

Joint Multiple Description Coding and FEC for Delay-Constrained Applications in Congested Networks

Xunqi Yu, James W. Modestino and Dian Fan
Electrical and Computer Engineering Department
University of Miami
Coral Gables, FL 33124

Email: xyu1@umsis.miami.edu, jmodestino@miami.edu, dfan@umsis.miami.edu

Abstract—Multiple description coding (MDC) and forward error control (FEC) are two frequently used techniques for improving transport performance on lossy packet networks. Typically these two techniques are used separately and independently with resulting performance dependent on network conditions, the choice of coding parameters and the applications being transported. In this paper we propose and analyze a joint MDC and FEC coding approach for delay-constrained applications on congested networks under some idealized modeling assumptions which are, nevertheless, sufficient to draw some qualitative conclusions. Specifically, for fixed delay constraints we show that under a range of operating parameters, there is an optimal FEC coding block size and assignment of MDC rates which can achieve significantly improved end-to-end performance, compared to the performance achieved by using only MDC or FEC alone.

I. INTRODUCTION

Forward error control (FEC) coding has often been proposed to combat network packet losses. FEC can help recover the lost packets through the use of redundant packets. However, from the network's perspective, the widespread use of FEC schemes by end nodes will increase the raw packet-loss rate in a network. Moreover, the additional delay caused by FEC encoding and decoding also need to be considered, which is an issue particularly important for delay-constrained applications, such as video telephony and video streaming. For delay-insensitive applications, the overall efficacy of FEC coding in combating network packet losses has been previously modeled and investigated in [1]–[4], among others.

Another widely use technique to improve network information transmission is multiple description coding (MDC). Using this technique, each source sample is encoded by several encoders and each encoder generates a separate description of the source sample, as shown in Fig. 1. Some descriptions may be lost in the network. At the destination, the more descriptions received, the lower the distortion that can be achieved. Research on this subject has been focused on the achievable rate-distortion regions and specific coder designs for actual audio/video applications.

In [5], the authors investigated the performance of MDC in congested networks and demonstrated that MDC can significantly improve the overall average end-to-end distortion, at

least for a range of network operating parameters. However, modeling of the network transport process was somewhat simplified and no use of FEC was employed.

In this paper, we investigate the joint use of FEC coding and MDC to improve information transmission over congested networks subject to a strict delay constraint. We propose a system model for the transmission system using both FEC coding and MDC and analytically investigate the resulting end-to-end system performance. The major contributions of this paper includes two parts. Firstly, we model the delay-constrained application using FEC and study the overall efficacy of FEC in improving delay-constrained applications in congested networks. Secondly, we model the information transmission system jointly using a combination of FEC and MDC, and show that joint FEC and MDC can improve the end-to-end performance for delay-constrained transmission in congested networks under appropriate choice of network conditions.

The paper is organized as follows. In Section II we describe a system model and assumptions for joint FEC and MDC for delay-constrained information transmission applications over congested networks. In Section III we describe the distortion evaluations for an *i.i.d.*, zero-mean, unit-variance Gaussian source in both single description coding (SDC) and double description coding (DDC) systems. In Section IV we describe the analysis of the decoded packet-loss rate after FEC decoding where we include both the network transport delay and the delay resulting from FEC encoding/decoding. In Section V we quantitatively demonstrate the resulting end-to-end performance using joint FEC and MDC by some selected numerical examples. In Section VI we provide a summary and conclusions.

II. SYSTEM MODELS AND ASSUMPTIONS

In this paper we limit attention to only two descriptions. Figure 2 illustrates the joint DDC and FEC system for delay-constrained applications in congested networks. For comparison, Fig. 3 illustrates the corresponding SDC system. In order to analytically study the use of the combination of DDC and FEC, we assume an abstract and simplified network model, as

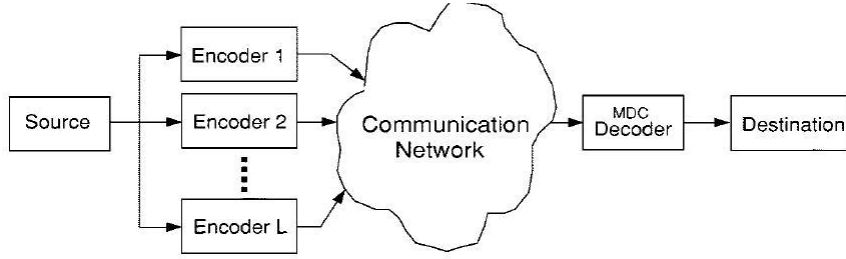


Fig. 1. A MDC transmission system.

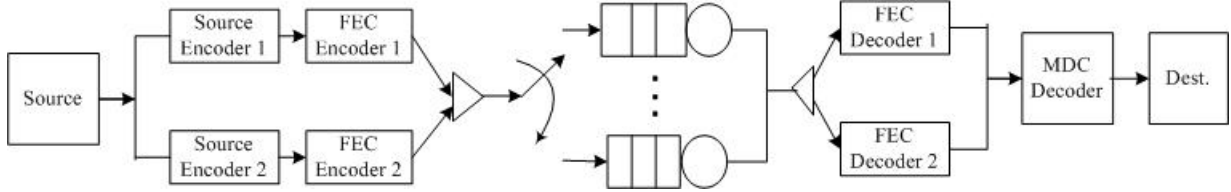


Fig. 2. A joint DDC and FEC transmission system.

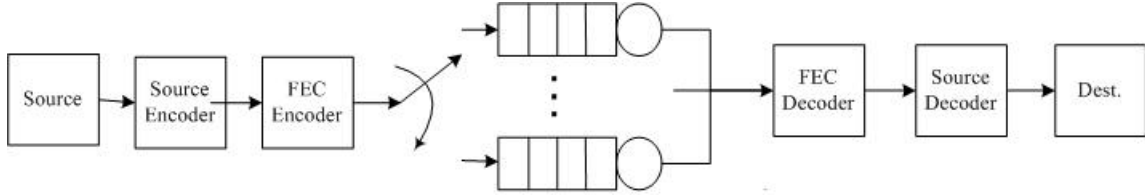


Fig. 3. A joint SDC and FEC transmission system.

in [5], so as to focus on the essential nature of the inherent tradeoffs. We assume the network is modeled in terms of a set of L parallel communication links and queues as shown in Fig. 2. Each link has a service rate C bits/sec and a limited buffer with a capacity of B bits. It should be noted that this is different from the assumptions in [5] where an infinite buffer is employed. We only consider wired networks and further assume the channels are noise-free. Each path is further modeled by a first come first served (FCFS) queue. We assume the arriving traffic to each queue follows a Poisson process with aggregate rate λ , which includes traffic from other sources. If the average packet size is R bits/packet, then the buffer can accommodate up to $K = B/R$ packets. An arriving packet will be discarded if it finds a full buffer.

We assume the source generates *i.i.d.*, zero-mean, unit-variance Gaussian variables. In the SDC system, each source sample is encoded with an average rate of R bits/sample and each codeword is transmitted in its entirety through the network. Assuming a uniform distribution of packets across paths, the resulting system load for each queue is then $\rho = \lambda * R / (L * C)$. In the DDC system, each source sample is encoded by two encoders, as shown in Fig. 2. Encoder 1 encodes the sample at average rate R_1 per sample and encoder 2 encodes the sample at average rate $R_2 = R - R_1$. The output of each encoder is then sent to the corresponding FEC encoders. As in [5], the system load in the DDC system is assumed identical

to the load of the SDC system.

We assume an interlaced Reed-Solomon $RS(n, k)$ coding scheme [6] is used to provide FEC. For every k information packets, an additional $n - k$ redundant packets are transmitted. The channel-coding rate is then given by $R_c = k/n$. As a result of the channel coding, the packet rate will increase by a ratio of n/k , *i.e.*, $\lambda' = \lambda(n/k)$. To avoid additional delay of the encoded packets, we assume that the packets are not held by the FEC encoder until redundant packets are generated. Instead, the packets produced by the source encoder are assumed available for transmission over the network immediately, but a copy of each packet will be made and sent to the FEC encoder to generate the redundant packets. Let T_e denote the encoding delay, *i.e.*, the processing time for the FEC encoder to execute the coding algorithm and generate redundant packets. For example, suppose the 1st information packet of an FEC coding block enters the network at time epoch $t = t_0$. Then the redundant packets of this coding block will be generated and are all available to enter the network at time epoch $t = t_0 + (k - 1) * \delta + T_e$, where δ is the packet interarrival time. We assume the output of FEC encoders is uniformly distributed between the queues and all servers will process a homogeneous traffic load.

At the receiver side, for the same reason, we assume the received source information packets are sent to the source decoder immediately upon arrival, but copies of the received

source information packets, together with all redundant packets, will also be sent to the FEC decoder to recover the lost packets. The FEC decoder groups all the received packets into blocks. We assume the FEC decoder works in a best-effort fashion, *i.e.*, it scans each group of packets corresponding to a block and whenever it finds there are enough received packets ($\geq n-k$) in a block, it begins to execute the decoding algorithm for that block and sends the recovered source packets to the source decoder buffer. Therefore, the total packet delay can be minimized. We assume the decoding delay (the processing time for the FEC decoder to execute the decoding algorithm) is T_d .

III. AVERAGE END-TO-END DISTORTION

Assume the source generates independent and identically distributed (*i.i.d.*), zero-mean, unit-variance Gaussian variables. For the SDC system, as in [5], the mean-square distortion for each SDC packet P with length R bits is

$$D_{SDC} = \begin{cases} 2^{-2R}, & \text{if } P \text{ received in time,} \\ 1, & \text{if } P \text{ is not received in time.} \end{cases} \quad (1)$$

Let PL denote the probability that P is not received in time, which is the effective packet loss rate for the SDC system. Then the average end-to-end distortion can be written as

$$\bar{D}_{SDC} = 2^{-2R}(1-PL) + PL. \quad (2)$$

For a DDC system, let P^1 and P^2 denote packets corresponding to the two descriptions and D_{DDC} denote the achievable distortion. Then we have, as in [5]

$$D_{DDC} = \begin{cases} d_0 = \frac{2^{-2(R_1+R_2)}}{2^{-2R_1} + 2^{-2R_2} - 2^{-2(R_1+R_2)}}, & \text{if both } P^1 \text{ and } P^2 \text{ are received in time,} \\ d_1 = 2^{-2R_1}, & \text{if only } P^1 \text{ received in time,} \\ d_2 = 2^{-2R_2}, & \text{if only } P^2 \text{ received in time,} \\ 1, & \text{if neither is received in time.} \end{cases} \quad (3)$$

It should be noted that, as mentioned in [5], this distortion only represents the boundary of the achievable R-D region for a DDC system. Let PL^1 and PL^2 denote the effective loss probabilities for P^1 and P^2 , respectively. Then the end-to-end average distortion can be written as

$$\bar{D}_{DDC} = d_0 * (1-PL^1) * (1-PL^2) + d_1 * (1-PL^1) * PL^2 + d_2 * PL^1 * (1-PL^2) + PL^1 * PL^2. \quad (4)$$

IV. ANALYSIS OF EFFECTIVE PACKET-LOSS RATE

We assume the length of transmitted codewords follows an exponential distribution. Therefore, in the SDC system, the service time for each queue is modeled by an exponential distribution and the queue is modeled as an $M/M/1/K$ queue. In the DDC system, the service time for each queue is modeled by a hyperexponential distribution and the queue is then modeled as an $M/G/1/K$ queue. Let PL_{raw}^l denote the raw packet-loss rate of an $M/G/1/K$ queue for packets from

encoder $l = 1, 2$ and $f_{T_n}^l(t|received)$ denote the corresponding pdf of the overall system delay, of received packets from encoder l , caused by the $M/G/1/K$ queue. The determination of PL_{raw}^l and $f_{T_n}^l(t|received)$ are outlined in the Appendix ¹.

Assume the delay threshold for each source information packet is Δ , *i.e.*, each source information packet is expected to arrive at the source decoder, either received from the network or recovered by the FEC decoder, within time Δ after it is sent into the network by the FEC encoder. In this section, we derive the effective packet-loss probability after FEC decoding, denoted by PL , *i.e.*, the effective packet-loss rate seen by the source decoder. Since the FEC encoding time T_e and decoding processing time T_d are hardware-dependent, and typically T_e and T_d are very small compared to the network delay T_n , in this paper we assume $T_e = 0$ and $T_d = 0$.

We derive PL by calculating the effective loss probability of each source information packet in an arbitrary block. Each FEC coding block has n packets, denoted by P_i , $i = 1, 2, \dots, n$, among which the first k packets are source information packets and the remaining $n-k$ packets are redundant packets. Let t_i^e , $0 \leq i \leq n$, denote the time epoch at which P_i is sent to the network by the source encoder. Suppose $t_1^e = t_0$, then according to the assumptions in Section II, we have

$$t_i^e = \begin{cases} t_0 + (i-1) * \delta & , \quad 1 \leq i \leq k \\ t_0 + (k-1) * \delta + T_e = t_0 + (k-1) * \delta & , \quad k+1 \leq i \leq n. \end{cases} \quad (5)$$

For each source information packet P_i , $1 \leq i \leq k$, suppose it is due at the source decoder at time epoch t_i^d . For a successful reception, assuming it is not lost, we require

$$t_i^d \leq t_i^e + \Delta = t_0 + (i-1) * \delta + \Delta, \quad 1 \leq i \leq k. \quad (6)$$

Let $Prob_i^a$, $1 \leq i \leq k$, denote the probability that P_i arrives from the network (not lost) before its due epoch t_i^d . According to the network model, we have

$$Prob_i^a = (1-PL_{raw}) * \int_0^{\Delta} f_{T_n}(\tau|received) d\tau. \quad (7)$$

Now let $Prob_i^r$, $1 \leq i \leq k$, denote the probability that P_i has not arrived from the network but can be recovered by the FEC decoder before its due epoch t_i^d . The effective loss probability of packet P_i , denoted as PL_i can then be expressed as

$$PL_i = 1 - Prob_i^a - Prob_i^r, \quad 1 \leq i \leq k. \quad (8)$$

Next we compute $Prob_i^r$. Define the set $\Omega = \{0, 1\}$ and $\mathcal{F} = \Omega^n$, *i.e.*, for each $w \in \mathcal{F}$, w is an n -tuple $w = (w_1, w_2, \dots, w_n)$, where $w_i = 0$ or 1 , $1 \leq i \leq n$.

We define an n -dimensional vector random process $I(t)$ as follows: for each time-epoch t , $I(t)$ is an n -dimensional random vector $I(t) = (I_1(t), I_1(t), \dots, I_n(t))$, where $I_j(t)$, $1 \leq j \leq n$, is the indicator function of the availability of the j -th packet P_j in the coding block at time-epoch t , *i.e.*,

$$I_j(t) = \begin{cases} 0, & \text{if } P_j \text{ arrives from the network before epoch } t, \\ 1, & \text{otherwise.} \end{cases} \quad (9)$$

¹From this point on for notational convenience we remove the conditioning on the encoder producing the packet with the understanding that a parallel development exists conditioned on the packet source.

We have then

$$\text{Prob}\{I_j(t) = 0\} = (1 - PL_{raw}) * \int_0^{t-t_j^e} f_{T_n}(\tau|received)d\tau, \quad (10)$$

and it follows that $\text{Prob}\{I_j(t) = 1\} = 1 - \text{Prob}\{I_j(t) = 0\}$.

For any $w = (w_1, w_2, \dots, w_n) \in \mathcal{F}$, due to the assumed independence of packet losses and delays, we have

$$\text{Prob}\{I(t) = w\} = \prod_{j=1}^n \text{Prob}\{I_j(t) = w_j\}. \quad (11)$$

If packet P_i has not arrived from the network by its due time t_i^d , it can be recovered only if at least k packets have arrived before epoch $t_i^d - T_d = t_i^d$ (again assuming $T_d = 0$). Therefore, we have

$$\text{Prob}_i^r = \sum_{w \in \mathcal{D}_i} \text{Prob}\{I(t_i^d) = w\}, \quad (12)$$

where $\mathcal{D}_i = \{w : w \in \mathcal{F}, w_i = 1 \text{ and } \sum_{j=1}^n w_j < n - k\}$. Then, from (7), (8) and (12), PL_i can be computed. The effective average packet-loss rate can then be expressed as

$$PL = \left(\sum_{i=1}^k PL_i \right) / k. \quad (13)$$

Then the end-to-end distortion can be computed from the effective packet-loss rates from ² (2) and (4).

V. NUMERICAL EXAMPLES

In this section we demonstrate some selected numerical examples to illustrate the effects of several system parameters on the end-to-end transport performance. Again, the purpose of this paper is to propose a model to analyze the FEC coding efficacy for delay-constrained transport applications, and to show that joint MDC and FEC approaches may outperform a system that uses only MDC or FEC alone. As mentioned above, in order to simplify the analysis in this paper we use an abstract and simplified network model. While the quantitative conclusions may not hold for more realistic network models, the qualitative conclusions are likely to hold.

We first study the efficacy of FEC coding on delay-constrained applications in an SDC system. Figure 4 shows the end-to-end normalized performance (in dB) as a function of FEC coding block size n with different source packet interarrival times δ . Generally, for fixed FEC coding block size n , the larger the source packet interarrival time is, the larger the decoding delay is. The FEC coding rate is fixed at $R_c = 2/3$. Other relevant system parameters are indicated in the figure. For comparison, the performance when no FEC is used is also indicated. It shows that generally for a given source packet interarrival time δ , there is an optimum coding block size n which can achieve the optimum end-to-end performance. Small coding block sizes are not powerful enough to recover the lost packets. A larger block size can achieve more powerful error correction capability, but it will also cause larger decoding delays.

²Again, we emphasize that a parallel development allows evaluation of PL^l , $l = 1, 2$ in (4).

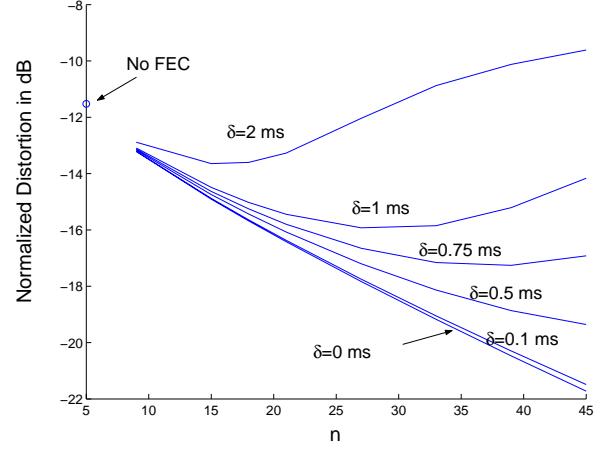


Fig. 4. End-to-end distortion in dB versus FEC coding block size n for different source packet interarrival times δ . SDC system, $R = 6$ bits, $C = 1000$ bits/s, $K = 5$, $R_c = 2/3$, $\Delta = 50$ ms, $\rho = 0.7$.

Figure 5 shows the system performance when a joint MDC and FEC approach is used. More specifically, it shows the end-to-end normalized distortion in dB as a function of the MDC rate ratio R_1/R for different FEC coding block sizes n . The FEC coding rate is again fixed at $R_c = 2/3$. The case when no FEC is used is also indicated. The results show that for each coding block size, there is an optimal MDC rate ratio which can achieve the best performance. Compared with the case when no FEC is used, it shows that joint MDC and FEC can improve the end-to-end performance significantly as far as this set of system parameters are concerned. When no FEC is used, the optimal MDC ratio is about 0.3 which can achieve optimal end-to-end performance -23 dB. When an $RS(30,20)$ code is used, the optimal MDC ratio is 0.2 which can achieve end-to-end performance -26 dB. The rate ratio $R_1/R = 0$ corresponds to the case of an SDC system. It should be noted that generally DDC with appropriately chosen rate ratio can achieve much better performance compared to the case when SDC is used. For example, for the curve labeled $RS(30,20)$ in Fig. 5, SDC ($R_1/R = 0$) can only achieve normalized distortion about -16 dB while DDC with ratio $R_1/R = 0.2$ can achieve about -26 dB.

Figure 5 seems to indicate that larger FEC coding block size can achieve improved performance. However, this is generally not the case. As indicated by Fig. 4, for each source packet interarrival time δ , there is an optimal FEC coding rate R_c . Figure 5 shows the case when $\delta = 1$ ms. Figure 6 shows the scenario when $\delta = 2$ ms, with all the other system parameters the same. In this case, the optimal FEC coding block size is $n = 21$ and the optimal MDC ratio is 0.3. Again, this is a reflection of the fact that for a fixed n , decoding delay increases with δ .

VI. SUMMARY AND CONCLUSIONS

In this paper we proposed a joint MDC and FEC coding approach for delay-constrained information transport over congested networks. We modeled and analyzed FEC efficacy

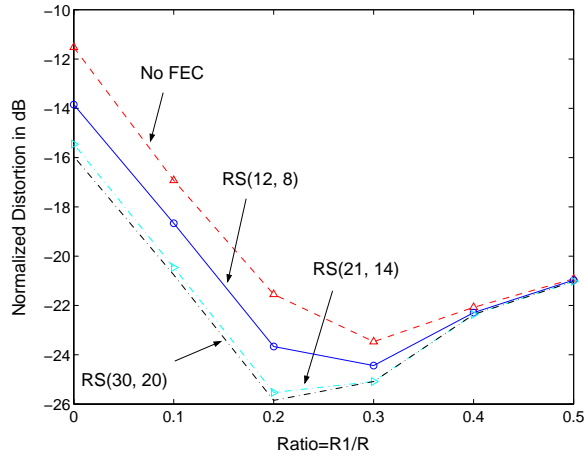


Fig. 5. End-to-end distortion in dB versus MDC rate ratio R_1/R for different FEC coding block sizes, DDC system, $R = 6$ bits, $C = 1000$ bits/s, $K = 5$, $R_c = 2/3$, $\Delta = 50$ ms, $\rho = 0.7$, $\delta = 1$ ms.

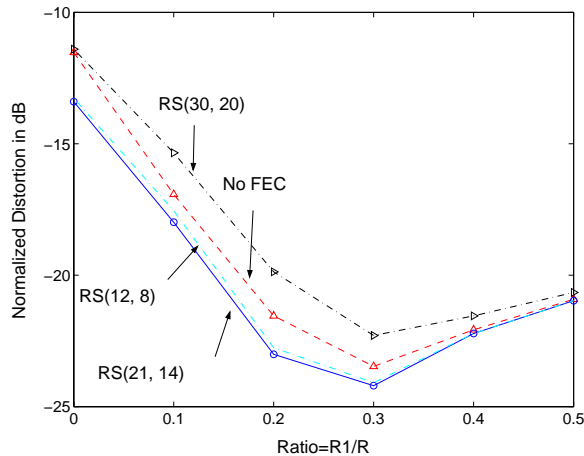


Fig. 6. End-to-end distortion in dB versus MDC rate ratio R_1/R for different FEC coding block sizes, DDC system, $R = 6$ bits, $C = 1000$ bits/s, $K = 5$, $R_c = 2/3$, $\Delta = 50$ ms, $\rho = 0.7$, $\delta = 2$ ms.

in improving delay-constrained network information transmission. We showed that under selected range of operating parameters, there is an optimal FEC coding block size n and MDC coding rate ratio R_1/R which can achieve significantly improved end-to-end performance compared to an SDC system.

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APPENDIX I

RAW PACKET-LOSS RATE AND PDF OF PACKET SYSTEM DELAY IN SDC AND DDC SYSTEM

A. Preliminary: $M/G/1/K$ queue

In this subsection we describe some preliminary background for a general $M/G/1/K$ queue, which is required for the next two subsections.

Let $B^*(s)$ denote the Laplace transform of the packet service time distribution and $\hat{B}^*(s)$ denote the Laplace transform of the residual packet service time distribution. The system delay T_n consists of two parts: the waiting time T_w and the service time T_s . If a packet sees i packets in the system when it arrives, the waiting time of this packet is composed of one residual packet service time and $i - 1$ independent packet service times. As a result, the total system delay consists of one residual packet service time and i independent packet service times. Let q_i , $0 \leq i \leq K$ denote the arrival-epoch system-size probability, *i.e.*, the probability that an arriving packet sees i packets in the system. Therefore, the raw packet-loss rate $PL_{raw} = q_K$. Let π_i , $0 \leq i \leq K - 1$, denote the probability that an arriving packet sees i packets in the system given this packet is not lost. Therefore, given the packet is not lost, the Laplace transform of packet total delay distribution $S^*(s)$ is:

$$S^*(s) = \sum_{i=0}^{K-1} S^*(s|i)\pi_i, \quad (14)$$

where

$$S^*(s|i) = \begin{cases} (B^*(s))^i * \hat{B}^*(s) & ; \text{if } i \geq 1, \\ B^*(s) & ; \text{if } i = 0. \end{cases} \quad (15)$$

In [7], [8], an algorithm to compute q_i and π_i is described, which is omitted in this paper because of limited space. Then the pdf of the packet system delay T_n can be determined by computing the inverse Laplace transform of $S^*(s)$.

B. SDC system

In an SDC system, the packet length is exponentially distributed with an average length of R . In this case, each link is modeled as an $M/M/1/K$ queue. For an $M/M/1/K$ queue, there is a closed-form expression for the system-size distribution p_i , $i = 0, 1, \dots, K$, in steady-state [9]:

$$p_i = \frac{\rho^i}{\sum_{i=0}^K \rho^i}, \quad 0 \leq i \leq K, \quad (16)$$

where ρ is the normalized system load with $0 \leq \rho \leq 1$.

According to the PASTA(Poisson arrivals see time averages) Theorem, for an $M/M/1/K$ queue, the arrival-epoch system-size distribution q_i , $0 \leq i \leq K$ is equal to the system-size distribution p_i in steady-state:

$$q_i = p_i = \frac{\rho^i}{\sum_{i=0}^K \rho^i}, 0 \leq i \leq K. \quad (17)$$

Then the arrival-epoch system-size distribution given the arriving packet is not lost, π_i , $0 \leq i \leq K-1$, is given by

$$\pi_i = \frac{q_i}{1 - q_K}. \quad (18)$$

For an $M/M/1/K$ queue, the residual packet service time distribution is the same as the packet service time distribution. Therefore, for an SDC system, in (15), $\hat{B}^*(s)$ is equal to $B^*(s)$. Then (14) and (15) can be simplified. Therefore, from (14), (15) and (18), we can obtain a closed-form expression for the Laplace transform of the pdf of the packet system delay in an SDC system.

C. DDC system

For the DDC system, we assume the average packet length for the packets from encoder 1 is R_1 and the average packet length for the packets from encoder 2 is R_2 , both with exponential distributions. Assume the average service time for packets from encoder l is $\mu_l = C/R_l$, $l = 1, 2$. In this case, the service time for a typical packet has a hyperexponential distribution.

$$f(t) = \frac{1}{2}f_1(t) + \frac{1}{2}f_2(t) = \frac{1}{2}\mu_1 e^{-\mu_1 t} + \frac{1}{2}\mu_2 e^{-\mu_2 t}. \quad (19)$$

Here, by "typical" we mean that the packet may come from either encoder 1 or encoder 2. Then each link is modeled as an $M/G/1/K$ queue, and the arrival-epoch system-size distribution given this arriving packet is not lost, π_i , $0 \leq i \leq K-1$, can be determined similarly as above.

According to (4) and the development in Section IV, for the DDC system, we need to derive the pdf of the system delays seen by either decoder. That is, we need to derive the pdf of the system delays given the packets coming from encoder l , $l = 1, 2$. Again, in this section, we only derive their Laplace transforms. The corresponding pdfs then can be obtained by performing the inverse Laplace transforms.

Let $B^*(s)$ denote the Laplace transform of $f(t)$, the service time distribution for a typical packet, and $\hat{B}^*(s)$ denote the Laplace transform of the residual packet service time distribution for a typical packet. Let $B_l^*(s)$ denote the Laplace transform of $f_l(t) = \mu_l e^{-\mu_l t}$, the service time distribution for packets from encoder l , $l = 1, 2$.

Since the total system delay T_n^l for packets from encoder l consists of the common waiting time T_w and the service time T_s^l for packets from encoder l , the Laplace transforms of their distributions have the following relationship:

$$S_l^*(s) = W^*(s) * B_l^*(s); l = 1, 2, \quad (20)$$

where $S_l^*(s)$ is the Laplace transform of the distribution of the total system delay for packets from encoder l and $W^*(s)$ is

the Laplace transform of the distribution of the waiting time for any packet.

Given a packet sees i packet in the system when it arrives, the waiting time consists of one residual packet service time for a typical packet and $i-1$ independent packet service time for a typical packet. Therefore, we have

$$W^*(s) = \sum_{i=0}^{K-1} W^*(s|i)\pi_i, \quad (21)$$

where

$$W^*(s|i) = \begin{cases} (B^*(s))^{i-1} * \hat{B}^*(s) & ; \text{if } i \geq 1, \\ 0 & ; \text{if } i = 0. \end{cases} \quad (22)$$

From (21) and (20), we can obtain the Laplace transform of the distribution of the total system delay for packets from encoder l . The pdf of the total system delay for packets from encoder l can then be obtained by performing an inverse Laplace transform.