A Pragmatic Approach to Coded Continuous-Phase Modulation

Sergio Benedetto, Guido Montorsi, Alberto Perotti, Alberto Tarable
Center for Multimedia Radio Communications
Politecnico di Torino, Torino, Italy
Email: {sergio.benedetto, guido.montorsi, alberto.perotti, alberto.tarable}@polito.it

***Abstract*— In this paper, we show that a "pragmatic" approach to coded CPM schemes suffers from a significant capacity loss. This loss can be greatly reduced by choosing an appropriate mapping different from natural or Gray, adopted so far. We propose to add to the CPM modulator a linear feedback, optimized through capacity arguments, that permits to achieve performance within 1 dB from the CPM capacity.**

I. Introduction

Continuous phase modulation (CPM) is a class of bandwidth efficient modulation schemes [1] that have found several applications in wireless systems (like for example GSM). Their characteristic of constant envelope makes CPM schemes particularly suited to fully exploit the output power of a nonlinear amplifier.

A CPM modulator is a (generally) finite-state device delivering to the channel a waveform that depends on its input symbol and internal state. In [2], the modulator with memory has been split into the so called Rimoldi decomposition, consisting of the cascade of a time-invariant convolutional encoder (continuous-phase encoder, CPE) operating on a ring of integers and of a time-invariant memoryless modulator (MM). After the discovery of turbo codes [3] and their celebrated iterative decoding algorithm, this decomposition has been exploited by inserting an outer convolutional encoder whose coded bits enter an interleaver and then the CPE, thus forming what is known in the literature as a serially-concatenated convolutional encoder (SCCC) [4]. Iterating between the outer encoder and the CPE through the interleaver yields rather good performance [5]–[7], which should be compared to the capacity of the CPM scheme. This capacity, in turn, can be evaluated through the techniques explained in [8], [9]. In the following, we will call this scheme SCCC-CPM.

In wireless communication systems, and, more in general, in any telecommunication system where the channel conditions can vary significantly with time, an efficient radio resource management requires the availability at the physical layer of adaptive coding-modulation. Flexible coded-modulation schemes are then required, capable of varying their characteristics of bandwidth and energy efficiency from frame to frame, following the channel rate of variation. This crucial versatility requirement has spurred an active research on bit-interleaved (also known as *pragmatic*) coded modulation (see [10] and references therein), which consists in cascading a highly performing binary encoder (typically, a turbo or low-density parity-check code) with several modulation schemes with increasingly large signal alphabets. An example of the obtainable results has been published in [11], which demonstrates a scheme based on SCCC and linear two-dimensional modulations yielding spectral efficiencies in a very wide range lying around 1 dB from the Shannon capacity limits. To the author knowledge, nothing has been published yet on pragmatic schemes employing CPM modulation (called P-CPM in the following).

This paper is a first attempt to fill the gap. In it, we first evaluate the pragmatic (or bit-interleaved) capacity of CPM modulation, and show that using the natural or Gray mapping as done so far (according to the case of linear modulations) it is rather far from the CPM capacity, thus explaining why the pragmatic approach has never been applied to CPM. We support these capacity results through bit error rate simulations.

In [12] an approach that incorporates an outer convolutional encoder into the CPE, and tries to optimize the overall trellis encoder based on Euclidean distances maximization has been proposed. In this paper, still adopting the Rimoldi decomposition, we modify the CPE through linear feedback and optimize it in order to maximize the pragmatic capacity, a concept to be explained later in connection with the CPM. We show, through capacity and simulation results, that large improvements can be obtained in this way, which move the pragmatic capacity very close to the CPM capacity, and then the convergence threshold of P-CPM within 1 dB from the CPM capacity. This approach does not require iterating with the CPE, since the CPM is treated exactly as a linear modulation in a bit-interleaved turbo-trellis coded modulation approach. As nice consequences:

- The overall CPE state complexity is not enhanced by the number of iterations, thus permitting to increase the bandwidth efficiency through the use of a larger number of CPE states
- The separation between the outer encoder and the CPM could pave the way to add a greater flexibility to P-CPM schemes.

In the paper, only a few examples are shown. The optimization of a broad class of CPM schemes for their use in P-CPM as well the problem of how to better cope with the versatility issue are left to a future publication.

II. The CPM modulator

A CPM modulator is a device with memory that generates continuous-phase, constant envelope modulated waveforms

$$x(t) = \sqrt{\frac{2E_s}{T}} e^{j\psi(t)} \quad (1)$$

whose phase

$$\psi(t) = 2\pi h \sum_{n=-\infty}^{\infty} a_n q(t - nT) \quad (2)$$

depends on the input information symbols $a_n \in \{\pm1, \pm3, \ldots, \pm(M-1)\}$, where $M = 2^m$ is the size of the input alphabet. Here, T is the symbol interval, E_s is the energy per symbol, $h = Q/P$ is the *modulation index* (Q and P are relatively prime integers), and $q(t)$ is the phase pulse, a continuous function with the following properties

$$q(t) = \begin{cases} 0 & t \le 0 \\ \frac{1}{2} & t \ge LT \end{cases}$$

The phase pulse is usually defined as the integral of a *frequency pulse* $s(t)$

$$q(t) = \int_{-\infty}^{t} s(\tau) d\tau$$

A CPM scheme is then defined by specifying its parameters M, h, L and $s(t)$.

A well known representation of CPM modulators is the Rimoldi decomposition [2]: according to this representation, the modulator is decomposed into the cascade of a continuous phase encoder (CPE) and a memoryless modulator (MM) (see Fig. 1).

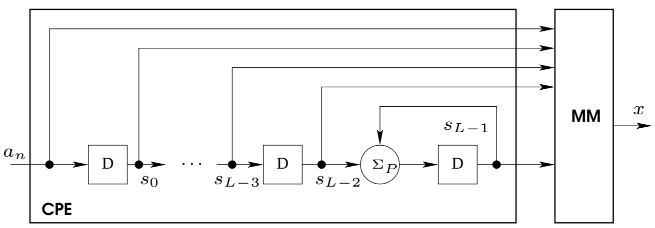

Fig. 1. Rimoldi decomposition of the CPM encoder. The block labelled Σ_P is a modulo P adder.

The CPE is, in general, a time-invariant convolutional encoder operating on a ring of integers. Therefore, it can be decoded using a trellis-based decoding algorithm (i.e., Viterbi for hard output or BCJR [13] for soft output). The MM is a device that performs a symbol-by-symbol mapping from CPE coded symbols to continuous-phase waveforms.

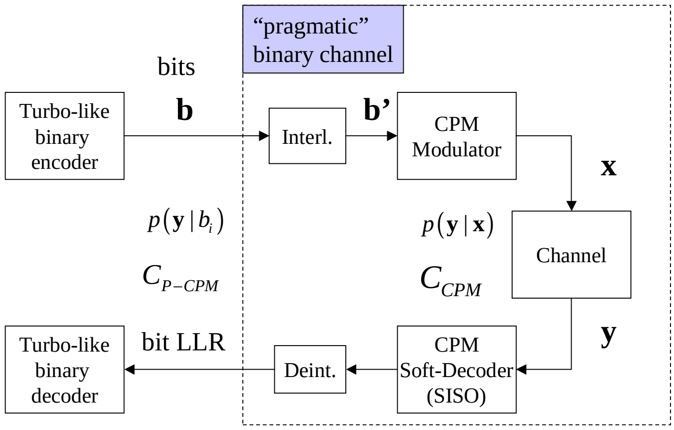

Fig. 2. Block diagram of an encoded CPM mo-demodulator used with a pragmatic approach. No decoding iteration is performed between the CPM demodulator and the binary decoder as in the SCCC-CPM scheme.

III. Pragmatic capacity of CPM modulation

The capacity of a CPM scheme (see Fig. 2 for notations) can be computed as

$$C_{CPM} = E_{\mathbf{y},\mathbf{x}} \left\{ \log_2 \frac{p(\mathbf{y}|\mathbf{x})}{p(\mathbf{y})} \right\} \text{ bits/channel use,} \quad (3)$$

where $\mathbf{x}$ is a CPM waveform and $\mathbf{y}$ is the received signal. Recently, new algorithms for the estimation of the capacity for channels with memory have been proposed [8], [9]. In particular, the algorithm presented in [9] has been developed for CPM capacity estimation. By using the BCJR algorithm, it computes the mutual information between the channel input and channel output as a function of the state and branch metrics. Here, this algorithm has been employed in order to compute the capacity of CPM schemes.

Since we are interested in a "pragmatic" use of CPM modulators, we should also consider their correspondent capacity. In the pragmatic approach, the LLRs of the m bits labeling each CPM symbol are provided to the binary decoder, which assumes them to be independent as if they were transmitted using a binary memoryless modulation (see Fig. 2).

Assuming equally likely independent binary inputs, the pragmatic capacity C_t^{P-CPM} is defined by summing up the capacities of the m equivalent binary channels:

$$C_t^{P-CPM} \triangleq \sum_{i=1}^{m} E_{\mathbf{y},\mathbf{x}} \left\{ \log_2 \frac{p(\mathbf{y}|b_{i,t}(\mathbf{x}))}{p(\mathbf{y})} \right\} \text{ bits/channel use,} \quad (4)$$

where $b_{i,t}(\mathbf{x})$ are the functions that associate to each waveform of the CPM modulator $\mathbf{x}$ the value of the i-th bit of its input label at time t. If the functions $b_{i,t}(\mathbf{x})$ are defined on the time-invariant trellis of the CPM modulator, C_t^{P-CPM} does not depend on the time index t.

Comparing (3) and (4), we observe that only the latter depends on the input mapping $b_{i,t}(\mathbf{x})$. It is then possible to optimize the pragmatic capacity by varying the mapping.

In Fig. 3, a 3REC binary CPM with $h = 1/2$ is considered. Its CPM capacity and pragmatic capacity are estimated. A gap of several dB between the two capacity curves can be observed in a wide range of E_b/N_0.

The region of achievable rates for P-CPM schemes is limited by the pragmatic capacity curve, hence it is not possible to approach the full capacity of the CPM modulation.

This can be observed from the results obtained by simulating a P-CPM scheme with an outer binary (16003, 7998) SCCC consisting of two 4-state, rate 1/2 punctured systematic convolutional encoders. The constituent encoders' feedback polynomial is $f(D) = 1 + D + D^2$ and their feedforward polynomial is $g(D) = 1 + D^2$. The interleaver connecting the outer encoder to the inner encoder is a spread interleaver [14]. Another spread inteleaver has been used to connect the SCCC to the CPM modulator. Decoding of the SCCC is performed by means of an iterative decoder implementing the BCJR algorithm; it has been configured to perform 10 iterations per block.

We observe that the information rate [8] of this scheme approaches the pragmatic capacity with a gap of less than 1 dB at information rates of 0.5 bits/channel use.

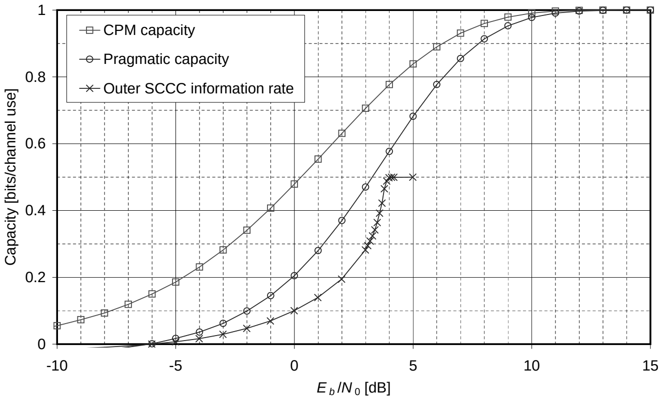

Fig. 3. Capacity of CPM and P-CPM.

IV. Dynamic mapping

The Rimoldi decomposition induces, through the definition of the CPE, a natural mapping of information symbols to waveforms. While the capacity of the CPM scheme depends only on the main CPM parameters (namely, h, L, $s(t)$, M), its pragmatic capacity depends also on the input symbol mapping (see (4)). Thus, it may possible to improve it, and therefore to extend the achievable capacity region of P-CPM schemes, by using different mappings.

The chosen approach consists in modifying the CPE structure by adding a state-dependent input mapping. This solution, besides preserving phase continuity of the modulated signal, allows us to vary the input symbol mapping as a function of the feedback coefficients. The resulting CPE structure is shown in Fig. 4.

The mapping function can be described by the following equation

$$\bar{a}_n = f_\sigma(a_n)$$

where f_σ is a one-to-one function and σ is the CPE state. It can be represented as a binary vector σ with the following number of elements

$$r = \lceil \log_2(N_s) \rceil = m(L-1) + \lceil \log_2 P \rceil$$

which derive from the binary representation of the M-ary symbols $s_0, \ldots, s_{L-2}$ and the P-ary symbol s_{L-1}. Here, $N_s = PM^{L-1}$ is the number of states of the CPE. Starting from the trellis of the CPE without feedback ($\bar{a}_n = a_n$), the mapping function f_σ corresponds to a permutation of the input labels on the sets of edges with the same starting state. Since the output labelling is not altered, phase continuity is preserved.

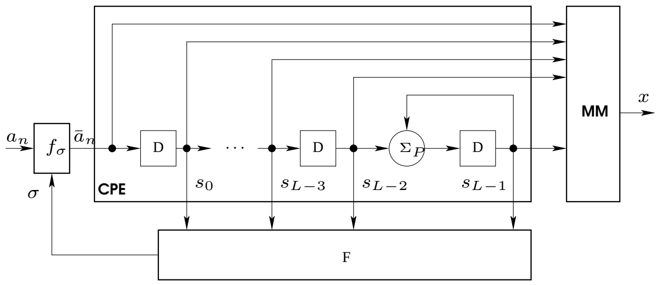

Fig. 4. CPM encoder with feedback.

The number of distinct mapping functions for the CPE in Fig. 4 is $(M!)^{N_s}$. An optimization over such set is considered in [15]. Here, we restrict the search to a smaller subset.

An effective technique consists in restricting f_σ to be a *linear function* over GF(2) of the state vector σ. In this case, a feedback symbol $\mathbf{b}$ is computed as

$$\mathbf{b} = \sigma \mathbf{F}'$$

where $\mathbf{F}$ is an $m \times r$ binary matrix. Then, $\mathbf{b}$ is added to the input symbol to obtain

$$\bar{a}_n = a_n \oplus b_n.$$

Here, the sum is performed over $\mathbb{Z}_2^m$, i.e., the m bits of $\mathbf{b}$ are added modulo 2 to the m bits of $\mathbf{a}$. The number of linear feedback functions is now reduced to $2^{rm} \geq N_s^m$, i.e., the number of distinct matrices $\mathbf{F}$.

This solution requires a very low additional complexity at the transmitter and no additional complexity at the receiver. In fact, the state and branch complexities of the CPE trellis are not increased.

Since the feedback symbol depends on the encoder state, hence on the information sequence, we call this mapping *dynamic*.

A. Feedback optimization

The optimization of the feedback coefficients (the $\mathbf{F}$ matrix) has been performed by maximizing the area below the pragmatic capacity curve. When only one or a few values of information rates are of interest, only the corresponding range of E_b/N_0 can be used for the area computation.

Although this method does not guarantee to provide the best feedback values, results show that those obtained in this way yield significant improvement in pragmatic capacity.

V. RESULTS

Fig. 5 shows the capacity curves for the binary CPM scheme of Fig. 3 (a 3REC binary CPM with $h = 1/2$). For this CPM scheme, the feedback optimization algorithm provided the value $\mathbf{F} = [1\ 1\ 1]$. The curve labelled “Pragmatic, F = [1 1 1]” shows the pragmatic capacity curve in this case, which is almost coincident with the CPM capacity curve. An improvement of more than 2 dB is observed for a wide range of capacity values.

In order to verify that the new capacity values can be achieved, the code described in Sec. III has been connected to the CPM in a P-CPM scheme. We observe that the capacity of this scheme approaches the improved pragmatic capacity with a gap of less than 1 dB, therefore resulting in an improvement of more than 2 dB with respect to the CPM with no feedback. This improvement can be observed also on the bit error rate curves of Fig. 6, which also show the good behaviour in terms of error floor.

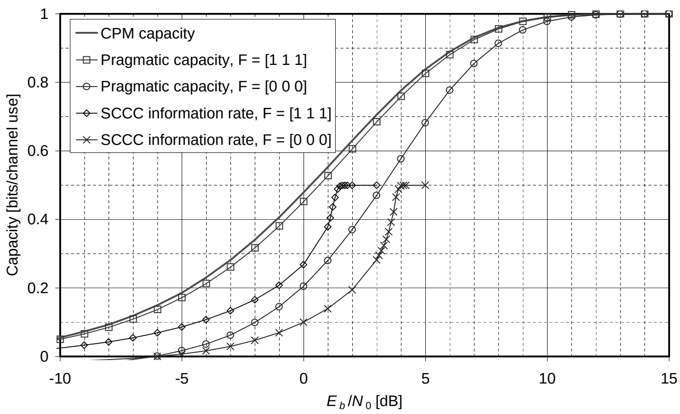

Fig. 5. Capacity of 3REC binary CPM with $h = 1/2$.

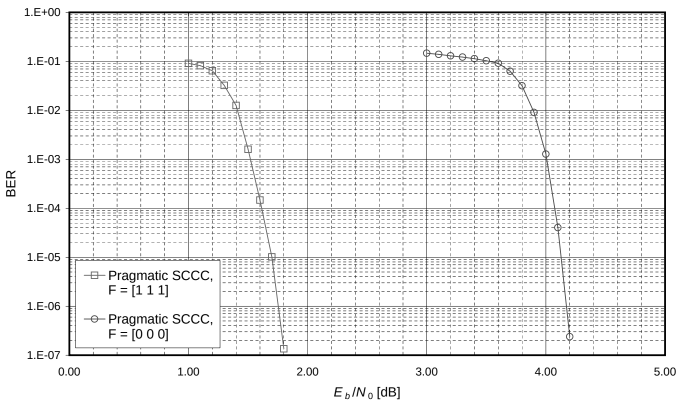

Fig. 6. Bit error rate for 3REC binary CPM with $h = 1/2$.

The same optimization has been performed for a 2REC quaternary CPM with $h = 1/4$. Results are shown in Fig. 7. The feedback optimization algorithm described above provided the following feedback coefficients

$$\mathbf{F} = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 \end{bmatrix}.$$

For compactness, in Fig. 7 labels, the columns of $\mathbf{F}$ have been represented as decimal values ($\mathbf{F} = [3\ 0\ 3\ 0]$).

In this case, although the resulting capacity curve is significantly better than the case without feedback, it does not approach the CPM capacity limit. So far, whether this residual gap is due to the intrinsic characteristics of the considered CPM scheme or to the constraints on the mapping function needs further investigation. Nevertheless, the achieved improvement exceeds 2 dB and is close to what can be obtained using the SCCC-CPM approach.

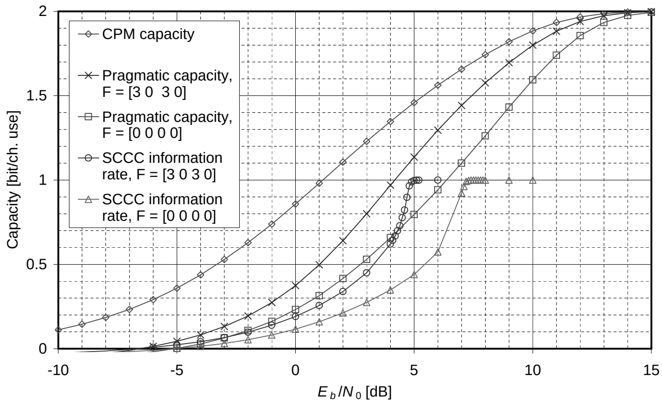

Fig. 7. Capacity of 2REC quaternary CPM with $h = 1/4$.

VI. CONCLUSIONS

The achievable capacity region of pragmatic CPM systems is limited by the pragmatic capacity of the CPM scheme. After observing that the pragmatic capacity is often far below the CPM capacity, we have shown that it can be greatly improved by choosing an appropriate input symbol mapping. To this purpose, a modified structure of CPM modulator and an algorithm for its optimization have been proposed.

The proposed solution results in a slight additional complexity at the modulator and requires no additional complexity at the receiver. Improvements of more than 2 dB have been obtained in the considered cases.

ACKNOWLEDGEMENTS

This work has been supported by Regione Piemonte under Contract E4.

REFERENCES

[1] J. B. Anderson, T. Aulin, and C.-E. Sundberg, *Digital Phase Modulation*, R. W. Lucky, Ed. New York, London: Plenum Press, 1986.

[2] B. E. Rimoldi, “A decomposition approach to CPM,” *IEEE Transactions on Information Theory*, vol. 34, pp. 260–270, March 1988.

[3] C. Berrou, A. Glavieux, and P. Thitimajshima, “Near Shannon limit error-correcting coding and decoding: turbo codes,” in *Proceedings of the International Conference on Communications*, vol. 2, Geneva, Switzerland, May 1993, pp. 1064–1070.

[4] S. Benedetto, D. Divsalar, G. Montorsi, and F. Pollara, “Serial concatenation of interleaved codes: Performance analysis, design adn iterative decoding,” *IEEE Transactions on Information Theory*, vol. 44, pp. 909–926, May 1998.

[5] P. Moqvist and T. M. Aulin, “Serially concatenated continuous phase modulation with iterative decoding,” *IEEE Transactions on Communications*, vol. 49, pp. 1901–1915, November 2001.

[6] K. R. Narayanan and G. L. Stüber, “Performance of trellis-coded CPM with iterative demodulation and decoding,” *IEEE Transactions on Communications*, vol. 49, pp. 676–687, April 2001.

[7] K. R. Narayanan, I. Altunbas, and R. S. Narayanaswami, “Design of serial concatenated MSK schemes based on density evolution,” *IEEE Transactions on Communications*, vol. 51, pp. 1283–1295, August 2003.

[8] D. Arnold, H.-A. Loeliger, P. A. Vontobel, A. Kavčić, and W. Zeng, “Simulation-based computation of information rates for channels with memory,” *IEEE Transactions on Information Theory*, vol. 52, pp. 3498–3508, August 2006.

[9] K. Padmanabhan, S. Ranganathan, S. P. Sundaravaradhan, and O. M. Collins, “General CPM and its capacity,” in *Proceedings of the International Symposium on Information Theory*, Adelaide, AU, September 2005, pp. 750–754.

[10] G. Caire, G. Taricco, and E. Biglieri, “Bit-interleaved coded modulation,” *IEEE Transactions on Information Theory*, vol. 44, pp. 927–946, May 1998.

[11] S. Benedetto, R. Garello, G. Montorsi, C. Berrou, C. Douillard, D. Giancristofaro, A. Ginesi, L. Giugno, and M. Luise, “MHOMS: high-speed ACM modem for satellite applications,” *IEEE Wireless Communications*, pp. 66–77, April 2005.

[12] M. R. Shane and R. D. Wesel, “Parallel concatenated turbo codes for continuous phase modulation,” in *IEEE Wireless Communications and Networking Conference*, Chicago (IL), USA, September 2000, pp. 147–152.

[13] L. R. Bahl, J. Cocke, F. Jelinek, and J. Raviv, “Optimal decoding of linear codes for minimizing symbol error rate,” *IEEE Transactions on Information Theory*, vol. 20, pp. 284–287, March 1974.

[14] S. Dolinar and D. Divsalar, “Weight distributions for turbo codes using random and nonrandom permutations,” *TDA Progress Report*, vol. 42-122, pp. 56–65, April-June 1995.

[15] S. Benedetto, G. Montorsi, A. Perotti, and A. Tarable, “Capacity-achieving coded CPM shcemes,” in preparation.