On the Broadcast Capacity of Wireless Networks

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***Abstract*— A fundamental problem in wireless networks is determining the *broadcast capacity*, i.e. the maximum data transfer rate from a given node to every other node in a relay network. This problem becomes more important as many network protocols rely on broadcast of certain control messages. In this paper, the scaling of the broadcast capacity with the number of nodes (N) in the network is studied. In the *high-density regime* (i.e. the node density goes to infinity; the network area is fixed), it is shown that the broadcast capacity is upper bounded by $\Theta(\log N)$. Schemes are provided that achieve i) $\Theta(\log N)$ throughput if the channel fading is *spatially continuous*; ii) $\Theta(\log \log N)$ throughput if the channel fading is *spatially i.i.d.*. The reasons for this drastic reduction in throughput and the connections with *multiuser diversity* are discussed. Analogous results are provided for the *extended-network model* (i.e. the node density is fixed; the network area goes to infinity).**

I. Introduction

In both wireless and wired networks, communication consists of not only transmissions between single source-destination pairs (unicast), but also transmissions from a source to a group of destinations (multicast, one-to-many) and from a group of sources to a destination (many-to-one). In the case of one-to-many communication, if the destination set corresponds to the entire network nodes, this transmission is also referred as *broadcast*.

For many wireless network applications, broadcasting and multicasting constitute a significant portion of network traffic, and they may cause performance bottlenecks. Several authors have studied designing broadcast protocols based on various criteria such as energy efficiency and reduction of retransmissions [1]. In contrast with traditional approaches, an important observation is that "*collisions*" between different transmitting nodes, which hinder point-to-point communication, may actually be *beneficial*, when the transmitting nodes are broadcasting the same message [2]. Motivated by this idea, we are interested in designing novel broadcasting protocols and fundamental performance bounds on practical schemes. The goal of this paper is to study the broadcast capacity of wireless networks.

In recent years, there has been a lot of interest in fundamental limits of wireless networks with unicast traffic [3]–[11]. These works were initiated by [3], where the authors study the capacity for a multihop wireless network with multiple source-destination pairs. Later, in [5], [6] schemes that exceed the performance of multihopping for wireless unicast communications are presented. Without going into details of these works, we would like to mention that taking an information theoretic perspective usually leads to sophisticated physical layer designs with better performance. In [12], [13], the authors have studied scaling laws of many-to-one networks.

A few works, [14]–[17], have studied the scaling laws for broadcast capacity of wireless networks. In [14], the authors assume that each node has a fixed transmission radius, and show that under multihop broadcasting, the broadcast capacity scales as $\Theta(1)$. In [15], the author studies a multihop relay network where node locations are modelled as a Poisson process. The capacity bounds are derived under the assumption that the link rates are governed by signal-to-interference ratio (SINR). In [16], the authors show that the broadcast capacity is $\Theta(1)$ under the protocol model [3]. As opposed to these works, we follow an information theoretic approach, find upper bounds to the broadcast capacity and provide schemes that achieve the same scaling as the upper bound in case of spatially-continuous fading.

In a recent work [17], the authors characterized the broadcast capacity for slowly fading channels. They considered a model where an outage is declared if any of the receivers fails to decode the source message, and the broadcast capacity is defined as the maximum data rate at which the outage probability converges to zero as the number of nodes goes to infinity. Theys showed that the broadcast capacity converges to $C = \log(1 + \frac{P}{N_0})$, where P is the sum power constraint on the network and N_0 is the noise power. This result is obtained under the assumption that there is i.i.d small scale fading between nodes, but there is no signal attenuation with distance. The achievability result is based on a two-phase cooperative broadcasting scheme.

In this paper, we study a network composed of a single source and N destinations. The destination nodes also serve as relays for others. The channel coefficients are assumed to be ergodic in time, and the metric of interest is the scaling of broadcast capacity with respect to the number of nodes. Our results are based on two different channel models: (i) spatially continuous fading; (ii) spatially i.i.d. fading.

In the first part of the paper, we study high-density networks (i.e. the node density goes to infinity while the network area is fixed) under the total power constraint. We show that the broadcast capacity is upper bounded by $\Theta(\log N)$ and provide schemes that achieve i) $\Theta(\log N)$ throughput if the channel fading is spatially continuous; ii) $\Theta(\log \log N)$ throughput if the channel fading is spatially i.i.d.. Under the spatially-continuous model it becomes possible to coherently combine the signals from a small group of nodes to other receivers as the network density increases. Coherent combining of a polynomial number of nodes leads to achievability of $\Theta(\log N)$ throughput. In the case of spatially i.i.d. fading, the gains are similar to multiuser diversity gains—a small fraction of nodes receive the source message correctly, but they retransmit and amplify it enough that every other node receives it successfully. As in the case of classical multiuser diversity in in single-hop broadcast/multiaccess setting, the achieved rate is only $\Theta(\log\log N)$.

Next, we study the extended networks (i.e. the network area goes to infinity while the node density is fixed) and provide a $\Theta(\log N)$ upper bound on the broadcast capacity under per-node power constraint. We also determine the transmission rate of the cooperative multistage broadcasting protocol introduced in [2] ($\Theta(1)$ scaling). Comparison of cooperative broadcast with conventional multihop communication is left as a future work.

The paper is organized as follows. In Section II, we introduce the network and system models. Section III summarizes the main results. The analysis of high density networks under spatially-continuous channel models is given Section IV and V, and under i.i.d. fading model is studied in Section VI. Section VII studies extended networks. The analysis of transmission rate for multistage cooperative broadcasting is studied in Section VIII. Finally, Section IX presents concluding remarks and future directions.

II. System Model and Problem Formulation

We consider a network of a single source and N destinations. We will denote the source node with index 0 and the index set of the destinations with $\mathcal{S} := \{1, \ldots, N\}$. We assume the nodes are half-duplex, i.e., they can not transmit and receive at the same time. We study two different regimes:

1) *High Density Network Model:* The nodes are uniformly and independently distributed in a given region with area A. The node density increases with an increase in the number of nodes.
2) *Extended Network Model:* The nodes are uniformly and independently distributed with a given node density. The network area increases with an increase in the number of nodes.

Each node communicates over a wireless channel which is effected by both signal attenuation and small scale fading. Let $\tilde{h}(\mathbf{x}_i, \mathbf{x}_k)$ denote the channel coefficient between i'th and k'th nodes located at $\mathbf{x}_i$ and $\mathbf{x}_k$, respectively:

$$\tilde{h}(\mathbf{x}_i, \mathbf{x}_k) = \frac{h(\mathbf{x}_i, \mathbf{x}_k)}{\|\mathbf{x}_i - \mathbf{x}_k\|^{\alpha/2}}, \quad (1)$$

where $h(\cdot, \cdot)$ denotes the small-scale fading function, and $i, k \in \mathcal{S}$. The parameter α denotes the pathloss exponent and usually lies in the interval $(2, 4)$. We assume small-scale fading is zero mean and unit variance, i.e., for any given node-pair $\{i, k\}$, $\mathbb{E}\{h(\mathbf{x}_i, \mathbf{x}_k)\} = 0$ and $\mathbb{E}\{|h(\mathbf{x}_i, \mathbf{x}_k)|^2\} = 1$. We assume that the source is located at the origin, i.e., $\mathbf{x}_0 = (0, 0)$. In the following, we will also use $h_{ik} := h(\mathbf{x}_i, \mathbf{x}_k)$ whenever appropriate, and $d_{ik} := \|\mathbf{x}_i - \mathbf{x}_k\|$. We will denote the time variation in the channel coefficients as $h_{ik}[m]$, where m denotes the m'th time instant. The received signal at the k'th node at time m is

$$y_k[m] = \sum_{i \neq k} h_{ik}[m] x_i[m] + z_k[m],$$

where $x_i[m]$ is the transmitted signal by the i'th node at time m and $z_k[m]$ denotes the i.i.d. additive white Gaussian noise at the k'th node at time instant m with power N_0.

We assume that the locations of the nodes are slowly-varying (fixed over the communication duration), and the small-scale fading is fast-varying. We are interested in the scaling of the ergodic capacity for both high density networks and extended networks. In the next subsection, we give detailed explanation of the considered channel models. We assume the total transmission power of the nodes is fixed:

$$\sum_{i=0}^{N} P_i \leq P, \quad (2)$$

where P_i denotes the average power for the i'th relay and $P < \infty$ is independent of the number of nodes N.

A. Fading Models

We consider different fading distributions over the space and assume $h_{ik}[m]$'s are ergodic in time. Each model represents different scenarios:

I) *Spatially-continuous Phase Fading:* Here, we assume that the small-scale channel variation between i'th and k'th nodes, $h(\mathbf{x}_i, \mathbf{x}_k)$, is only due to the phase difference, that is:

$$h(\mathbf{x}_i, \mathbf{x}_k) = e^{j\theta(\mathbf{x}_i, \mathbf{x}_k)},$$

where $\theta(\cdot, \cdot)$ represents the phase variation as a function of node locations. We assume $\theta(\mathbf{x}_i, \mathbf{x}_k)$'s are uniformly distributed in $(0, 2\pi]$. The function $\theta(\cdot, \cdot)$ is assumed to be uniformly continuous:

A1) $\forall \delta > 0$, $\exists \epsilon > 0$ s.t. $\forall\ \mathbf{x}_1, \mathbf{x}_2, \mathbf{y} \in \mathbb{R}^2$,

$$\text{if} \ \ \|\mathbf{x}_1 - \mathbf{x}_2\| < \epsilon, \ \ \text{then} \ \ |\theta(\mathbf{x}_1, \mathbf{y}) - \theta(\mathbf{x}_2, \mathbf{y})| < \delta,$$

almost surely.

This assumption essentially means that as two nodes get closer and closer, their channel coefficients become highly correlated. The main motivation behind this model is its simplicity which helps understanding the intuition behind the results we obtain.

II) *Spatially-Continuous Small-Scale Fading Model:* Here, we consider small-scale fading with a certain correlation model. The correlation is such that $h(\cdot, \cdot)$ is uniformly continuous and satisfies the following conditions:

A2) $\forall \delta > 0$, $\exists \epsilon > 0$ s.t. $\forall \mathbf{x}_1, \mathbf{x}_2, \mathbf{y} \in \mathbb{R}^2$, if $\|\mathbf{x}_1 - \mathbf{x}_2\| < \epsilon$, then $|h(\mathbf{x}_1, \mathbf{y}) - h(\mathbf{x}_2, \mathbf{y})| < \delta$, almost surely.

A3) $\forall \delta > 0$, $\exists \epsilon > 0$ s.t. $\forall \mathbf{x}, \mathbf{y}_1, \mathbf{y}_2 \in \mathbb{R}^2$, if $\|\mathbf{y}_1 - \mathbf{y}_2\| < \epsilon$, then $|h(\mathbf{x}, \mathbf{y}_1) - h(\mathbf{x}, \mathbf{y}_2)| < \delta$, almost surely.

A4) $\forall \mathbf{x} \in \mathbb{R}^2$, $h(\mathbf{x}, \mathbf{x}) = 1$.

III) *Independent and Identically Distributed Small-Scale Fading:* Here, we assume that $h(\mathbf{x}, \mathbf{y})$'s are independent and identically distributed for different values of $(\mathbf{x}, \mathbf{y})$ pairs.

III. Summary of Main Results

In the following sections, we propose novel network schemes that coordinate the transmissions from source and relays to the destinations. Let T denote the number of channel uses for the network protocol. A rate R_s(bits/sec) is said to be achievable under the proposed protocols if there exists a sequence of $(M := 2^{nR_s}, n)$ codes such that the probability of error of decoding P_e converges to zero as $n \to \infty$. Define $C_s := \frac{nR_s}{T}$ (bits/channel use) as the broadcast rate.

We assume each node utilizes a Gaussian codebook such that each message $m \in \{1, 2, \ldots, 2^{nR_s}\}$ is mapped to a codeword $\mathbf{X}_m$ of length n, where $\mathbf{X}_m \sim \mathcal{N}(0, \mathbf{I})$. We assume the distribution over different messages is uniform. Each node transmits $\mathbf{X}_m$ for message m.

- Under models I & II, the broadcast capacity for high density networks scales as
$$C_1^{high} = \Theta(\log N),$$
for large N with probability approaching 1 (w.p.a. 1).
- Under model III, the broadcast capacity for high density networks satisfies
$$K_1 \log(\log N) \leq C_2^{high} \leq K_2 \log N,$$
for large N, w.p.a. 1.
- For extended networks, the broadcast capacity is upper bounded as
$$C^{ext} \leq K_3 \log(\log N)$$
for large N, w.p.a. 1.

IV. Scaling of Broadcast Capacity under Spatially-Continuous Phase Fading

In this section, we show that under spatially-continuous phase fading model, the cooperative broadcast capacity is
$$C_1^{high} = \Theta(\log N), \tag{3}$$
w.p.a. 1, for large N. Although validity of spatially-continuous phase fading is debatable, this model leads to a simplified analysis and helps understand the intuition behind more complicated models. In order to prove (3), we provide an upper bound for C_1^{high} that scales as $\log(N)$ in Section IV-A, and we describe a novel scheme with achievable rate that scales as $\log(N)$ in Section IV-B.

A. Upper Bounds on the Cooperative Broadcast Capacity

Lemma 1: The broadcast rate C_1^{high} under spatially-continuous phase fading model is bounded above as
$$C_1^{high} \leq K \log(N), \quad \text{for} \quad N \to \infty,$$
where K is a constant.

Proof: The proof is based on the cut-set bound theorem [18, Chapter 14, page 445]. We are interested in only the cut that separates the source from the rest of the network. Based on this cut, the broadcast rate is bounded by the capacity of the single-input multiple-output (SIMO) channel where the relays act as co-located antennas at the receiver. For details, see Appendix B. ∎

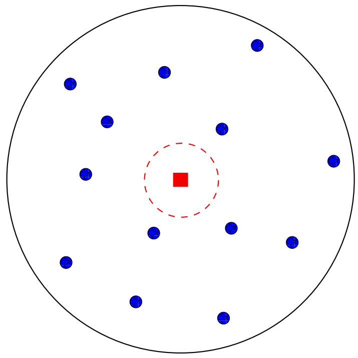

Fig. 1. SIMO cut: source is denoted with a square; nodes are denoted by filled circles; SIMO cut is dashed circle.

B. Proposed Cooperative Broadcasting Protocol

In this section, we provide a scheme that has achievable rate which scales as $\log(N)$ for large N under spatially-correlated phase fading. This result is due to the spatial correlation of the fading model which lets the neighboring nodes coherently retransmit the source message to every other node in the network. The proposed scheme is composed of two phases: (i) source broadcasts the message; (ii) *close-by neighbors* (detailed explanation is given below) of the source node retransmit the source message. The transmissions in different phases are time-duplexed, *i.e.,* Phase-1 occurs during $[0, T/2)$ and Phase-2 occurs during $[T/2, T)$ where T is the transmission period. We provide a detailed explanation of these two phases in the following.

- *Phase-1:* In the first phase, source node broadcasts the message. Under the spatially-correlated phase fading model, the channel gain from the source to the k'th node is
$$G_{0k} = \left| \frac{e^{j\theta(\mathbf{x}_0, \mathbf{x}_k)}}{\|\mathbf{x}_0 - \mathbf{x}_k\|^{\alpha/2}} \right|^2 = \|\mathbf{x}_0 - \mathbf{x}_k\|^{-\alpha}.$$
Let the transmission rate of the source node be $R_I = \frac{1}{2}\log(1 + N^b)$. Consider the nodes that lie within the disc with radius $r := N^{-b/\alpha}$ around the source node. These nodes have channel gains $G_{0k} \geq N^b$, hence can decode the message correctly. We will name these nodes

as the *close-by neighbors* of the source node. The number of *close-by neighbors* is

$$N_r := \frac{N}{A}\pi r^2 = N^{-2b/\alpha+1}, \quad (4)$$

where the total area is assumed to be $A = \pi$ for simplicity. We would like $-2b/\alpha + 1 > 0$ which implies $b < \alpha/2$ (satisfied for small b's).

- *Phase-2:* In the second phase, the nodes within the radius r retransmit the source message. For convenience, we assume that the index set of these nodes is $\mathcal{S}_r := \{1, \ldots, N_r\}$. The effective channel gain at the k'th node ($k \in \mathcal{S}\backslash\mathcal{S}_r$) due to transmission of the nodes in $\mathcal{S}_r$ is given by:

$$G_k = \left|\sum_{i=1}^{N_r} \sqrt{P_i} e^{j\theta(\mathbf{x}_i,\mathbf{x}_k)} \|\mathbf{x}_i - \mathbf{x}_k\|^{-\alpha/2}\right|^2.$$

Using the definition $d_{ik} := \|\mathbf{x}_i - \mathbf{x}_k\|$, we can rewrite the gain G_k as

$$\begin{aligned} G_k &= \sum_{i_1=1}^{N_r}\sum_{i_2=1}^{N_r} \sqrt{P_{i_1}P_{i_2}} e^{j(\theta(\mathbf{x}_{i_1},\mathbf{x}_k)-\theta(\mathbf{x}_{i_2},\mathbf{y}))} d_{i_1 k}^{-\alpha/2} d_{i_2 k}^{-\alpha/2} \\ &= \sum_{i_1=1}^{N_r} (P_{i_1} d_{i_1 k}^{-\alpha} + 2 d_{i_2 k}^{-\alpha/2} \times \\ &\qquad \sum_{i_2>i_1}^{N_r} \sqrt{P_{i_1}P_{i_2}} \cos(\Delta_{i_1 i_2}^k) d_{i_2 k}^{-\alpha/2}), \end{aligned}$$

where $\Delta_{i_1 i_2}^k := |\theta(\mathbf{x}_{i_1}, \mathbf{x}_k) - \theta(\mathbf{x}_{i_2}, \mathbf{x}_k)|$. Under assumption A1), we know that $||\mathbf{x}_1 - \mathbf{x}_2|| < r$ implies that $|\theta(\mathbf{x}_1, \mathbf{y}) - \theta(\mathbf{x}_2, \mathbf{y})| < \delta_r < \pi/2$ almost surely. Using this fact, we can lower bound the effective gain as

$$G_k \geq \sum_{i_1=1}^{N_r} (P_{i_1} d_{i_1 k}^{-\alpha} + 2 d_{i_1 k}^{-\alpha/2} \cos(\delta_r) \sum_{i_2>i_1}^{N_r} \sqrt{P_{i_1}P_{i_2}} d_{i_2 k}^{-\alpha/2}).$$

The distances d_{ik} are upper bounded by $d_{max} := 1$ (since we assumed the total area $A = \pi$). Then,

$$\begin{aligned} G_k &\geq \sum_{i_1=1}^{N_r} (P_{i_1} d_{max}^{-\alpha} + 2 d_{max}^{-\alpha/2} \cos(\delta_r) \sum_{i_2>i_1}^{N_r} \sqrt{P_{i_1}P_{i_2}} d_{max}^{-\alpha/2}) \\ &= \sum_{i_1=1}^{N_r} (P_{i_1} + 2\cos(\delta_r) \sum_{i_2>i_1}^{N_r} \sqrt{P_{i_1}P_{i_2}}) \end{aligned}$$

In the following, we assume $P_i = \frac{P}{N}$ which satisfies the total power constraint (2).

$$\begin{aligned} G_k &\geq PN_r \cos(\delta_r) + P(1 - \cos(\delta_r)) \\ &= PN^{1-4b/\alpha} \cos(\delta_r) + PN^{-2b/\alpha}(1 - \cos(\delta_r)) \end{aligned}$$

Then, one can easily find a constant K such that

$$R_{II} = \min_k \frac{1}{2} \log(1 + \frac{G_k}{N_0}) \geq K \log(1 + N), \quad (5)$$

for large N.

Since both R_I and R_{II} scales as $\log(N)$ for large N, the described two-phase protocol is optimal under the correlated phase fading model.

V. Scaling of Broadcast Capacity under Spatially-Continuous Small-Scale Fading Model

In this section, the small-scale fading is assumed to be ergodic in time, and the message experiences many realizations of the fading. In the following, we analyze the scaling behavior of the ergodic capacity, which is the maximum achievable rate on average.

Similar to Section IV, first we provide an upper bound for the ergodic capacity, and then we describe a novel scheme with achievable rate that scales as $\log(N)$. We will use the notation $h_{ik} = h(\mathbf{x}_i, \mathbf{x}_k)$ for convenience.

Lemma 2: The ergodic broadcast capacity C_1^{high} under spatially-correlated small-scale fading is bounded above as

$$C_1^{high} \leq K \log N,$$

w.p.a. 1, for large N.

Proof: See Appendix C. ■

Next, we provide a scheme that has achievable rate which scales as $\log(N)$ for large N. The transmissions happen in two phases:

- *Phase I:* In the first phase, source node broadcasts the message. The channel gain from the source to the k'th node is

$$G_{0k} = d_{0k}^{-\alpha} |h_{0k}|^2,$$

where $h_{0k} = h(\mathbf{x}_0, \mathbf{x}_k)$. Let the transmission rate be $R_I = \frac{1}{2}\log(1 + N^b)$. Consider the nodes that lie within the disc with the radius $r := N^{-b/\alpha}$ (close-by neighbors of the source). The channel gains from the source to these nodes are lower bounded as

$$G_{0k} \geq N^b |h_{0k}|^2.$$

A given node which lies in radius r receives the source transmission with channel gain greater and equal to N^b almost surely. This follows from the assumption A4). The number of nodes that can decode the source message correctly is

$$N_r \geq \frac{N}{A}\pi r^2 = N^{-2b/\alpha+1}, \quad (6)$$

almost surely. Here the total area is assumed to be π. For $b < \alpha/2$, $-2b/\alpha + 1 > 0$, and hence, N_r increases with the total number of nodes N. Let $\mathcal{S}_r := \{1, \ldots, N_r\}$ denote the set of successful nodes.

- *Phase II:* In the second phase, the nodes within the set $\mathcal{S}_r$ retransmit the message. The instantaneous channel gain at the k'th node ($k \in \mathcal{S}\backslash\mathcal{S}_r$) due to transmission of the nodes in $\mathcal{S}_r$ is given by:

$$G_k = \left|\sum_{i=1}^{N_r} \sqrt{P_i} h(\mathbf{x}_i, \mathbf{x}_k) d_{ik}^{-\alpha/2}\right|^2,$$

and the achievable rate in the second phase is given by

$$R_{II} = \min_k \frac{1}{2}\mathbb{E}\{\log(1 + \frac{G_k}{N_0})\}.$$

Note that $\log((1-z)+ze^x)$ is a convex function in x for $0 < z < 1$ and $x > 0$. Using this fact and Jensen's Inequality, we derive a lower bound on R_{II}. For $N_0 \geq 1$,

$$R_{II} \geq \min_k \frac{1}{2}\log\left((1-\frac{1}{N_0}) + \frac{1}{N_0}e^{\mathbb{E}\log(G_k+1)}\right). \quad (7)$$

For $N_0 < 1$, using the fact that the capacity increases with the inverse of the noise variance, we can lower bound R_{II} as

$$R_{II} \geq \min_k \frac{1}{2}\mathbb{E}\{\log(G_k+1)\}, \quad \text{for } N_0 < 1. \quad (8)$$

We will lower bound the $\mathbb{E}\{\log(G_k+1)\}$ which will allow us to lower bound C_1^{high}:

$$\mathbb{E}\{\log(G_k+1)\} \geq \mathbb{E}\{\log(G_k+1)|G_k \geq N^a\}\cdot \Pr\{G_k \geq N^a\}. \quad (9)$$

We claim that $\Pr\{G_k \geq N^a\} \rightarrow p > 0$, in the limit $N \rightarrow \infty$. The proof of this claim is given in the Appendix under Lemma 8. Based on this claim,

$$\mathbb{E}\{\log(G_k+1)\} \geq \log(1+N^a)p,$$

which implies

$$\begin{aligned} R_{II} &\geq \begin{cases} \frac{1}{2}\log\left((1-\frac{1}{N_0}) + \frac{(1+N^a)^p}{N_0}\right) & \text{if } N_0 \geq 1 \\ \frac{1}{2}\log\left((1+N^a)p\right) & \text{otherwise} \end{cases} \\ &\geq K\log(N), \end{aligned}$$

as $N \rightarrow \infty$ for some constant K.

Since both R_I and R_{II} scales as $\log N$ for large N,

$$C_1^{high} = \Theta(\log N),$$

as $N \rightarrow \infty$.

VI. Scaling of Broadcast Capacity under i.i.d. Small-scale Fading

In this section, we assume the channel coefficients are spatially independent and identically distributed. We will also assume that the channel coefficient $h(\mathbf{x},\mathbf{y}) \sim \mathcal{N}_c(0,1)$. A brief description of the two-phase scheme is as follows. In the first phase, the source node broadcasts with rate $\Theta(\log\log N)$. In order for a node to be able to decode this message; it should have a channel gain which scales like $\log N$. The important point is that, although source node transmits with a high rate, there is always a group of node which can decode this message. This idea follows from the i.i.d. fading and is very similar to multiuser diversity [19]. In the second phase, successful nodes retransmit the message. Note that each node has a neighbor who has been able to decode the message reliably in the first phase. The nodes that were unsuccessful in the first phase receive from its successful neighbor in the second phase. Hence, every node can decode the message. The details are as follow.

Lemma 3: The capacity under spatially i.i.d. small-scale fading is upper bounded as

$$C_2^{high} \leq K\log N,$$

w.p.a. 1, where K is a constant which is independent of N.

Proof: The proof is the same as the proof of Lemma 2. ■

In the following, we describe a scheme of rate $C_2 = \Theta(\log(\frac{\log N}{K}))$ for $K > d_{max}^{\alpha}$, where $d_{max} := \max_k d_{0k}$. The scheme is composed of two phases:

- *Phase 1:* After source transmission with rate $R_I = \frac{1}{2}\log(1+\frac{\log N}{K})$, a given node $\mathbf{x}_k$ receives the source message correctly with probability

$$\begin{aligned} P_c &= \Pr\left\{\frac{|h(\mathbf{x}_0,\mathbf{x}_k)|^2}{d_{0k}^{\alpha}} > \frac{\log N}{K}\right\} \\ &\geq \Pr\left\{\frac{|h(\mathbf{x}_0,\mathbf{x}_k)|^2}{d_{max}^{\alpha}} > \frac{\log N}{K}\right\} \\ &= \exp\left(-\frac{d_{max}^{\alpha}\log(N)}{K}\right) \\ &= \frac{1}{N^{d_{max}^{\alpha}/K}}. \end{aligned}$$

  Then, the number of nodes that decodes source message correctly scales as

$$N_1 := NP_c \geq \frac{N}{N^{d_{max}^{\alpha}/K}} = N^{1-d_{max}^{\alpha}/K},$$

  w.p.a. 1.
- *Phase 2:* In the second phase, nodes that have received source message correctly retransmits. Let's pick a node, call it k'th node, that has not received source message correctly and consider a disc around this node with radius ϵ. The nodes that have received source message correctly in the first phase in this disc retransmit the message. Total number of these nodes is lower bounded as

$$N_{\epsilon} \geq N_1\epsilon^2 = \epsilon^2 N^{1-d_{max}^{\alpha}/K},$$

  the total area is assumed to be $A = \pi$, and $d_{max} = 1$. Note that sum of independent Gaussian random variables is a Gaussian with a variance equal to the sum of the variances. Using this fact, the received signal power at the k'th nodes is

$$G_k \geq \frac{1}{\epsilon^{\alpha}}N_{\epsilon}\frac{P}{N_{\epsilon}}.$$

  Let $\epsilon = N^{-\delta/\alpha}$, for some $\delta > 0$; then we obtain

$$G_k = N^{\delta}.$$

  Hence each node can decode the source message correctly.

In summary, in the first phase, similar to multiuser diversity, a group of lucky nodes with good channel realizations receive the message. In the second phase, these nodes retransmit and amplify signal power for their nearest neighbors.

VII. Extended Network

In this section, we consider a network with N nodes (uniformly and randomly distributed) with a fixed node density $\rho = 1$. We look at the asymptote as the number of nodes increases, i.e. $N \to \infty$ and the total area increases. We will analyze both one-dimensional and two-dimensional configurations. The upper bounds provided on the broadcast capacity is valid for both correlated and i.i.d. fading models.

In the following, we will use a result which can be also be found in a related work [7, Lemma 4.1]. Suppose that we randomly place N identical balls into N boxes (assuming it is equally likely to place any ball into any of the boxes). Let N_k be the number of balls that falls into the k'th box, for $k = 1 \dots N$. Note that N_k is random. Then,

$$N_k \leq \log N, \tag{10}$$

w.p.a. 1.

A. Linear Network

Lemma 4: Under total power constraint (2), the broadcast capacity for the extended network is upper bounded as

$$C_1^{ext} \leq K \log(\log N),$$

w.p.a. 1.

Proof: The proof is based on the cut-set bound theorem [18]. We are interested in only the cut that separates source from the rest of the network. Based on this cut, the broadcast rate is bounded by the capacity of the single-input multiple-output (SIMO) channel where the relays act as co-located antennas at the receiver:

$$C_1^{ext} \leq \log(1 + \frac{P_0}{N_0} \sum_{i=1}^{N} |h(\mathbf{x}_0, \mathbf{x}_i)|^2),$$

where N_0 is the variance of the AWGN channel. Using the properties of expectation and Jensen's inequality,

$$\begin{aligned} C_1^{ext} &\leq \log(1 + \frac{P_0}{N_0} \sum_{i=1}^{N} \frac{1}{d_{0i}^{\alpha}} \mathbb{E}\{|h_{0i}|^2\}) \\ &= \log(1 + \frac{P_0}{N_0} \sum_{i=1}^{N} \frac{1}{d_{0i}^{\alpha}}). \end{aligned} \tag{11}$$

We are going to divide the network into intervals of length $r := 1/\rho$ such that the i'th interval is $((i-1)r, ir)$ for $i = 1 \dots N$. Let N_i denote the number of nodes that lie in the i'th interval. In order to lower bound d_{0i}, we will move all the nodes that lie in the i'th interval to the location ri (see Fig. 2).

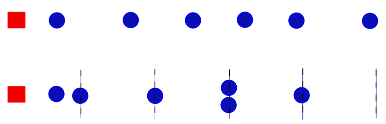

Fig. 2. Extended Network: Upper bound derivation - one dimensional network

Using the new topology, we can upper bound $\sum_{i=1}^{N} \frac{1}{d_{0i}^{\alpha}}$ as

$$\sum_{i=1}^{N} \frac{1}{d_{0i}^{\alpha}} \leq K_1 + \sum_{i=1}^{N-1} \frac{N_{i+1}}{r^{\alpha} i^{\alpha}}, \tag{12}$$

where $K_1 := \sum_{i \in (0,r)} \frac{1}{d_{0i}^{\alpha}}$, where $i \in (0,r)$ means that the i'th node lies in the interval $(0, r)$. Using the result (10), we can further upper bound $\sum_{i=1}^{N} \frac{1}{d_{0i}^{\alpha}}$ as

$$\sum_{i=1}^{N} \frac{1}{d_{0i}^{\alpha}} \leq K_1 + \frac{\zeta(\alpha)}{r^{\alpha}} \log N, \tag{13}$$

w.p.a. 1. Then, using (18) and the total power constraint (2), we can upper bound C_1^{ext} as

$$\begin{aligned} C_1^{ext} &\leq \log(1 + \frac{P}{N_0}(K_1 + \frac{\log N \zeta(\alpha)}{r^{\alpha}})) \\ &\leq \Theta(\log \log N), \end{aligned} \tag{14}$$

w.p.a. 1. ■

B. Planar Network

Lemma 5: Under total power constraint (2), the broadcast capacity for the extended network is upper bounded as

$$C_2^{ext} \leq K \log(\log N),$$

w.p.a. 1.

Proof: Similar to the linear network, the broadcast rate is bounded by the capacity of the single-input multiple-output (SIMO) channel where the relays act as co-located antennas at the receiver [18]:

$$C_2^{ext} \leq \log(1 + \frac{P_0}{N_0} \sum_{i=1}^{N} |h(\mathbf{x}_0, \mathbf{x}_i)|^2),$$

where N_0 is the variance of the AWGN channel. Using the properties of expectation and Jensen's inequality,

$$\begin{aligned} C_2^{ext} &\leq \log(1 + \frac{P}{N_0} \sum_{i=1}^{N} \frac{1}{d_{0i}^{\alpha}} \mathbb{E}\{|h_{0i}|^2\}) \\ &= \log(1 + \frac{P}{N_0} \sum_{i=1}^{N} \frac{1}{d_{0i}^{\alpha}}). \end{aligned} \tag{15}$$

Divide the network into rectangles such that each rectangle is of unit area (see Figure 3). Note that based on the result (10), the number of nodes in each rectangle is $\leq \log N$, w.p.a. 1. Let $r = \sqrt{\frac{1}{N\pi\rho}}$ and let $\mathcal{C}_i$ denote the circle with radius $r_i = ir$. Consider circles $\mathcal{C}_i, i = 1 \dots N$. We will move the nodes that lie in the ring between the circles $\mathcal{C}_i$ and $\mathcal{C}_{i+1}$ onto the circle $\mathcal{C}_i$ for $i = 1 \dots N$ (see Fig. 3).

Note that the i'th circle can contain at most $\pi r^2 i^2$ rectangles; hence, the number of nodes in the i'th circle is upper bounded as

$$M_i \leq (\pi r^2 i^2) \log N,$$

w.p.a. 1.

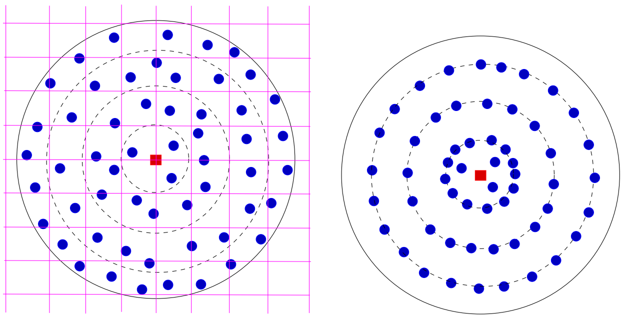

Fig. 3. Extended Network: Upper bound derivation - two dimensional network

Then, we can upper bound $\sum_{i=1}^{N}\frac{1}{d_{0i}^{\alpha}}$ as

$$\sum_{i=1}^{N}\frac{1}{d_{0i}^{\alpha}} \leq K_1+\sum_{i=2}^{N}\frac{M_i-M_{i-1}}{r^{\alpha}(i-1)^{\alpha}}, \quad (16)$$

$$\leq K_1+\sum_{i=1}^{N-1}\frac{\pi(2i+1)}{r^{\alpha-2}i^{\alpha}}\log N, \quad (17)$$

where $K_1:=\sum_{i\in\mathcal{C}_1}\frac{1}{d_{0i}^{\alpha}}$, and $i\in\mathcal{C}_1$ means that the i'th node lies inside the circle $\mathcal{C}_1$. We will first study the the scenario where the pathloss exponent $\alpha>2$:

$$\sum_{i=1}^{N}\frac{1}{d_{0i}^{\alpha}} \leq K_1+\frac{\pi}{r^{\alpha-2}}(2\zeta(\alpha-1)-\zeta(\alpha))\log N. \quad (18)$$

On the other hand, for $\alpha=2$, the summation in (16) can be rewritten as

$$\sum_{i=1}^{N}\frac{1}{d_{0i}^{\alpha}} \leq K_1+\sum_{i=1}^{N-1}\frac{\pi(2i+1)}{i^2}\log N,$$

$$\leq K_1+\pi(2(1+\log(N-1))+\zeta(2))\log N. \quad (19)$$

This follows from the fact that $\sum_{i}^{N}\frac{1}{i}\leq 1+\int_1^N\frac{1}{x}dx$.

Using (15), (18), and (19), under total power constraint (2), we obtain

$$C_2^{ext}\leq K\log(\log(N)), \quad (20)$$

w.p.a. 1, for some finite K independent of N.

Note that it is straight forward to extend the above results to the case where we replace the total power constraint with a per-node power constraint. Assume that each node can transmit with an average power P, i.e.,

$$P_i\leq P, \quad i=0\ldots N. \quad (21)$$

Then,

$$C_2^{ext}\leq K\log(N), \quad (22)$$

w.p.a. 1. ∎

VIII. MULTISTAGE COOPERATIVE BROADCAST

In this section, we describe and analyze a scheme which achieves $\Theta(1)$ scaling in an extended network, for pathloss exponent $\alpha=2$ under a power density constraint, which is equivalent to per node power constraint. Based on the analysis in the previous section, it is straight forward to see that the broadcast capacity in this case is upper bounded by $\Theta(\log N)$. Here, we follow the seminal work [3], and use the physical layer model based on signal-to-interference ratio. Next, we briefly describe the scheme.

Suppose that N nodes are uniformly and randomly distributed within $\mathbb{S}=\{(x,y):x^2+y^2\leq R^2\}$ and the source node is located at the origin. The scheme is as follows in detail. The source node initiates the transmission by sending a message with power P_s. We will assume the noise is of unit power. After source transmission, the group of nodes that receives the message with sufficient SNR τ will be called level-1 nodes. We will denote the location of level-1 nodes by the set $\mathcal{S}_1=\{(x,y)\in\mathbb{S}:\frac{P_s}{x^2+y^2}\geq\tau\}$. We assume that the message is channel coded so that the nodes with received SNR greater than or equal to τ can decode the message correctly. Let P_r denote the transmission power of each relay. After the transmission of the nodes in $\mathcal{S}_1$, the nodes that receive the message with SNR greater and equal to τ will be called level-2 nodes. We assume that the group transmissions are synchronized and each node uses the same Gaussian codebook. It is assumed that each relay accumulates signals from one previous levels. The set of locations of level-k nodes $\mathcal{S}_k$ is given as

$$\mathcal{S}_k=\{(x,y)\in\mathbb{S}\backslash\bigcup_{i=1}^{k-1}\mathcal{S}_i:\sum_{(x',y')\in\mathcal{S}_{k-1}}\frac{P_r}{(x-x')^2+(y-y')^2}\geq\tau\}, \quad (23)$$

for $k\geq 2$.

We analyze the performance of this scheme for a single shot transmission, that is, the source node sends a single message in [2]. There, we were interested in determining the critical SNR threshold such that the message propagates to the entire network. In order to obtain the results in [2], we first considered a random network in which the node locations are randomly and uniformly distributed, and we obtained a continuum model from the random network by letting the number of nodes go to infinity while fixing the total relay power. Let $\rho=N/\text{Area}(\mathbb{S})$ be the density (node/unit area) of relays within the region $\mathbb{S}$. Define the *relay power per unit area* as $\bar{P}_r\triangleq P_rN/\text{Area}(\mathbb{S})=P_r\rho$.

Under the continuum model, each level becomes a disc with inner radius r_{k-1} and outer radius r_k, *i.e.,* the level-k set $\mathcal{S}_k$ can be approximated by the region $\mathcal{A}_k=\{(x,y):r_{k-1}^2<x^2+y^2\leq r_k^2\}$ [2, Theorem 1, Lemma 1]. We explicitly determined level discs, *i.e.,* $\{r_k\}$ and analyzed network dynamics as a function of decoding threshold τ, relay power density $\bar{P}_r$, and source power P_s. Furthermore, we showed that there exists a *phase transition* in the network behavior: if the SNR threshold is below a *critical value*, the

message is delivered to the whole network. Otherwise, only a fraction of the nodes is reached proportional to the source transmit power. That is,

$$\lim_{k\to\infty} r_k \to \begin{cases} \infty & \text{if } \tau \le \pi \ln(2)\bar{P}_r \\ K & \text{if } \tau > \pi \ln(2)\bar{P}_r \end{cases} \tag{24}$$

where $K < \infty$ depends on $P_0, \tau/\bar{P}_r$ and AWGN noise is assume to be unit power. The result (24) is obtained under the pathloss attenuation model $\ell(r) = 1/r^2$.

In this work, we study the scenario where the source node continuously transmits. Let T_s denote the transmission period of the source node. Notice that periodic source transmission adds interference into the picture which was ignored in the single shot analysis [2].

Due to the symmetric nature of the pathloss attenuation model and uniform relay power allocation, the nodes receive messages in the order of their distances from the source node. For a given node k, we define *downstream* nodes as the ones that receive the source message before node k and the nodes that receive the message after node k are called the *upstream* nodes. Notice that nodes can easily cancel the upstream interference, since they have already decoded the corresponding message. This is based on the assumption that the nodes can estimate the channel coefficients perfectly at the receiver side. Let $P(x,y)$ denote the transmission power of the node located at (x,y). Then, the nodes with sufficient signal to interference plus noise ratio (SINR) can decode the messages. Assuming source starts transmission at time zero, the SINR at location (x,y) at time k is given as

$$\gamma_k(x,y) = \frac{\sum_{(x',y')\in\mathcal{S}_{k-1}} \frac{P(x',y')}{(x-x')^2+(y-y')^2}}{1+\sum_{(x',y')\in\mathcal{U}_k} \frac{P(x',y')}{(x-x')^2+(y-y')^2}}, \tag{25}$$

where $\mathcal{U}_k = \bigcup_{i\in L_k} \mathcal{S}_i$, L_k is the set of levels that transmit at time k. Let $\nu := mod(i, T_s)$, then $I_i = \{\nu, \nu+T_s, \nu+2T_s, \ldots, i-T_s\}$, and $mod(a,b)$ denotes the remainder of the division of a by b. We assume that the nodes with sufficient SINR can decode the message correctly [3]. Then, the set of locations of level-k nodes $\mathcal{S}_k$ is given as

$$\mathcal{S}_k = \{(x,y) \in \mathbb{S} \setminus \bigcup_{i=1}^{k-1} \mathcal{S}_i : \sum_{(x',y')\in\mathcal{S}_{k-1}} \gamma_k(x',y') \ge \tau\}, \tag{26}$$

where $\mathcal{S}_1 = \{(x,y) \in \mathbb{S} : \frac{P_0}{x^2+y^2} \ge \tau\}$.

We claim that in order for the message to propagate under period source transmission, the transmission levels (regions under continuum model) should increase in size exponentially. This will help suppressing the interference caused by the downstream nodes. We will show that this is a sufficient condition for the message to propagate. Define

$$L := \lim_{k\to\infty} \frac{a_{k-1}}{a_k} < 1, \tag{27}$$

where $a_k = r_k^2$, and r_k is the outer radius of the kth level.

Lemma 6: Let $T_s = 1$. Using the continuum model, the critical threshold τ_c can be derived as

$$\tau_c \ge \arg\max_{L\in(0,1)} \frac{\log(1+L)}{\log(\mu)-\log(1-L^2)},$$

where $\bar{P}_r$ is the relay power density and $\mu = exp\left(1/\pi\bar{P}_r\right)$.

Proof: In the continuum, each level becomes a disc. This can be obtained by generalizing the derivation in [2, Section II.c]. Let r_{k-1} and r_k denote the inner and outer radius of level-k disc, respectively. The received power at a distance x from the source due to transmission of level-k nodes is given by

$$G_k(x) := \pi\bar{P}_r \log\frac{|x^2-r_{k-2}^2|}{|x^2-r_{k-1}^2|}.$$

See [2] for details of the derivation.

Let $a_k = r_k^2$. The solution for a_k can be find by solving the following non-linear dynamical equation:

$$\pi\frac{\bar{P}_r}{\tau}\log\left(\frac{a_k-a_{k-2}}{a_k-a_{k-1}}\right) = 1+\pi\bar{P}_r\log\left(\frac{a_k}{a_k-a_{k-2}}\right)+\frac{P_s}{a_k},$$

with initial conditions $a_0 = 0$ and $a_1 = \frac{P_s}{\tau}$. In order to solve a_k, we ignore the effect of source transmission $\frac{P_s}{a_k}$ for large k, and obtain a simpler form:

$$\frac{a_k-a_{k-2}}{a_k-a_{k-1}} \approx \left(\frac{a_k}{a_k-a_{k-2}}\right)^\tau \mu^\tau, \text{ for large } k. \tag{28}$$

Using definition (27) and (28), we find that

$$\tau(L) = \frac{\log(1+L)}{\log(\mu)-\log(1-L^2)}.$$

Note that $\tau(L)$ is a concave function in L, and has a single maximum in the interval $L \in (0,1)$. Then the proof of the theorem follows. ■

Then, the transmission rate of multistage cooperative broadcast is

$$R_s = \log(1+\tau_c) \ge \Theta(1).$$

IX. CONCLUSIONS

A fundamental problem in large scale wireless networks is determining the maximum transfer rate from a source to the whole network. The previous works on the subject focus on multihop strategies, for which the broadcast capacity scales at most as $\Theta(1)$. We consider both high density networks and extended networks for spatially correlated and i.i.d. channel models. We determine the upper and lower bounds on the broadcast capacity. Currently, we are working on designing schemes with rates that approaches these upper bounds for the extended network. Table I and II summarize the scaling of the upper and lower bounds on the broadcast capacity.

TABLE I
UPPER BOUNDS ON BROADCAST CAPACITY

	Spatially-correlated fading	I.i.d fading
high-density networks	$\log N$	$\log N$
extended networks	$\log(\log N)$	$\log(\log N)$

TABLE II

LOWER BOUNDS ON BROADCAST CAPACITY

	Spatially-correlated fading	I.i.d fading
high-density networks	$\log N$	$\log(\log N)$
extended networks	constant	constant

APPENDIX

A. Behavior of d_{min} for large number of nodes N

Lemma 7: Consider a disc network with radius $R = 1$. Assume that the source node is located at the center and the rest of the nodes (N of them) are distributed uniformly in this circular region. Let d_{min} denote the minimum distance from the source to every other node in the network. Then,

$$\Pr\{|\frac{1}{d_{min}} - N^{\delta}| < \epsilon\}\to 1, \quad \text{as} \quad N\to\infty, \tag{29}$$

where $\delta > 0$ and $\epsilon > 0$.

Proof: For a disc network of radius $R = 1$ with the source node located at the center, the probability distribution function of d_{min} is given as

$$F_{d_{min}}(x) = \Pr\{d_{min} \leq x\} = 1 - \left(1 - x^2\right)^N .$$

Then, the proof follows easily. ■

B. Proof of Lemma 1

The proof is based on the cut-set bound theorem [18, Chapter 14, page 445]. We are interested in only the cut that separates source from the rest of the network. Based on this cut, the broadcast rate is bounded by the capacity of the single-input multiple-output (SIMO) channel where the relays act as co-located antennas at the receiver:

$$C_1^{high} \leq \log(1 + \frac{P_0}{N_0}\sum_{i=1}^{N}|h(\mathbf{x}_0, \mathbf{x}_i)|^2),$$

where N_0 is the variance of the AWGN channel. Let $d_{min} := \min_i \|\mathbf{x}_0 - \mathbf{x}_i\|$, then under spatially-continuous phase fading model

$$|h(\mathbf{x}_0, \mathbf{x}_i)|^2 = \frac{1}{\|\mathbf{x}_0 - \mathbf{x}_i\|^{\alpha}} < \frac{1}{d_{min}^{\alpha}} < N^{\delta_1 \alpha},$$

with high probability as $N\to\infty$ for some small δ_1. Then,

$$C_1^{high} \leq \log(1 + \frac{P}{N_0}N^{\delta_1\alpha+1}) \leq K \log N,$$

w.p.a. 1, for some K independent of N.

C. Proof of Lemma 2

Similar to Lemma 1, we will use the cut-set bound theorem [18] based on the cut that separates the source from the rest of the network. Then,

$$C_2^{high} \quad \leq \quad \mathbb{E}\{\log(1 + \frac{P_0}{N_0}\sum_{i=1}^{N}\frac{1}{d_{0i}^{\alpha}}|h_{0i}|^2)\}.$$

Using the properties of expectation and Jensen's inequality,

$$\begin{aligned} C_2^{high} &\leq \log(1 + \frac{P_0}{N_0}\sum_{i=1}^{N}\frac{1}{d_{0i}^{\alpha}}\mathbb{E}\{|h_{0i}|^2\}) \\ &= \log(1 + \frac{P_0}{N_0}\sum_{i=1}^{N}\frac{1}{d_{0i}^{\alpha}}) \\ &\leq \log(1 + \frac{P}{N_0}\frac{N}{d_{min}^{\alpha}}), \end{aligned}$$

where $d_{min} := \min_i d_{0i}$ denotes the minimum distance from the source node to every other node in the network. Using Lemma 7, we obtain

$$\begin{aligned} C_2^{high} &\leq \log(1 + \frac{P}{N_0}N^{(\delta_1)\alpha+1}), \\ &\leq K\log(N), \end{aligned}$$

w.p.a. 1.

D. Claim and its proof

Lemma 8: As $N\to\infty$,

$$\Pr\{G_k \geq N^a\}\to p > 0.$$

Proof: We can rewrite the instantaneous channel gain as

$$\begin{aligned} G_k &= \sum_{i_1=1}^{N_r}\sum_{i_2=1}^{N_r}\sqrt{P_{i_1}P_{i_2}}h(\mathbf{x}_{i_1}, \mathbf{x}_k)h^*(\mathbf{x}_{i_2}, \mathbf{x}_k)d_{i_1k}^{-\alpha/2}d_{i_2k}^{-\alpha/2} \\ &= \sum_{i_1=1}^{N_r}(P_{i_1}d_{i_1k}^{-\alpha}|h(\mathbf{x}_i, \mathbf{x}_k)|^2 + 2d_{i_1k}^{-\alpha/2} \cdot \\ &\quad \sum_{i_2>i_1}^{N_r}\sqrt{P_{i_1}P_{i_2}}Re\{h(\mathbf{x}_{i_1}, \mathbf{x}_k)h^*(\mathbf{x}_{i_2}, \mathbf{x}_k)\}d_{i_2k}^{-\alpha/2}). \end{aligned}$$

Since, the nodes that lie within a radius r have transmitted in the first phase $\|\mathbf{x}_{i_1} - \mathbf{x}_{i_2}\| < r$ implies $|h(\mathbf{x}_{i_1}, \mathbf{x}_k) - h(\mathbf{x}_{i_2}, \mathbf{x}_k)| < \delta_r$ almost surely. Using this fact, we know that

$$\begin{aligned} 2Re\{h(\mathbf{x}_{i_1}, \mathbf{x}_k)h^*(\mathbf{x}_{i_2}, \mathbf{x}_k)\} &\geq \\ &\underbrace{|h(\mathbf{x}_{i_1}, \mathbf{x}_k)|^2 + |h(\mathbf{x}_{i_2}, \mathbf{x}_k)|^2 - \delta_r^2}_{:=\mathcal{A}(\mathbf{x}_{i_1}, \mathbf{x}_{i_2}, \mathbf{x}_k)}. \end{aligned} \tag{30}$$

Using (30), we can derive an lower bound for $Pr\{G_k \geq N^a\}$:

$$P_{LB} := Pr\{\mathcal{B} \geq N^a\},$$

where

$$\begin{aligned} \mathcal{B} := &\sum_{i_1=1}^{N_r}(P_{i_1}d_{i_1k}^{-\alpha}|h(\mathbf{x}_{i_1}, \mathbf{x}_k)|^2 + d_{i_1k}^{-\alpha/2} \cdot \\ &\sum_{i_2>i_1}^{N_r}\sqrt{P_{i_1}P_{i_2}}\mathcal{A}(\mathbf{x}_{i_1}, \mathbf{x}_{i_2}, \mathbf{x}_k)d_{i_2k}^{-\alpha/2}). \end{aligned} \tag{31}$$

Let's condition on the event $|h(\mathbf{x}_i, \mathbf{x}_k)| > \delta_h > \frac{\delta_r}{\sqrt{2}}$, $\forall k$. Then,

$$\begin{aligned} P_{LB} &\geq Pr\{\mathcal{B} \geq N^a \mid |h(\mathbf{x}_k, \mathbf{y})| > \delta_h, \forall k\} \cdot \\ &\qquad \Pr\{|h(\mathbf{x}_k, \mathbf{y})| > \delta_h, \forall k\} \\ &\geq Pr\{N_r \delta_h^2 + (N_r^2 - N_r)(\delta_h^2 - \delta_r^2/2) > \frac{N^{a+1} d_{max}^{\alpha}}{P} \mid \\ &\qquad |h(\mathbf{x}_k, \mathbf{y})| > \delta_h, \forall k\} \cdot \\ &\qquad \Pr\{|h(\mathbf{x}_k, \mathbf{y})| > \delta_h, \forall k\}. \end{aligned}$$

Note that under the condition

$$a < 1 - 4b/\alpha \tag{32}$$

for large enough N,

$$P_{LB} \geq \Pr\{|h(\mathbf{x}_k, \mathbf{y})| > \delta_h, \forall k\} \tag{33}$$

If we choose δ_h small enough, then we find that

$$p := \Pr\{|h(\mathbf{x}_k, \mathbf{y})| > \delta_h, \forall k\} > 0.$$

Then, $\mathbb{E}\{\log(G_k + 1)\} \geq \log(1 + N^a)p$.

■

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