(t) \tag{4}$$

where $0 \le t \le MT$ and $p = 1, 2, \ldots, M$. Our problem is now reduced to determining the indexes k such that $\{|\alpha_k|\}_{k=1}^{K}$ exceed an "activity threshold" τ_{A}, based on model (4).

Since $z(t)$ is white Gaussian noise, the components of $y(t)$ orthogonal to the subspace spanned by the signatures are irrelevant to our detection problem [8]. As a consequence, we might in principle adopt the signatures themselves, and their delayed versions, as an expansion basis for such a subspace. Alternatively, we may use the $L-$dimensional orthonormal basis

$$\bigcup_{\ell=0}^{L-1} \big\{\phi\big(t-\ell T_c-(p-1)T\big)\big\} \tag{5}$$

to expand the signal in the interval $\big[(n-1)T, nT\big)$. The two approaches are obviously equivalent, but the latter is mandatory in situations where the discovering node has no prior information as to the signatures of other users: although we do not deal blind ND in this paper, we choose this one due to its inherent flexibility.

Defining the scalar products

$$\mathbf{y}_{i,p} \triangleq \int_{(p-1)T}^{pT} y(t)\phi^*\big(t-(i-1)T_c-(p-1)T\big)\,dt \tag{6}$$

with * denoting conjugation, we obtain a vector representation $\mathbf{y}_p \triangleq [y_{1,p}, y_{2,p}, \ldots, y_{L,p}]'$ of the signal received in $\big[(p-$

[1] For simplicity, we disregard the more general case of nodes that can be in an *idle* state, i.e., they are neither receiving nor transmitting.

[2] We consider node 0 to be the reference node. Since all nodes are at the same hierarchical level, the same analysis applies to any node.

[3] The signatures are assumed to have unit energy.

[4] Notice that the index n refers to consecutive time slots, while p refers to the time scale defined by the silent phase of node "0".

$1)T, pT)$:

$$\mathbf{y}_p = \sum_{k=1}^{K} \psi_{k,p}\alpha_k \mathbf{s}_k + \mathbf{n}_p = \mathbf{S}\mathbf{\Psi}_p \boldsymbol{\alpha} + \mathbf{z}_p \tag{7}$$

where $\mathbf{s}_k \triangleq \frac{1}{\sqrt{L}}[s_{1,k}, s_{2,k}, \ldots, s_{L,k}]^T$, $\mathbf{S} \triangleq [\mathbf{s}_1, \mathbf{s}_2, \ldots, \mathbf{s}_K]$, $\mathbf{\Psi}_p \triangleq \text{diag}(\psi_{1,p}, \psi_{2,p}, \ldots, \psi_{K,p})$, $\boldsymbol{\alpha} \triangleq [\alpha_1, \alpha_2, \ldots, \alpha_K]^T$, $\mathbf{z}_p \triangleq [z_{1,p}, z_{2,p}, \ldots, z_{L,p}]^T$, and

$$z_{i,p} \triangleq \int_{(p-1)T}^{pT} z(t)\phi^*\big(t - (i-1)T_c - (p-1)T\big)\, dt \tag{8}$$

The ND problem now consists of assessing, after observing the set of M vectors $\mathbf{y}_{1:M} \triangleq \{\mathbf{y}_1, \ldots, \mathbf{y}_M\}$, which ones, among $|\alpha_1|, \ldots, |\alpha_K|$, exceed the "activity threshold" τ_A.

III. ND ALGORITHMS

A sensible criterion for the selection of a ND algorithm consists of minimizing the probability of choosing, among the K network nodes under scrutiny, an erroneous set of neighbors of node 0. Since there are 2^K such sets, each corresponding to one hypothesis H, this error probability is minimized by the maximum a posteriori (MAP) decision rule:

$$\widehat{H} = \arg\max_{H} P(H) p(\mathbf{y}_{1:M} \mid H) \tag{9}$$

where $P(H)$ is the a priori probability of hypothesis H, and $p(\mathbf{y}_{1:M}|H)$ is the probability density of the observations given H.

Now, $p(\mathbf{y}_{1:M} \mid H)$ depends on the actual pattern of transmit/receive intervals of each node, denoted $\mathbf{\Psi}_{1:M}$. Since this is unknown under our assumption that the transmission of signatures is not coordinated, it should be obtained from the marginalization

$$\sum_{\mathbf{\Psi}_{1:M}} p(\mathbf{y}_{1:M} \mid H, \mathbf{\Psi}_{1:M}) \mathbb{P}\{\mathbf{\Psi}_{1:M}\}$$

which has a complexity that grows exponentially with KM.

To overcome this complexity obstacle, the decision on the neighbor set works as follows. We first obtain estimates of the instantaneous powers $\widehat{|\alpha_i|^2}$ of all nodes, next we decide that a node is a neighbor by comparing each of them with a threshold, i.e.,

$$\widehat{|\alpha_i|^2} \underset{H_0}{\overset{H_1}{\gtrless}} \tau_i^2 \tag{10}$$

where

H_1: The received instantaneous power exceeds τ_A^2.
H_0: The received instantaneous power is below τ_A^2.

The performance of this test can be expressed through its probability $P_F^{(i)}$ of a *false-alarm* and its probability $P_M^{(i)}$ of a *miss*, defined as:

$$\begin{aligned} P_F^{(i)} &= \mathbb{P}\left\{|\widehat{\alpha}_i|^2 > \tau_i^2 \mid |\alpha_i| < \tau_A\right\} \\ P_M^{(i)} &= \mathbb{P}\left\{|\widehat{\alpha}_i|^2 < \tau_i^2 \mid |\alpha_i| > \tau_A\right\} \end{aligned} \tag{11}$$

These are related to the overall error probability through[5]

$$P^{(i)}(e) = P_F^{(i)} \mathbb{P}\{|\alpha_i| < \tau_A\} + P_M^{(i)} \mathbb{P}\{|\alpha_i| > \tau_A\} \tag{12}$$

Now, the maximum-likelihood (ML) estimators of the instantaneous powers can be obtained by jointly estimating $\boldsymbol{\alpha}$ and the matrix sequence $\mathbf{\Psi}_{1:M}$. Straightforward calculations show that the ML estimates of $\boldsymbol{\alpha}$ and $\mathbf{\Psi}_{1:M}$ result from the solution of the 2^{KM} linear systems — each corresponding to an outcome $\mathbf{\Psi}_{1:M,i}$ of the matrix sequence $\mathbf{\Psi}_{1:M}$:

$$\left(\sum_{p=1}^{M} \mathbf{\Psi}_{p,i} \boldsymbol{S}^\dagger \boldsymbol{S} \mathbf{\Psi}_{p,i}\right) \boldsymbol{\alpha} = \left(\sum_{p=1}^{M} \mathbf{\Psi}_{p,i} \boldsymbol{S}^\dagger \boldsymbol{y}_p\right) . \tag{13}$$

Computing

$$\widehat{\boldsymbol{\alpha}}_{ML} = \arg\min_{i=1:,\ldots,2^{KM}} \sum_{p=1}^{M} \| \boldsymbol{y}_p - \boldsymbol{S}\mathbf{\Psi}_{p,i}\widehat{\boldsymbol{\alpha}}_i \|^2 \tag{14}$$

with $\widehat{\boldsymbol{\alpha}}_i$ the solution corresponding to $\mathbf{\Psi}_{1:M,i}$, and recalling that ML estimates commute under nonlinear transformations, test (10) can be implemented by using $\widehat{|\alpha_i|^2} = |\widehat{\alpha}_{ML,i}|^2$,

Even with this receiver, implementation complexity would be unrealistic, and hence a further simplification is called for. Instead of dealing with the receive/transmit pattern related to the whole discovery session, we rather obtain estimates based on a single T-interval observation, which are then combined according to a suitable integration strategy.

A. Suboptimum ND algorithms

Consider again model (7). The ML estimate of $\mathbf{\Psi}_p\boldsymbol{\alpha}$, based upon the observation $\boldsymbol{y}_p$ available in slot p, is

$$\widehat{\mathbf{\Psi}_p\boldsymbol{\alpha}} = (\boldsymbol{S}^\dagger\boldsymbol{S})^{-1}\boldsymbol{S}^\dagger\boldsymbol{y}_p = \boldsymbol{S}^+\boldsymbol{y}_p \tag{15}$$

where $\boldsymbol{S}^+$ denotes the pseudo-inverse of the tall matrix $\boldsymbol{S}$.

A closer look at this solution reveals that, since

$$\boldsymbol{S}^+\boldsymbol{y}_p = \boldsymbol{S}^+\boldsymbol{S}\mathbf{\Psi}_p\boldsymbol{\alpha} + \boldsymbol{S}^+\boldsymbol{z}_p = \mathbf{\Psi}_p\boldsymbol{\alpha} + \boldsymbol{w}_p \tag{16}$$

with $\text{E}[\boldsymbol{w}_p\boldsymbol{w}_p^\dagger] = 2N_0(\boldsymbol{S}^\dagger\boldsymbol{S})^{-1}$, the interference from the other users is completely eliminated, at the price of some noise enhancement, reflecting the increase of the variance of its ith component by the factor $\{(\boldsymbol{S}^\dagger\boldsymbol{S})^{-1}_{i,i}\}_{i=1}^K$. It is interesting to notice that this estimate is noise-limited, but not interference-limited, implying that any receiver based on (15) is *asymptotically efficient* [10]; likewise, *near-far resistance* is granted [10].

Since there are M sensing phases, the M estimates resulting from repeated application of (15) should be combined to yield the final test statistic. Borrowing techniques from radar detection theory, reasonable combination criteria are *coherent integration* (CI), wherein an estimate of the instantaneous power is obtained as

$$\widehat{|\alpha_i|^2}_{CI} \triangleq \left|\frac{1}{M}\sum_{p=1}^{M}(\boldsymbol{S}^+\boldsymbol{y}_p)_i\right|^2 \tag{17}$$

[5]In what follows, the superscripts will be skipped whenever no confusion is induced by this notational simplification.

and *incoherent integration* (II)

$$\widehat{|\alpha_i|^2}_{II} \triangleq \frac{1}{M}\sum_{p=1}^{M} |(\boldsymbol{S}^{+}\boldsymbol{y}_p)_i|^2 \tag{18}$$

Notice that

$$\begin{aligned} \mathrm{E}\left[\widehat{|\alpha_i|^2}_{CI}\middle| M, |\alpha_i|^2\right] &= \varepsilon_i|\alpha_i|^2\left[\varepsilon_i + \frac{1-\varepsilon_i}{M}\right] \\ &\quad + \frac{2N_0(\boldsymbol{S}^{\dagger}\boldsymbol{S})^{-1}_{i,i}}{M} \qquad (19) \\ \mathrm{E}\left[\widehat{|\alpha_i|^2}_{II}\middle| M, |\alpha_i|^2\right] &= \varepsilon_i|\alpha_i|^2 + 2N_0(\boldsymbol{S}^{\dagger}\boldsymbol{S})^{-1}_{i,i} \qquad (20) \end{aligned}$$

implying that both $\widehat{|\alpha_i|^2}_{II}$ and $\widehat{|\alpha_i|^2}_{CI}$ can be interpreted as biased estimators of the instantaneous power received in each slot from node i: biases can however be absorbed in the detection thresholds τ_i, while what matters here is that they are both *consistent* in the mean square sense, a property that will be exploited later on. Inserting (17) and (18) into (10), and skipping factors that can be absorbed in the detection thresholds, we obtain the *coherent detector* (CD)

$$\begin{cases} \left|\sum_{p=1}^{M}(\boldsymbol{S}^{+}\boldsymbol{y}_p)_i\right|^2 > \tau_i^2 \rightarrow \text{node } i \text{ is a neighbor} \\ \left|\sum_{p=1}^{M}(\boldsymbol{S}^{+}\boldsymbol{y}_p)_i\right|^2 < \tau_i^2 \rightarrow \text{node } i \text{ is not a neighbor} \end{cases} \tag{21}$$

and the *incoherent detector* (ID):

$$\begin{cases} \sum_{p=1}^{M}|(\boldsymbol{S}^{+}\boldsymbol{y}_p)_i|^2 > \tau_i^2 \rightarrow \text{node } i \text{ a neighbor} \\ \sum_{p=1}^{M}|(\boldsymbol{S}^{+}\boldsymbol{y}_p)_i|^2 < \tau_i^2 \rightarrow \text{node } i \text{ is not a neighbor} \end{cases} \tag{22}$$

Notice how the CD can also be interpreted in a different way. Indeed, it may be obtained by first pre-processing the observations so as to form the cumulative sum:

$$\begin{aligned} \mathbf{y} &\triangleq \sum_{p=1}^{M}\mathbf{y}_p \\ &= \sum_{p=1}^{M}\left(\sum_{k=1}^{K}\psi_{k,p}\alpha_k\mathbf{s}_k + \mathbf{z}_p\right) \\ &= \sum_{k=1}^{K}\nu_k\alpha_k\mathbf{s}_k + \mathbf{z} \\ &= \mathbf{S}\mathbf{V}\boldsymbol{\alpha} + \mathbf{z} \end{aligned}$$

where

$$\nu_k \triangleq \sum_{p=1}^{M}\psi_{k,p} \qquad \mathbf{z} \triangleq \sum_{p=1}^{M}\mathbf{z}_p \tag{23}$$

and $\mathbf{V} \triangleq \mathrm{diag}(\nu_1, \ldots, \nu_K)$, then multiplying the new observation by $\boldsymbol{S}^{+}$ and finally extracting the ith component to form the test statistic (21). Rewriting equation (23) in the form:

$$\mathbf{y} = \underbrace{\nu_i\alpha_i\mathbf{s}_i}_{\text{useful signal}} + \underbrace{\sum_{k\neq i}\nu_k\alpha_k\mathbf{s}_k}_{\text{interference}} + \underbrace{\mathbf{z}}_{\text{noise}} \tag{24}$$

with $\mathbf{z} \sim \mathcal{N}_c(0, 2N_0M\mathbf{I})$, the CD is easily seen to be a member of the family of *linear ND tests* (LNDT), wherein a decision on the proximity of user i is made based on the rule:

$$|\mathbf{c}_i^{\dagger}\mathbf{y}|^2 \underset{H_0}{\overset{H_1}{\gtrless}} \tau_i^2 \tag{25}$$

Thus, the CD (21) can be also interpreted as the zero-forcing (ZF) member of the family (25), obtained as the unique solution to the constrained minimization problem:

$$\begin{cases} \mathbf{c}_{i,ZF} = \arg\min_{\mathbf{c}_i} \mathrm{E}\left[|\mathbf{c}_i^{\dagger}\sum_k \alpha_k\nu_k\mathbf{s}_k|^2\right] \\ \mathbf{c}_{i,ZF}^{\dagger}\mathbf{s}_i = \beta^2 \end{cases} \tag{26}$$

with $\beta \neq 0$, which yields[6]

$$\mathbf{c}_{i,ZF} = \left(\mathbf{I}_L - \mathbf{S}_i\mathbf{S}_i^{+}\right)\mathbf{s}_i = \mathcal{P}_i\mathbf{s}_i \tag{27}$$

where $\mathbf{I}_L$ is the $L \times L$ identity matrix, $\mathbf{S}_i$ is the $L \times (K-1)$ matrix obtained skipping the $i-$th column from $\mathbf{S}$ and $\mathcal{P}_i$ denotes the projector onto the orthogonal complement of the column span of $\mathbf{S}_i$. For future reference we remind here that [10]

$$|\mathbf{c}_{i,ZF}^{\dagger}\mathbf{s}_k|^2 = \begin{cases} 0 & \text{if } k \neq i \\ |\mathbf{s}_i^{\dagger}\mathcal{P}_i\mathbf{s}_i|^2 = \parallel \mathbf{s}_{i,\perp} \parallel^4 & \text{if } k = i \end{cases} \tag{28}$$

where $\mathbf{s}_{i,\perp}$ denotes the projection of $\mathbf{s}_i$ on the above orthogonal complement: needless to say, since $\parallel \mathbf{s}_{i,\perp} \parallel^2 = 1/[(\boldsymbol{S}^{\dagger}\boldsymbol{S})^{-1}_{i,i}]$, the noise power is enhanced by a factor $(\boldsymbol{S}^{\dagger}\boldsymbol{S})^{-1}_{i,i}$.

The vector $\mathbf{c}_i$ can be designed according to a number of different criteria. For example, in [12] an LNDT based on conventional matched filtering (MF), i.e., assuming

$$\mathbf{c}_{i,MF} \triangleq \mathbf{s}_i \tag{29}$$

has been proposed and analyzed for ND. MF is indeed simple, but it results into interference-limited performance, as we shall prove soon, nor does it retain the near-far resistance property granted by ML-based detectors.

A possible alternative to the ZF criterion is offered by the minimum-mean-output-energy (MMOE) strategy, first introduced in [11], wherein the vector $\mathbf{c}_i$ is obtained as the unique solution to the following constrained minimization problem:

$$\begin{cases} \mathbf{c}_{i,MMOE} = \arg\min_{\mathbf{c}_i} \mathrm{E}\left[\left|\mathbf{c}_i^{\dagger}\left(\sum_k \alpha_k\nu_k\mathbf{s}_k + \mathbf{z}\right)\right|^2\right] \\ \mathbf{c}_{i,MMOE}^{\dagger}\mathbf{s}_i = 1 \end{cases} \tag{30}$$

namely:

$$\mathbf{c}_{i,MMOE} = \frac{\mathbf{M}_{\mathbf{yy}}^{-1}\mathbf{s}_i}{\mathbf{s}_i^{\dagger}\mathbf{M}_{\mathbf{yy}}^{-1}\mathbf{s}_i} \tag{31}$$

where $\mathbf{M}_{\mathbf{yy}} \triangleq \sum_{k=1}^{K} 2\sigma_k^2 E[\nu_k^2]\mathbf{s}_k\mathbf{s}_k^{\dagger} + 2N_0M\mathbf{I}_L$. Due to the invariance of the decision rule to any positive scaling of the test statistic, an equivalent detector relies upon setting

$$\mathbf{c}_{i,MMOE} = \mathbf{M}_{\mathbf{yy}}^{-1}\mathbf{s}_i \tag{32}$$

[6]Notice from (27) that the parameter β^2 has been set to $\frac{1}{(\boldsymbol{S}^{\dagger}\boldsymbol{S})^{-1}_{i,i}}$

It might be worth recalling here that, since

$$\lim_{N_0 \to 0} \mathbf{M}_{\mathbf{yy}}^{-1} \mathbf{s}_i \propto \mathcal{P}_i \mathbf{s}_i \tag{33}$$

MMOE is itself asymptotically efficient. Likewise, it retains the near-far resistance property since the projection direction $\mathbf{c}_{i,MMOE}$ tends to become orthogonal to those signatures whose amplitudes become increasingly large [11]. The advantage of (32) over ZF is that it easily lends itself to adaptive implementations in situations where the signatures of the active users are unknown. Even though we do not deal with adaptive ND in this paper, we anticipate that a number of reduced complexity algorithms, ranging from the $\mathcal{O}(L)$-complex Least Mean Squares to the $\mathcal{O}(L^2)$-complex Recursive Least Squares, can be easily applied for adaptive MMOE implementation.

IV. Analysis

From now on we assume that the node to be detected is node "1". Consider first the ID. The conditional false-alarm and miss probabilities in assessing the proximity of node 1 can be written as:

$$P_M = \mathbb{P}(\chi_1 < \tau_1^2 \big| |\alpha_1| > \tau_A, \mathbf{\Psi}_{1:M}) \tag{34}$$

$$P_F = \mathbb{P}(\chi_1 > \tau_1^2 \big| |\alpha_1| < \tau_A, \mathbf{\Psi}_{1:M}) \tag{35}$$

with $\chi_1 \triangleq \sum_{p=1}^{M} |(\mathbf{S}^{+}\mathbf{y}_p)_1|^2$. Given $|\alpha_1|$ and $\mathbf{\Psi}_{1:N}$, χ_1 is noncentral chi-square distributed with $2M$ degrees of freedom and parameters $\nu_1|\alpha_1|^2$ and $\sigma_{n,1}^2 = (\mathbf{S}^{\dagger}\mathbf{S})_{1,1}^{-1} N_0$, implying

$$\mathbb{P}(\chi_1 > \tau_1^2 \mid |\alpha_1|, \mathbf{\Psi}_{1:M}) = Q_M\left(\frac{\sqrt{\nu_1}|\alpha_1|}{\sigma_{n,1}}, \frac{\tau_1}{\sigma_{n,1}}\right) \tag{36}$$

where $Q_M(\cdot,\cdot)$ is the Marcum function of order M. Using the series expansion of modified Bessel functions

$$I_n(x) = \sum_{k=0}^{\infty} \frac{(x/2)^{n+2k}}{k!\Gamma(n+k+1)} \tag{37}$$

we obtain

$$\begin{aligned} &\mathbb{P}(\chi_1 > \tau_1^2 \mid |\alpha_1|, \mathbf{\Psi}_{1:M}) \qquad (38) \\ &= e^{-\frac{|\alpha_1|^2 \frac{\nu_1}{\sigma_{n,1}^2}}{2}} \sum_{k=0}^{\infty} \frac{\left(|\alpha_1|\sqrt{\frac{\nu_1}{\sigma_{n,1}^2}}\right)^{2k}}{2^k k! \Gamma(M+k)} \Gamma\left(M+k; \frac{\tau_1^2}{2\sigma_{n,1}^2}\right) \end{aligned}$$

$$\begin{aligned} &\mathbb{P}(\chi_1 > \tau_1^2, |\alpha_1| > \tau_A \big| \mathbf{\Psi}_{1:M}) \qquad (39) \\ &= \frac{1}{1+\nu_1\rho_1} \sum_{k=0}^{\infty} \left(\frac{\nu_1\rho_1}{1+\nu_1\rho_1}\right)^k Q\left(M+k; \frac{\tau_1^2}{2\sigma_{n,1}^2}\right) \\ &\times \; Q\left(k+1; \frac{\tau_A^2}{2\sigma_1^2}(1+\nu_1\rho_1)\right) \end{aligned}$$

where

$$\Gamma(k;x) \triangleq \int_x^{\infty} t^{k-1} e^{-t} dt \, , \qquad Q(k;x) = \frac{\Gamma(k;x)}{\Gamma(k)} \tag{40}$$

are the upper incomplete Gamma function and its regularized version, respectively, while ρ_1 is the signal-to-noise ratio after decorrelation, i.e.:

$$\rho_1 = \frac{\sigma_1^2}{\sigma_{n,1}^2} = \frac{\sigma_1^2}{N_0(\boldsymbol{S}^{\dagger}\boldsymbol{S})_{1,1}^{-1}} \tag{41}$$

We thus obtain the conditional measure

$$\begin{aligned} P_M &= 1 - \frac{e^{\frac{\tau_A^2}{2\sigma_1^2}}}{1+\nu_1\rho_1} \sum_{k=0}^{\infty} \left(\frac{\nu_1\rho_1}{1+\nu_1\rho_1}\right)^k Q\left(M+k; \frac{\tau_1^2}{2\sigma_{n,1}^2}\right) \\ &\times \; Q\left(k+1; \frac{\tau_A^2}{2\sigma_1^2}(1+\nu_1\rho_1)\right) \qquad (42) \end{aligned}$$

which should be averaged over ν_1 to yield the conditional probability of a miss given M. Similar developments hold for P_F, yielding

$$\begin{aligned} &\mathbb{P}(\chi_1 > \tau_1^2, |\alpha_1| < \tau_A \big| \mathbf{\Psi}_{1:M}) \qquad (43) \\ &= \frac{1}{1+\nu_1\rho_1} \sum_{k=0}^{\infty} \left(\frac{\nu_1\rho_1}{1+\nu_1\rho_1}\right)^k Q\left(M+k; \frac{\tau_1^2}{2\sigma_{n,1}^2}\right) \\ &\times \; P\left(k+1; \frac{\tau_A^2}{2\sigma_1^2}(1+\nu_1\rho_1)\right) \qquad (44) \end{aligned}$$

where

$$P(k;x) = \frac{\gamma(k;x)}{\Gamma(k)} \, , \qquad \gamma(k;x) \triangleq \int_0^x t^{k-1} e^{-t} dt \tag{45}$$

are the lower incomplete Gamma function and its regularized version, respectively. Finally, from (43) we easily obtain

$$\begin{aligned} P_F &= \frac{e^{\frac{\tau_A^2}{4\sigma_1^2}}}{2(1+\nu_1\rho_1)} \operatorname{csch}\left(\frac{\tau_A^2}{4\sigma_1^2}\right) \sum_{k=0}^{\infty} \left(\frac{\nu_1\rho_1}{1+\nu_1\rho_1}\right)^k \qquad (46) \\ &\times \; Q\left(M+k; \frac{\tau_1^2}{2\sigma_{n,1}^2}\right) P\left(k+1; \frac{\tau_A^2}{2\sigma_1^2}(1+\nu_1\rho_1)\right) \end{aligned}$$

Consider now the test family (25). Notice that, since

$$g_1 = \mathbf{c}_1^{\dagger}\mathbf{y} = \underbrace{\nu_1 \mathbf{c}_1^{\dagger}\mathbf{s_1}\alpha_1}_{\text{useful signal}} + \underbrace{\sum_{k=2}^{K} \nu_k \mathbf{c}_1^{\dagger}\mathbf{s_k}\alpha_k + \mathbf{c}_1^{\dagger}\mathbf{z}}_{\text{interference+noise}} \tag{47}$$

$|g_1|^2$ is conditionally chi-square with two degrees of freedom, given α_1, $\{\nu_i\}_{i=1}^{K}$ and M, with non-centrality parameter $|\nu_1\mathbf{c}_1^{\dagger}\mathbf{s_1}\alpha_1|^2 = |\alpha_1|^2\nu_1^2\mathbf{c}_1^{\dagger}\mathbf{s_1}\mathbf{s_1}^{\dagger}\mathbf{c}_1$ and scale parameter

$$\begin{aligned} \Sigma^2(\mathbf{c}_1) &\triangleq \sum_{k=2}^{K} |\nu_k \mathbf{c}_1^{\dagger}\mathbf{s_k}|^2 \sigma_k^2 + MN_0\|\mathbf{c_1}\|^2 \\ &= \sum_{k=2}^{K} \sigma_k^2 \nu_k^2 \mathbf{c}_1^{\dagger}\mathbf{s_k}\mathbf{s_k}^{\dagger}\mathbf{c}_1 + MN_0\|\mathbf{c}_1\|^2 \end{aligned}$$

whereby, reproducing the same steps leading to (42) and (46), we obtain

$$P_M = 1 - \frac{e^{\frac{\tau_A^2}{2\sigma_1^2}}}{1+\nu_1^2\rho_{eq}} \sum_{k=0}^{\infty} \left(\frac{\nu_1^2 \rho_{eq}}{1+\nu_1^2\rho_{eq}} \right)^k \times Q\left(k+1; \frac{\tau_1^2}{2\Sigma^2(\mathbf{c}_1)}\right) Q\left(k+1; \frac{\tau_A^2}{2\sigma_1^2}(1+\nu_1^2\rho_{eq})\right) \quad (48)$$

$$P_F = \frac{e^{\frac{\tau_A^2}{4\sigma_1^2}}}{2(1+\nu_1^2\rho_{eq})} \text{csch}\left(\frac{\tau_A^2}{4\sigma_1^2}\right) \sum_{k=0}^{\infty} \left(\frac{\nu_1^2 \rho_{eq}}{1+\nu_1^2\rho_{eq}} \right)^k \times Q\left(k+1; \frac{\tau_1^2}{2\Sigma^2(\mathbf{c}_1)}\right) P\left(k+1; \frac{\tau_A^2}{2\sigma_1^2}(1+\nu_1^2\rho_{eq})\right) \quad (49)$$

where ρ_{eq} represents the signal-to-interference-plus-noise ratio (SINR) at the output of the linear filter, i.e.:

$$\rho_{eq} = \frac{\sigma_1^2 \mathbf{c}_1^\dagger \mathbf{s_1} \mathbf{s_1}^\dagger \mathbf{c}_1}{\Sigma^2(\mathbf{c}_1)} \quad (50)$$

Relationships (48) and (49) are quite reminiscent of (42) and (46), respectively, one major difference being the dependency of the performance on $\nu_1^2\rho_{eq}$, rather than $\nu_1\rho_1$. Of course, the quadratic factor in ν_1 stems from the fact that linear detectors operate on a coherent combination of the observations, while ID combines the slot-by-slot estimates incoherently. Notice, however, that the above relationships represent conditional measures, given M (i.e., given ν_0) and $\{\nu_i\}_{i=1}^K$. If the discovery session is long enough, so that the matrix sequence $\mathbf{\Psi}_{1:M}$ may exhibit its typical behavior, namely, if $N(1-\varepsilon_0) \gg 1$, then the ν_k's tend in probability to $M\varepsilon_k$, whereby the unconditional performances may be obtained by averaging the corresponding conditional measures on the typical set of values of $\{\nu_k\}_{k=1}^K$ and M only, implying

- $M \simeq N(1-\varepsilon_0)$;
- $\nu_k \simeq M\varepsilon_k = N\varepsilon_k(1-\varepsilon_0)$.

In this limiting situation, it is interesting to notice the relationship between the "cumulated" SNR's for ID and CD (i.e., the ZF of (27)), i.e. (see also (28) and subsequent comments):

$$\nu_1^2\rho_{eq} = \frac{\sigma_1^2\nu_1^2 \parallel \mathbf{s}_{1,\perp} \parallel^4}{MN_0 \parallel \mathbf{s}_{1,\perp} \parallel^2} = \frac{\nu_1^2\sigma_1^2}{MN_0(\boldsymbol{S}^\dagger\boldsymbol{S})_{1,1}^{-1}} \simeq \varepsilon_1\nu_1\rho_1 \quad (51)$$

Thus, in terms of cumulated signal-to-noise ratio and for large N, ID seems to be preferable to CD, even though a global superiority cannot be claimed due to the different forms assumed by the respective false-alarm and miss probabilities.

So far no criterion has been given to select the decision threshold τ_1. Notice, however, that the consistency of the estimates (17) and (18) allows devising the asymptotically optimum thresholds (those achieving minimum error probability for large N) from (19) and (20) in the form

$$\tau_{1,CD}^2 = N(1-\varepsilon_0)\big[\varepsilon_1\tau_A^2[N(1-\varepsilon_0)\varepsilon_1 + (1-\varepsilon_1)] + 2N_0(\boldsymbol{S}^\dagger\boldsymbol{S})_{1,1}^{-1}\big] \quad (52)$$

$$\tau_{1,ID}^2 = N(1-\varepsilon_0)\big[\tau_A^2\varepsilon_1 + 2N_0(\boldsymbol{S}^\dagger\boldsymbol{S})_{1,1}^{-1}\big] \quad (53)$$

For short discovery sessions, and under known activity factors of nodes to be discovered, optimum detection thresholds can be obtained by evaluating numerically the unconditional error probability, and then determining the points where it has a minimum.

V. Results

We consider here a fully loaded network with $K+1 = 7$, each node being assigned a length-7 m-sequence. As in previous section, we assume that node 0 has to decide on the proximity of node 1. Figure 2 assumes $\text{SNR}_1 \triangleq \sigma_1^2/N_0 = 0$ dB, $N = 100$, a power-controlled scenario wherein all nodes are received with the same average power, uniform activity factor ($\varepsilon_k = \varepsilon = 0.5$), and an activity threshold equal to the median of the fading amplitude distribution, i.e., such that $\mathbb{P}(|\alpha_1| > \tau_A) = 0.5$. The figure represents the pair P_M, P_F for the various receivers examined so far. Interestingly, "conventional" MF [12] suffers from the presence of the other nodes even in this rather benign situation, while MMOE, ZF and CD take advantage of their asymptotic efficiency.

The reliability of the asymptotic approximation for long discovery sessions can be assessed through figures 3-4 for the CD, and through figures 5-6 for the ID, which refer to the same scenario as in Fig. 2.

The curves of these figures represent

- The unconditional false alarm and miss probabilities obtained by simulation.
- The same pair obtained by a semi-analytical method, i.e., by estimating the averages of their conditional counterparts.
- The asymptotic approximation.

From the plots, it is evident that the asymptotic approximation tends to overestimate the performances in the interesting region of low error probabilities, while coming closer and closer to the true performance as N increases: notice that the approximation is extremely tight for $N = 500$, a realistic value indeed in real applications, which, for $\epsilon_0 = 0.5$, corresponds to $M \simeq 250$. However, it should be kept in mind that, for larger activity factors of the discovering node, the minimum value of N for the asymptotic behavior to be reached inevitably increases.

Fig. 7 is aimed at comparing CD and ID. It represents the error probability versus the signal-to-noise ratio SNR_1 using the optimal thresholds for both receivers, and assuming again $\varepsilon = 0.5$, $N = 500$, and τ_A as before. It is interesting to notice that CD outperforms ID for small signal-to-noise ratios, while ID is preferable for medium-to-large values of SNR_1.

VI. Conclusions

We have examined the problem of discovery which nodes are neighbors in a wireless network operating over a fading channel. The optimum Bayesian decision rule has been derived, showing that its complexity is practically prohibitive. Two suboptimum neighbor-discovery algorithms have been introduced, based on standard techniques of coherent and incoherent integration. We show how coherent integration may be viewed as a particular case of a family of algorithm akin to Linear Neighbor Discovery Tests (LNDT). Theoretical analysis allows one to understand the design of a system employing such algorithms according to constraints on error rate, signal-to-noise ratio and discovery session duration. Finally, algorithm optimization was considered, and formulas were derived for asymptotical optimum threshold.

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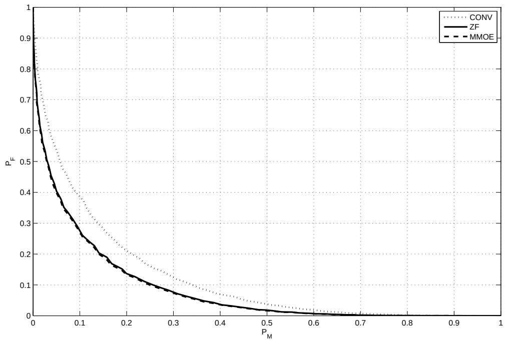

Fig. 2. *Performance of various ND algorithms under perfect power control,* $2\sigma_1^2 = 2N_0 = 1$ *(SNR*$_1$*=0 dB),* $N = 100$*, fully-loaded network.*

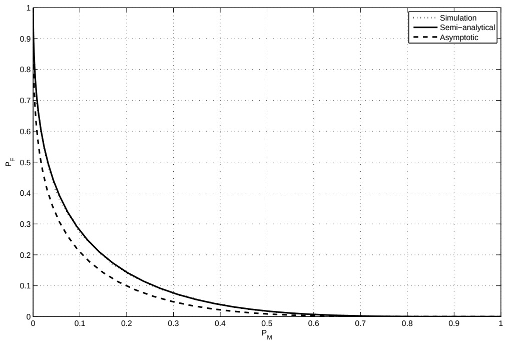

Fig. 3. *Performance of the CD for* $2\sigma_1^2 = 2N_0 = 1$ *(SNR*$_1$*=0 dB),* $N = 100$

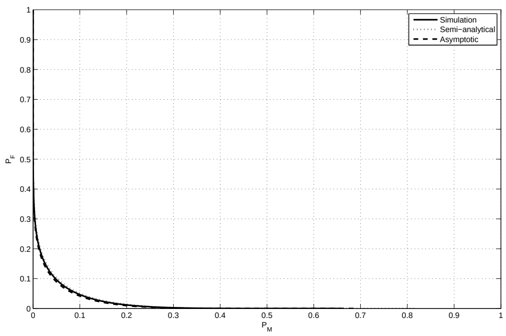

Fig. 4. *Performance of the CD for* $2\sigma_1^2 = 2N_0 = 1$ *(SNR*$_1$*=0 dB),* $N = 500$

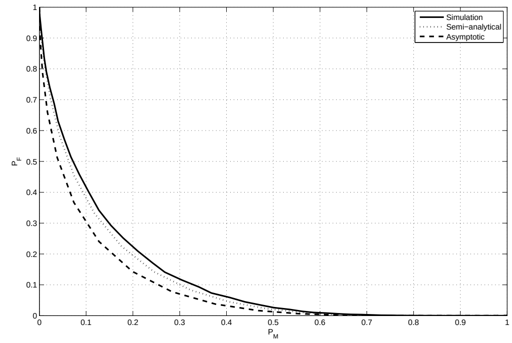

Fig. 5. *Performance of the ID for* $2\sigma_1^2 = 2N_0 = 1$ *(SNR*$_1$*=0 dB),* $N = 100$.

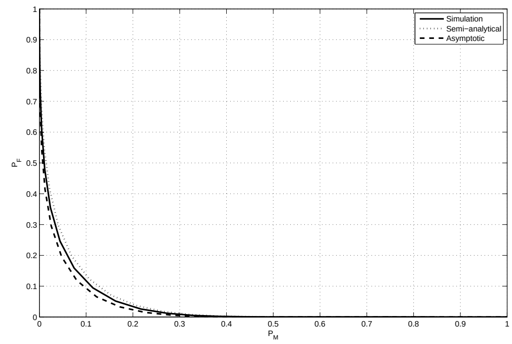

Fig. 6. *Performance of the ID for* $2\sigma_1^2 = 2N_0 = 1$ *(SNR*$_1$*=0 dB),* $N = 500$.

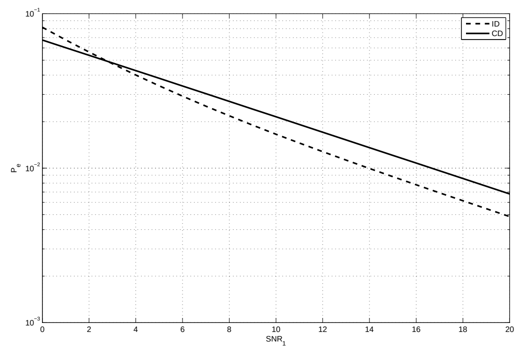

Fig. 7. *Global comparison between CD and ID,* $N = 500$, $\varepsilon = 0.5$, $2\sigma_1^2 = 1$.