On Cooperative Lattice Coding and Decoding

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***Abstract*—We propose novel lattice coding/decoding schemes for half-duplex outage-limited cooperative channels. These schemes are inspired by the cooperation protocols of Azarian *et al.* and enjoy an excellent performance-complexity tradeoff. More specifically, for the relay channel, we devise a novel variant of the dynamic decode and forward protocol, along with a lattice-coded implementation, which enjoys a near-optimal diversity-multiplexing tradeoff with a low encoding/decoding complexity. On the other hand, for the cooperative multiple-access channel, we present a lattice-coded implementation of the optimal non-orthogonal amplify and forward protocol. Finally, we establish the performance gains of our proposed protocols via a comprehensive simulation study.**

I. Introduction

The design of cooperation protocols for half-duplex outage-limited channels was pioneered by the work of Laneman *et al.* [1]. Inspired by a high signal-to-noise ratio (SNR) analysis, Azarian *et al.* obtained more efficient protocols that achieve a superior diversity-multiplexing tradeoff (DMT) [2]. The practical implementation of Azarian *et al.* cooperation schemes was recently considered by Yang and Belfiore where they presented a low complexity scheme that achieves the optimal DMT of the amplify and forward relay channel. Here, building on our work in [2], [3], we construct low complexity lattice coding schemes which efficiently exploit the available cooperative diversity in the relay and cooperative multiple access (CMA) channels. In particular, our work considers the following distinct scenarios:

1) For the decode and forward (DF) relay channel, we first devise a novel variant of the dynamic decode and forward (DDF) protocol [2] which, through the judicious use of orthogonal space-time constellations, reduces the channel seen by the destination to a single-input single-output (SISO) time-selective channel. This variant achieves the excellent DMT of the DDF protocol, while reducing the decoding complexity at the destination. We further modify this variant by limiting the relay to start transmission only at a finite number of time instants. As argued in the sequel, this modification allows for a significant reduction in complexity while still achieving a near-optimal DMT. We then present a lattice-coded implementation of this variant and evaluate its performance through a comprehensive simulation study.
2) For the CMA channel, we present a lattice-coded implementation of the DMT-optimal nonorthogonal amplify and forward (CMA-NAF) protocol proposed in [2]. This implementation employs the minimum mean square error decision feedback equalizer (MMSE-DFE) Fano decoder of [3], to achieve near maximum-likelihood (ML) performance at a much lower complexity.

II. Notation and System Model

In our work, all channels are assumed to be flat Rayleigh-fading and quasi-static, i.e., the channel gains remain constant during one codeword and change independently from one codeword to the next. Furthermore, the channel gains are mutually independent with unit variance. The additive noises at different nodes are zero-mean, mutually-independent, circularly-symmetric, and white complex-Gaussian. The variances of these noises are proportional to one another such that there are always *fixed* offsets between the different SNRs. In particular, we define the ratio of the destination noise variance to that of the relay (or cooperating node in the CMA case) by c, i.e., $c \triangleq \sigma_v^2/\sigma_w^2$, where σ_v^2 denotes the noise variance at the destination and σ_w^2 the noise variance at the relay (or cooperating node). All nodes have the same power constraint, have a single antenna, and operate synchronously. Only the receiving node of any link knows the channel gain; no feedback to the transmitting node is permitted. All cooperating partners operate in the half-duplex mode, i.e., at any point in time, a node can either transmit or receive, but not both. This constraint is motivated by, e.g., the typically large difference between the incoming and outgoing signal power levels.

Our work relies heavily on the notion of diversity-multiplexing tradeoff (DMT) posed by Zheng and Tse in [4]. In this formulation, we consider a family of codes (one for every SNR ρ), such that the code corresponding to ρ has a rate of $R(\rho)$ bits per channel use (BPCU) and error probability $P_e(\rho)$. For this family, the multiplexing gain r and the diversity gain d are defined as

$$r \triangleq \lim_{\rho\to\infty} \frac{R(\rho)}{\log \rho}, \qquad d \triangleq -\lim_{\rho\to\infty} \frac{\log P_e(\rho)}{\log \rho}. \tag{1}$$

In the sequel, we say that protocol A *uniformly dominates* protocol B if, for any multiplexing gain r, $d_A(r) \geq d_B(r)$. Furthermore, protocol A is said to be *Pareto optimal*, if there is no protocol B dominating protocol A in the Pareto sense. Protocol B is said to dominate protocol A in the Pareto sense if there is some r_0 for which $d_B(r_0) > d_A(r_0)$, but no r such that $d_B(r) < d_A(r)$.

III. Decode and Forward (DF) Cooperation

For exposition purposes, we limit our discussions below to the single relay scenario. In the DDF protocol the source transmits data at a rate of R bits per channel use (BPCU) during every symbol-interval in the codeword. A codeword is defined as M consecutive sub-blocks, where each sub-block is composed of T symbol-intervals. All channel gains are assumed to remain fixed during the length of a codeword. The relay, on the other hand, listens to the source for enough number of sub-blocks such that the mutual information between its received signal and source signal exceeds MTR. It then decodes and re-encodes the message using an independent codebook and transmits the encoded symbols for the rest of the codeword. We denote the signals transmitted by the source and relay by $\{x_k\}_{k=1}^{MT}$ and $\{\tilde{x}_k\}_{k=M'T+1}^{MT}$, respectively, where M' is the number of sub-blocks that the relay waits before starting transmission given by [2].

$$M' = \min\left\{M, \left\lceil \frac{MR}{\log_2(1+|h|^2 c\rho)} \right\rceil\right\}, \tag{2}$$

where h is the source-relay channel gain. In this expression, $c = \sigma_v^2/\sigma_w^2$ denotes the ratio of the destination noise variance, to that of the relay. The following result from [2], describes the DMT achievable by the DDF protocol as $T \to \infty$ and $M \to \infty$.

Theorem 1: ([2]) The DMT achieved by the DDF protocol is given by

$$d(r) = \begin{cases} 2(1-r) & \text{if} \quad \frac{1}{2} \geq r \geq 0 \\ (1-r)/r & \text{if} \quad 1 \geq r \geq \frac{1}{2} \end{cases}. \tag{3}$$

In [2], the achievability result in (3) was established using independent and random codebooks at the source and relay nodes. This approach may not be practically feasible due to the prohibitive decoding complexity required at the destination. Allowing the relay node to start transmission at the beginning of any sub-block (based on the instantaneous value of the source-relay channel gain), is another potential source for complexity. In practice, this requires the source to use a very high-dimensional constellation (with a very low code-rate) to ensure that the information stream is uniquely decodable, even after one sub-block, given that the source-relay channel is good enough. It also impacts the amount of overhead in the relay-destination packet, since the destination needs to be informed of the starting time of the relay. Here, we introduce two modifications of the original DDF protocol that aim to lower the complexities associated with these two aspects.

1) After successfully decoding, the relay can correctly anticipate the future transmissions from the source (i.e., x_k for $MT \geq k \geq M'T+1$) since it knows the source codebook. Based on this knowledge, the relay implements the following scheme, i.e.

$$\tilde{x}_k = \begin{cases} x_{k+1}^* & \text{for} \quad k = M'T+1, M'T+3, \cdots \\ -x_{k-1}^* & \text{for} \quad k = M'T+2, M'T+4, \cdots \end{cases}, \tag{4}$$

which reduces the signal seen by the destination for $MT \geq k \geq M'T+1$ to an Alamouti constellation.

2) We allow the relay to transmit only after the codeword is halfway through, i.e., we replace the rule in (2) with

$$M' = \min\left\{M, \max\left\{\frac{M}{2}, \left\lceil \frac{MR}{\log_2(1+|h|^2 c\rho)} \right\rceil\right\}\right\}, \tag{5}$$

Fortunately, these modifications do not entail any loss in performance (at least from the DMT perspective) as formalized in the following lemma.

Lemma 2: The modified DDF protocol (with modifications given by (4) and (5)), still achieves the DMT in Theorem 1.

It is now evident that the channel seen by the destination in the modified DDF protocol is a time-selective SISO. This greatly reduces the decoding complexity at the destination, as it facilitates leveraging standard SISO decoding architectures (e.g., belief propagation, Viterbi/Fano decoders). In addition, restricting the relay to transmit only after $M' \geq M/2$ implies that the constellation size can be chosen such that the information stream is uniquely decodable only after $M' = M/2$.

The next result investigates the effect of limiting the relay to start transmission only at a finite number of time-instants. These time-instants partition the code word into $N+1$ segments which are not necessarily equal in length. We let the j-th segment ($N+1 \geq j \geq 1$) span sub-blocks $M_{j-1}+1$ through M_j, with $M_0 \triangleq 0$ and $M_{N+1} \triangleq M$. We further define the set of waiting fractions $\{f_j\}_{j=0}^{N+1}$ by $f_j \triangleq \frac{M_j}{M}$. Thus

$$f_0 = 0 < f_1 < \cdots < f_N < f_{N+1} = 1.$$

The question now is how to choose $\{f_j\}_{j=1}^{N}$, for a finite N, such that the protocol achieves the *optimal* DMT. The following lemma shows that this problem does not have a uniformly optimal solution and characterizes a Pareto optimal set of waiting fractions.

Lemma 3: For the DDF protocol with a finite N,

1) there exists no *uniformly* dominant set of fractions $\{f_j^u\}_{j=1}^{N}$.
2) let $f_1^p = \frac{1}{2}$ and

$$f_j^p = \frac{1-f_{j-1}^p}{2-(1+\frac{1}{f_N^p})f_{j-1}^p}, \quad \text{for} \quad N \geq j > 1 \tag{6}$$

then the set of fractions $\{f_j^p\}_{j=1}^{N}$ is Pareto optimal, with

$$d^p(r) = 1 - r + (1 - \frac{r}{f_N^p})^+. \tag{7}$$

In the following simulation study, we use the low complexity variant of the DDF protocol suggested by Lemmas (2) and (3). Furthermore, we consider construction-A lattice codes obtained from systematic convolutional codes (CCs) with generator polynomials over $\mathbb{Z}_Q$. The generator polynomials are of constraint length 4 and chosen at random (the optimization of generator polynomials is beyond the scope of this work). Unless otherwise stated, we choose the SNR level of the source-relay channel to be 3 dB higher than the SNR at the destination. In all scenarios, we use the MMSE-DFE Fano decoder with a bias $b_F = 1.2$ and a step-size $\Delta = 5$ [3]. The frame length for the coded bit-stream is 128. For the range of transmission rates considered in the sequel, it turns out that increasing the number of segments beyond 3 provides negligible increase in performance. Figure 1 shows the outage probability of the low-complexity DDF variant, when the codeword is partitioned into $2, 3$ and 4 segments. As seen from the figure, the gap between the outage curves is negligible. Therefore, we consider only the variant of DDF relay protocol with 3 segments. Moreover, we choose the waiting fractions $\{f_j^p\}_{j=1}^2$ according to Lemma 3, i.e., $\{\frac{1}{2}, \frac{2}{3}\}$. Figure 2 compares the low complexity DDF variant with Yang-Belfiore implementation of the NAF protocol for 2 and 3 BPCU. As seen from this figure, the proposed DDF strategy offers a gain of about 4 dB (and about 6 dB) over the NAF scheme, for 2 (and 3) BPCU.

IV. The Cooperative Multiple Access (CMA) Channel

In the CMA-NAF protocol, each of the two sources transmits once per cooperation-frame, where a cooperation-frame is defined by two consecutive symbol-intervals. Each source, when active, transmits a linear combination of the symbol it intends to send and the (noisy) signal it received from its partner during the last symbol-interval. For source j and cooperation-frame k, we denote the broadcast and repetition gains by a_j and b_j, respectively, the symbol to be sent by $x_{j,k}$, and the transmitted signal by $t_{j,k}$. At startup the transmitted signals will take the form

$$t_{1,1} = a_1 x_{1,1} \tag{8}$$

$$t_{2,1} = a_2 x_{2,1} + b_2(h t_{1,1} + w_{2,1}) \tag{9}$$

$$t_{1,2} = a_1 x_{1,2} + b_1(h t_{2,1} + w_{1,1}) \tag{10}$$

$$t_{2,2} = a_2 x_{2,2} + b_2(h t_{1,2} + w_{2,2}) \tag{11}$$

where h denotes the inter-source channel gain and $w_{j,k}$ the noise observed by source j during the cooperation-frame k. (We assume that $w_{j,k}$ has variance σ_w^2.) The corresponding signals received by the destination are

$$y_{1,1} = g_1 t_{1,1} + v_{1,1} \tag{12}$$

$$y_{2,1} = g_2 t_{2,1} + v_{2,1} \tag{13}$$

$$y_{1,2} = g_1 t_{1,2} + v_{1,2} \tag{14}$$

$$y_{2,2} = g_2 t_{2,2} + v_{2,2} \tag{15}$$

where g_j is the gain of the channel connecting source j to the destination and $v_{j,k}$ the destination noise of variance σ_v^2. The broadcast and repetition gains $\{a_j, b_j\}$ are (experimentally) chosen to minimize outage probability at the destination. As a consequence of symmetry, a_1 and a_2, as well as b_1 and b_2, will have the same optimal value. Thus, we assume that broadcast and repetition gains are the same at each source and omit the subscripts, yielding $\{a, b\}$. We assume the codewords of each source to be of length N. Notice that it takes $2N$ symbol-intervals, or equivalently N cooperation frames for the two sources to transmit their codewords. In [2], it was shown that the CMA-NAF protocol achieves the optimal DMT of the cooperative multiple access channel.

In order to apply our lattice decoding framework to the CMA-NAF protocol, we exploit the linearity of the CMA-NAF protocol over the field of complex numbers, to describe the joint effect of lattice coding at the sources and cooperation among them, by one extended generator matrix. This results in a typical setting in which the MMSE-DFE Fano decoder is expected to be efficient in recovering the two information streams jointly at the destination. In particular, by examining (8) through (15), we realize that the received signal at the destination can be written as

$$\tilde{\mathbf{y}}^c = \tilde{\mathbf{H}}^c \mathbf{x}^c + \mathbf{B}^c \mathbf{w}^c + \mathbf{v}^c, \tag{16}$$

where $\tilde{\mathbf{y}}^c \triangleq [y_{1,1}, y_{2,1}, \cdots, y_{1,N}, y_{2,N}]^T$ denotes the vector of received signals at the destination, and $\mathbf{x}^c \triangleq [x_{1,1}, x_{2,1}, \cdots, x_{1,N}, x_{2,N}]^T$ denotes the vector formed by multiplexing the two sources' codewords (i.e., $\mathbf{x}_j^c \triangleq [x_{j,1}, \cdots, x_{j,N}]^T, j \in \{1, 2\}$) in an alternate fashion. $\mathbf{w}^c \sim \mathcal{N}_C(\mathbf{0}, \sigma_w^2 \mathbf{I}_{2N-1})$ and $\mathbf{v}^c \sim \mathcal{N}_C(\mathbf{0}, \sigma_v^2 \mathbf{I}_{2N})$ denote noise vectors observed by the two sources and the destination, respectively. Finally, matrices $\tilde{\mathbf{H}}^c \in \mathbb{C}^{2N \times 2N}$ and $\mathbf{B}^c \in \mathbb{C}^{2N \times (2N-1)}$ are given by

$$\tilde{\mathbf{H}}^c = a\mathbf{G}^c \begin{bmatrix} 1 & & & \\ bh & 1 & & \\ \vdots & \vdots & \ddots & \\ (bh)^{2N-1} & (bh)^{2N-2} & \cdots & 1 \end{bmatrix}, \tag{17}$$

and

$$\mathbf{B}^c = b\mathbf{G}^c \begin{bmatrix} 0 & & & \\ 1 & 0 & & \\ \vdots & \vdots & \ddots & \\ (bh)^{2N-3} & (bh)^{2N-4} & \cdots & 0 \\ (bh)^{2N-2} & (bh)^{2N-3} & \cdots & 1 \end{bmatrix}, \tag{18}$$

with $\mathbf{G}^c \triangleq \mathbf{I}_N \otimes \operatorname{diag}(g_1, g_2)$. The noise component in (16), i.e. $\tilde{\mathbf{z}}^c \triangleq \mathbf{B}^c \mathbf{w}^c + \mathbf{v}^c$, is colored and is whitened in the front-end of the receiver (before our lattice decoding framework can be applied). The details of the proposed MMSE-DFE Fano decoder is reported in [5].

In the following simulation study, we demonstrate the excellent performance of the lattice-coded CMA-NAF protocol, through comparing it against other schemes. As before, we take construction-A lattice codes as our coding scheme. The frame length is set to $N = 128$. Figures 3 and 4 compare the FER performance of the lattice coded CMA-NAF protocol and Yang-Belfiore implementation of the NAF relay protocol, for 2 BPCU and 4 BPCU, respectively. The figures also show the performance when the CMA-NAF protocol is used with uncoded QAM transmission. Both coded and uncoded transmission with the CMA-NAF protocol perform significantly better

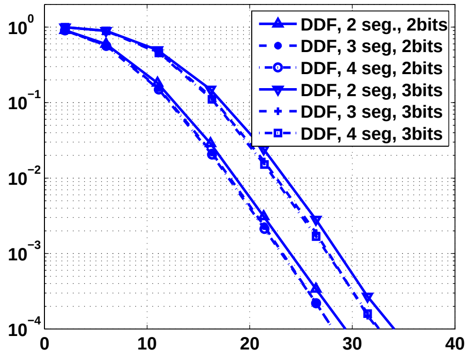

Fig. 1. Outage probability of Pareto optimal DDF protocol with $2, 3$ and 4 segments.

than the NAF-relay protocol. The performance gap between the two schemes widens as the transmission rate increases. This can be explained by the superior DMT of the CMA-NAF protocol, compared to the NAF-relay protocol. For comparison purposes, Figures 3 and 4 also give the performance curves when the two sources do not cooperate, i.e., each source transmits independently. Here too, we consider both coded and uncoded transmission.

V. Conclusions

We presented novel lattice-coded protocols for the half-duplex outage-limited cooperative channels. The proposed protocols exhibit attractive performance-complexity tradeoffs. For the DF relay channel, we first devised a novel variant of the DDF protocol which enjoys a comparable DMT to the original DDF with a much lower complexity. We then presented a lattice-coded implementation of this variant and evaluated its performance through simulation. For the CMA channel, we presented a low complexity lattice-coded implementation of the CMA-NAF protocol. Our results establish the natural matching between the optimal CMA-NAF protocol and the MMSE-DFE Fano decoder.

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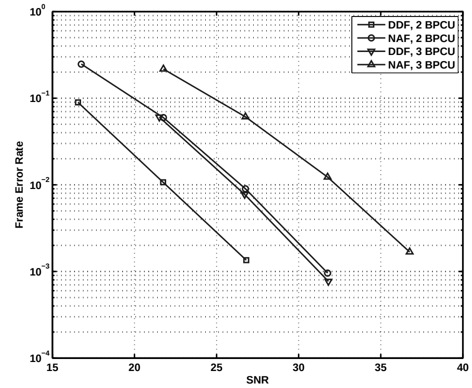

Fig. 2. FER of DDF relay protocol (with 3 segments) versus NAF relay protocol.

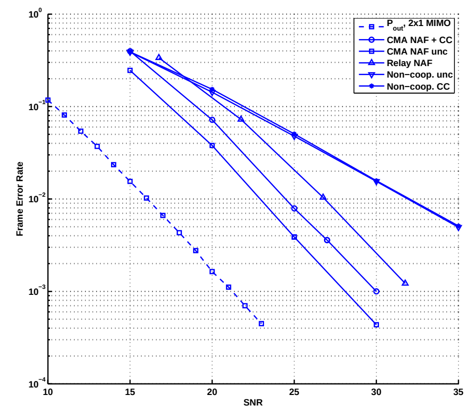

Fig. 3. FER of CMA-NAF, Relay NAF and non-cooperative protocols (2 BPCU).

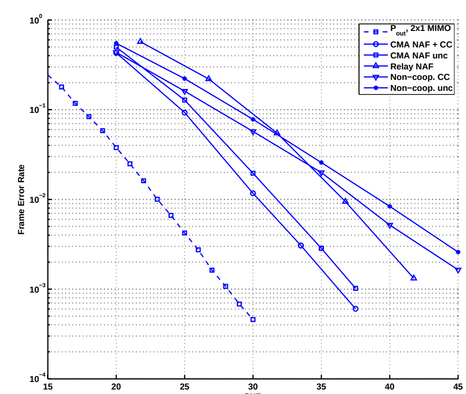

Fig. 4. FER of CMA-NAF, Relay NAF and non-cooperative protocols (4 BPCU).