Space-Time Mesh Codes for the Multiple-Access Relay Network: Space vs. Time Diversity Benefits

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***Abstract*—We consider a wireless multiple access network where the sender nodes are aided by a number of relay nodes. A transmission of bit-messages is completed in two phases: in the first phase each sender node originates its message which is overheard at the relay node, and in the second phase each relay node transmits the parity bit calculated from the overheard bit-messages. The low-density parity-check codes are used at the sender nodes in the time-domain, and the low-density generator-matrix code is formed across the spatial domain. At the access node, the received bits from multiple sender nodes and relay nodes are thus encoded in both the time and the spatial domain. We call this combination a space-time *mesh* code here. In this paper, an iterative decoding scheme is designed for the mesh code and its BER performance in AWGN, fast Rayleigh fading, and quasi-static Rayleigh fading channels are investigated. We note that there is an important trade-off relation between the time-domain and the spatial domain coding. Namely, the time-domain coding is desired when the channel exhibits fast fading; while the spatial domain coding is preferred when the channel is in quasi-static fading state.**

***Index Terms*— Iterative decoding, LDPC codes, LDGM codes, Network coding,**

I. Introduction

Since the seminal paper by Ahlswede, Cai, Li, and Yeung [1], the idea of *network coding* has drawn a lot of interests from the research community. In the *network coding* framework, an intermediate relay node can be configured to transmit the result of linear combination of its incoming messages over a finite field. It has been shown that the use of this network coding can increase the traffic carrying capacity of certain wired networks in the multicast application [1][9]. Specifically, if senders and relays are only allowed to cope with binary messages, the linear combination operation is reduced to a simple modulo-2 addition. That is, a relay transmits the result of the binary parity-check operation of its incoming bit-messages. We call this the *parity-checking* network coding in this paper. The parity-checking network coding has a further application in the wireless multiple-access relay network. Since all bits from the senders, information bits, and relays, the parity-check bits, are collected at the access node, the access node can virtually treat all the received bits as a codeword of a linear block code. Thus, the access node can utilize the *built-in* spatial-domain coding offered by the multiple-access relay network to improve the reception.

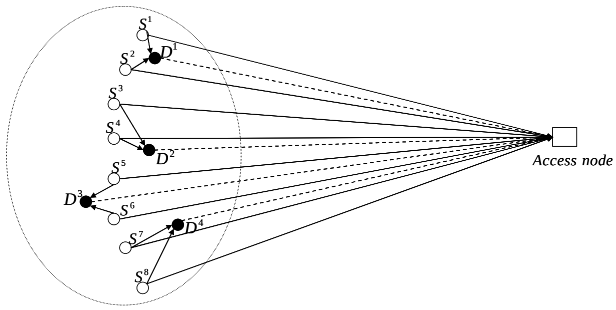

Figure 1: Example of a multiple-access relay network, where $S^1 \cdots S^8$ are sender nodes and $D^1 \cdots D^4$ are relay nodes.

Bao and Li proposed a two-phase-transmission scenario for the multiple-access network similar to ours (refer to section II for a detailed description), and showed that the parity-checking network coding is better than simple routing in simulations [4]. In [6], the authors further investigated the spatial-domain diversity offered by implementing the parity-checking network coding. However, they assume that each sender processes only a single information bit to be transmitted at a time, rather than a coded bit stream. Thus, the time-domain diversity is not utilized in this scenario.

On the other hand, Hausl *et. al*. in [3] considered the time-domain coding, rather than a single-bit transmission, in a similar multiple access relay network. But they considered a rather limited cooperation scheme in which there are only two senders each of which employs low-density parity-check (LDPC) code and investigated the performance of the iterative receiver at the access node in simulation. In this scenario, the time-domain diversity is utilized, but the spatial-domain diversity is not fully explored, due to lack of senders.

In this paper, we propose the idea of *space-time mesh code.* The space-time mesh code can utilize both the spatial and time diversity which might available in the channel. The other effect is that the block length of the code can be increased by combining signaling over the both dimensions. We provide an iterative decoder for this code, and present its bit error rate (BER) simulation results. We show that with the proposed coding framework we can investigate the trade-off relationship between the spatial and the time domain coding. Utilizing this tradeoff relation the network code can adapt to different channel condition in an optimal manner.

The rest of this paper is organized as followings. Section II provides the model for the wireless multiple-access relay network. Section III gives the space-time mesh code and its iterative decoder. In section IV, simulation results and detailed discussions are presented. Finally, we make a conclusion and discuss future works in section V.

II. Wireless Multiple-access Relay Network Model

A. System of interest

The wireless multiple-access relay network is depicted in Figure 1. There is a single *access node* depicted as the square box. The white nodes are the *traffic-originating* sender nodes and the black nodes are the *relay* nodes. Each sender node transmits *k* information bits independently generated from other sender nodes. This information bits are individually encoded with a low-density parity-check (LDPC) code. Here, we adopt the LDPC code due to its ability not only to achieve channel capacity [2], but also to cooperate with the spatial systematic low-density generator matrix (LDGM) code (details in section III). An LDPC codeword of length *n* is to be transmitted to the access node through each sender's dedicated wireless channel. The dedicated channels mean that each sender has its own transmission channel multiplexed into either a different time, a different frequency or a different spreading code, so that no inter-user signal interference occurs at the access node. To better define the system, we assume

1. The sender and the relay nodes form a cluster such that these nodes within the cluster are randomly but closely located with each other while the access node is located far from the cluster. Hence the channel conditions, such as signal-to-noise ratios and fading rates, from each node within the cluster to the access node are regarded all the same.
2. Each relay node has the capability to listen to signals from any sender nodes within the cluster. A particular relay is able to pick up on a number of channels on which the reception quality is good. Hence, error-free wireless links from the picked sender nodes to the relay node are assumed.

The sender nodes and the relay nodes cooperate in the following *two phase transmission scheme* which we adopted from [4][5]: In the first phase, each sender node transmits a single coded bit out of its coded bit-stream to the access node. Meanwhile, a relay node gets a number of "error-free" 1-bit messages from the sender nodes which provide good receptions. In the second phase, each relay node transmits the calculated single parity check bit, by summation on its incoming bit-messages under mod-2 operation, to the access node through its dedicated channel (and hence no interference incurred). We assume that the single common access node can provide the necessary synchronization and channel assignment among sender and relay nodes for this two phase transmission.

B. Network channel model

Each sender's signal sent in different channels can be collected at the access node. The received signal $y_{s,t}$ at the access node is written as

$$y_{s,t} = \sqrt{E_S}\,\alpha_{s,t} x_{s,t} + w_{s,t}, \tag{1}$$

for $s = 1, 2, \cdots, N_S, \cdots, (N_S + N_D)$, and $t = 1, 2, \cdots, n$. The index *s* is for the spatial-channels, and *t* for the time index. E_S is the transmitted symbol energy at each sender or relay; $x_{s,t}$ is the binary phase shift keying symbol for the t^{th} time-epoch of the s^{th} transmitter. It either refers to the signal sent by the senders if the spatial index *s* is less than the number of senders N_s, i.e. for $s = 1, 2, \cdots, N_S$, or the signal sent by the relays if $s = N_S + 1, \cdots, (N_S + N_D)$. We assume perfect phase de-rotation. The fading gain is denoted as $\alpha_{s,t}$, samples of $\alpha_{s,t}$ are drawn from the Rayleigh distribution. For any fixed spatial-index *s*, the channel is called *quasi-static fading* when $\alpha_{s,t}$ is held as a constant during the whole codeword length (i.e., $\alpha_{s,t}$ is fixed once chosen for the duration of whole transmission period, $t = 1, 2, \cdots, n$). It independently varies from one period of the codeword to the other. On the other hand, the *fast fading* channel is implemented by having $\alpha_{s,t}$ independently varied at every time index t . We assume all the spatial channels are independent and undergo the same type of fading, i.e., all undergo either the quasi-static or the fast fading channel condition. It shall be noted that we can let $\alpha_{s,t} = 1$ for the AWGN channel.

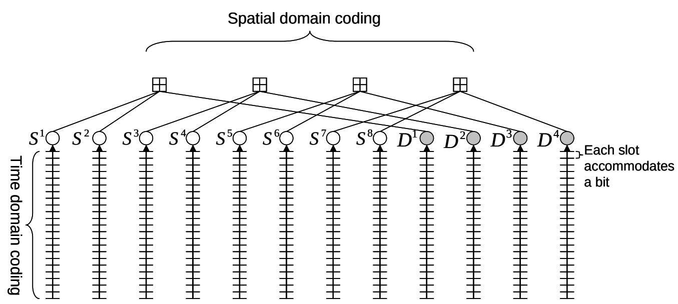

Figure 2. Space-time mesh code seen at the access node in the multiple-access relay network

III. Space-time Mesh code and Iterative Decoders at Access node

To visualize the relationship among transmitted message bits and parity bits received at the access station, we use Figure 2 which depicts an example of the *Tanner graph* for the space-time mesh code. The Tanner graph shown in the horizontal axis is for the coding done across the spatial domain. It represents the parity-check equations, generated through the two phase transmission scheme. The bits related by a parity-check equation sum up to zero under the mod-2 operation.

$$
\mathbf{H} = \begin{bmatrix}
\left[\mathbf{h}_{sp,1}\ \mathbf{h}_{sp,2} \cdots \mathbf{h}_{sp,N_S} ; \mathbf{I}_{N_D \times N_D}\right]_1 & \mathbf{0}_{N_D \times (N_S+N_D)} & \cdots & \mathbf{0}_{N_D \times (N_S+N_D)} \\
\mathbf{0}_{N_D \times (N_S+N_D)} & \left[\mathbf{h}_{s,1}\ \mathbf{h}_{s,2} \cdots \mathbf{h}_{s,N_S} ; \mathbf{I}_{N_D \times N_D}\right]_2 & \cdots & \mathbf{0}_{N_D \times (N_S+N_D)} \\
\vdots & \vdots & \ddots & \vdots \\
\mathbf{0}_{N_D \times (N_S+N_D)} & \mathbf{0}_{N_D \times (N_S+N_D)} & \cdots & \left[\mathbf{h}_{s,1}\ \mathbf{h}_{s,2} \cdots \mathbf{h}_{s,N_S} ; \mathbf{I}_{N_D \times N_D}\right]_n \\
\left[\mathbf{h}^1_{t,1}\ \mathbf{0}_{v,2} \cdots \mathbf{0}_{v,N_S} ; \mathbf{0}_{(n-k)\times N_D}\right]_1 & \left[\mathbf{h}^1_{t,2}\ \mathbf{0}_{v,2} \cdots \mathbf{0}_{v,N_S} ; \mathbf{0}_{(n-k)\times N_D}\right]_2 & \cdots & \left[\mathbf{h}^1_{t,n}\ \mathbf{0}_{v,2} \cdots \mathbf{0}_{v,N_S} ; \mathbf{0}_{(n-k)\times N_D}\right]_n \\
\left[\mathbf{0}_{v,2}\ \mathbf{h}^2_{t,1} \cdots \mathbf{0}_{v,N_S} ; \mathbf{0}_{(n-k)\times N_D}\right]_1 & \left[\mathbf{0}_{v,2}\ \mathbf{h}^2_{t,2} \cdots \mathbf{0}_{v,N_S} ; \mathbf{0}_{(n-k)\times N_D}\right]_1 & \cdots & \left[\mathbf{0}_{v,2}\ \mathbf{h}^2_{t,n} \cdots \mathbf{0}_{v,N_S} ; \mathbf{0}_{(n-k)\times N_D}\right]_1 \\
\vdots & \vdots & \cdots & \vdots \\
\left[\mathbf{0}_{v,2}\ \mathbf{0}_{v,3} \cdots \mathbf{h}^{N_S}_{t,1} ; \mathbf{0}_{(n-k)\times N_D}\right]_1 & \left[\mathbf{0}_{v,2}\ \mathbf{0}_{v,3} \cdots \mathbf{h}^{N_S}_{t,2} ; \mathbf{0}_{(n-k)\times N_D}\right]_1 & \cdots & \left[\mathbf{0}_{v,2}\ \mathbf{0}_{v,3} \cdots \mathbf{h}^{N_S}_{t,n} ; \mathbf{0}_{(n-k)\times N_D}\right]_1
\end{bmatrix}_{n(N_D+N_S)-kN_S \times n(N_D+N_S)} \quad (4)
$$

The graph is formed across the transmitting and relay nodes for a single time-epoch. For each of the senders $S^1 \cdots S^8$, there is a vertical arrow which represents the time domain LDPC code. It has its own corresponding *Tanner graph*, although it is not shown there for simplicity. Thus, each bit transmitted either in time or space is related with some others. Due to this entanglement across time and space, we call this space-time *mesh* code in this paper.

It shall be noted that the arrows for delays $D^1 \cdots D^4$ are the derivatives for the coded bit-streams of $S^1 \cdots S^8$, and are the consequences of the two phase transmission operation for a single epoch *t*. Moreover, we notice that the code in the spatial domain is in the form of Low-density Generator Matrix (LDGM) codes.

In general, the parity-check matrix $\mathbf{H}_{sp}$ of the spatial LDGM code for total N_S senders and N_D relays can be described by

$$
\mathbf{H}_{sp} = \begin{bmatrix} | & | & & | & \\ \mathbf{h}_{sp,1} & \mathbf{h}_{sp,2} & \cdots & \mathbf{h}_{sp,N_S} & ; I_{N_D \times N_D} \\ | & | & & | & \end{bmatrix}_{N_D \times (N_S+N_D)}, \quad (1)
$$

where the *j*-th position of 1's in the N_D by 1 vector $\mathbf{h}_{sp,k}$, $k = 1, \cdots, N_S$, represents there exists a error-free data link from the *k*-th sender to *j*-th relay. For each sender, $s = 1, 2, \ldots, N_S$, the parity-check matrices $\mathbf{H}^s_t$ of the time domain LDPC codes with code length *n* and code rate *k*/*n* is given by

$$
\mathbf{H}^s_t = \begin{bmatrix} | & | & & | & & | \\ \mathbf{h}^s_{t,1} & \mathbf{h}^s_{t,2} & \cdots & \mathbf{h}^s_{t,k} & \ldots & \mathbf{h}^s_{t,n} \\ | & | & & | & & | \end{bmatrix}_{(n-k)\times n} \quad (2)
$$

for $s = 1, 2, \ldots, N_S$. Given the parity-check matrices $\mathbf{H}_{sp}$ and $\mathbf{H}^s_t, s = 1, 2, \ldots, N_S$, and consider a codeword of the space-time code by concatenating bits in Figure 2 row by row, it can be shown that the parity-check matrix $\mathbf{H}$ for the space-time mesh code is given by the above matrix (4). We denote that $\mathbf{0}_{v,k}$, $k = 2, \cdots, N_S$, is an *n* by 1 all zero vector.

The code rate of the mesh code is defined as the ratio of the total number of information bits to the total transmitted coded bits such that

$$
R_{Mesh} = \frac{kN_S}{n(N_S + N_D)}. \quad (5)
$$

One thing that can be noted here is that the length of the mesh code can become very large, proportionately increasing with the number of the cooperating sender nodes, and that of the relay nodes. Thus, the space-time mesh code can be used to subsume both the traditional time-domain coding and the emerging spatial network coding.

After having the parity-check matrix of a typical space-time mesh code (4), the access node can apply the standard message-passing iterative decoding algorithm to decode the received message from all senders. In this paper, we adopt the Gallager's sum-product algorithm [1][7] for decoding of the mesh code, and refer to it as the *mesh decoder*. Based on the check equations embedded in (4), the mesh decoder updates the extrinsic log-likelihood ratios (LLRs) between the check nodes and the bit nodes.

One alternative decoding strategy is to divide and conquer. From Figure 2, we know that the access node will get each row of the mesh code one at a time. The access node can choose to decode the LDGM-code coded bits row by row upon it receives each of them. In fact, without making hard decisions on the log-likelihood ratios (LLRs), they can be forwarded to the next step. After all rows have been processed, decoding over the time domain coding can be initiated at each column. This phase of decoding is based on the code graph of the time-domain LDPC code. The extrinsic LLRs from the first step can be used to initiate this decoding process.

This option can shorten the decoding latency and may lead to a reduced complexity implementation solution thanks to a shortened decoding block at each decoder. However, if we only allow a one way "message flow," i.e., only a single flow from the spatial-domain to the time-domain, the method will suffer from performance loss. In fact, our simulation results confirm that the performance of this method can be as much as about 3dB worse than that of the mesh code decoder. The performance of the second method would probably be improved by further iterations between the spatial and the temporal decoders. However, we note that this iterative decoder would still be inferior to the mesh decoder in terms of the performance.

In the sequel, therefore, we assume the use of the mesh decoder, and focus on the determination of the performance of the mesh decoder under different channel settings.

IV. Experimental Performance Analysis

In this section, we first compare the performance of the *mesh* code in various temporal and spatial domain settings. One benefit of this study is that we will be able to adjust the parameters of the mesh code according to the variation of the channel's fading state. In this paper, we show our results on the following four settings:

1. N_S=20, N_D=10, *ICR*=8, *n*=200 LDPC codes
2. *N*s=200, N_D=100, *ICR*=8, *n*=20 LDPC codes
3. N_S=20, N_D=0, *ICR*=0, *n*=200 LDPC codes
4. N_S=200, N_D=0, *ICR*=0, *n*=20 LDPC codes

where N_S is the number of senders, N_D is the number of relays, *ICR* is the number of Incoming Connections per Relay. For example, the *ICR* in Figure 1 is two. We use 5 iterations for the mesh code decoder. Figure 3 shows the extensive computer simulation results of these settings. Here we let each and every relay/sender has the same incoming/outgoing connections. The parameter *n* is the codeword length of the time-domain LDPC codes. We use the ensemble of the Gallager's (3, 6) LDPC codes, i.e., three 1's in each column and six 1's in each row of the parity-check matrix. In addition, each sender has its own parity-check matrix

Based on the code rate defined in (5), the BER curves are calibrated with respect to E_b / N_0, the ratio of the information bit energy to the power-spectral density, for fair comparison. Settings 1 and 2 represent the performance of the space-time mesh codes, whereas settings 3 and 4 represent the performance, averaged over all senders, of a single LDPC decoder (N_D = 0). As expected, the space-time entangled mesh code easily out-performs the single LDPC code. It should be noted that the both entangled mesh codes in settings 1 and 2 are of code length 6000 which is much longer than 200 and 20 of the settings 3 and 4 respectively.

Let us consider the settings 1 and 2 more closely. They both have the same size parity-check matrix of the form given in (4), and thus have the same code length and the same code rate. An interesting observation is that the performance of the two mesh codes is however very different. Setting 2 tends to produce an error floor in BER curves while Setting 1 does not. This phenomenon is in fact somewhat expected and can be explained from the parity-check matrix of the mesh codes.

Consider the extreme case for n = 1 and only 1-bit information is to be transmitted at each sender such that there is no time-domain LDPC codes applied. Then, the dimension of the parity-check matrix given in (4) becomes N_D by $N_D + N_S$. This matrix is exactly in the form of LDGM code. That is, the parity-check matrix is in the systematic form, $\mathrm{H}_{LDGM} = [\mathbf{P}; \mathbf{I}]$, where **P** is a sparse matrix and **I** is the identity matrix. It is pointed out in [8] that the minimal distance of regular LDGM codes are equal to the *number of 1's in a column of the P matrix* plus *one*, i.e., *degree*+1, and this small minimal distance causes a significant error floor. For a fixed code length, as we observe from (4), the higher the ratio N_D / n, the more the code looks and behaves like an LDGM code (e.g. Setting 2). On the other hand, the lower the ratio N_D / n, the more the code looks and behaves like an LDPC code, i.e., the proportion of the identity matrix in (4) becomes smaller (e.g. Setting 1). It shall be noticed that the minimum distance of an LDPC code increases in proportion to the code length [2]. Thus, Setting 1 shows much better BER performance than Setting 2 does.

Now we investigate the first two settings in different channel conditions, such as AWGN, the fast Rayleigh fading, and the quasi-static Raleigh fading channels and see how the BER performance changes in these different channels. The results shown in Figure 4 indicate that in both AWGN and fast Rayleigh fading channels, the LDPC-like mesh code, setting 1, is better than the LDGM-like code, setting 2. On the other hand, for the quasi-static Rayleigh channel the results of the LDGM-like mesh code, setting 2, is better. This phenomenon can be explained in the following way. For the fast fading channel, each and every redundant bit, either in time or spatial domain, suffers an independent fading coefficient drawn from the same Rayleigh distribution. Thus, statistically, there is no difference as to placing the redundant bits either in the time or in the spatial domain. Even though the total number of redundant bits are the same in Setting 1 and Setting 2, placing more redundant bits in the time domain makes the mesh code shaped more like an LDPC code, which out-performs the LDGM-like mesh code formed by setting 2. For the quasi-static fading channel, however, the situation takes a different form. Under quasi-static fading, if a sender suffers a deep fade, then all of its bit-messages are likely to be lost. There is no time-diversity benefit at all. Thus, it is more beneficial to put coding effort more in the spatial domain.

From the results so far, it is worth to note that the LDGM-like mesh code would be beneficial in certain network situations where some links from senders/relays to the access node are completely blocked (erased) perhaps by their surrounding building, but the other links from the senders/relays to the access node are clean.

In addition, it is worthy to mention that the *ICR* determines, and is proportional to, the degree of the spatial domain LDGM code. Also, recall that the minimum distance of the LDGM code is proportional to the degree. Thus, for LDGM-like mesh codes such as Setting 2, choosing a higher ICR value will have the error floor lowered. Our simulation results show that by increasing *ICR* to 8 from 4 in Setting 2, the error floor is lowered by as much as 1dB at BER 10^{-4}.

Finally, we have considered ways to pick a better mesh code from the ensemble. We first notice that if we let all the sender nodes employ exactly the same parity-check matrix for their LDPC codes, instead of varying them one from another, the parity-check matrix of the mesh code (4) becomes quite regular. This regularity may cause short cycles, which limit the extrinsic information flow in the iterative decoding and hence degrade the BER performance. However, by randomly choosing a single parity-check matrix $\mathbf{H}_t$ and applying it to all sender nodes, our experimental result shows that the BER performance is only 0.5 dB worse at the BER 10^{-4} in the setting *N*s=8, *N*d=4, *ICR*=2, and *n*=30. At the expense of this much performance loss, one possible benefit is the reduction in hardware complexity of the iterative decoder thanks to the regular structure. One interesting question is that if it is possible to find the optimal parity-check matrix such that the degradation is minimized.

V. Conclusion and Future work

We propose the idea of the space-time mesh codes for the multiple-access relay network, and present detailed discussions on the BER performance of the codes, under the sum-product iterative decoding algorithm. Given a fixed length of the mesh code, the parity-check matrix can be varied from a LDGM-like code to a LDPC-like code by choosing an appropriate parametric setting of the mesh code for the multiple-access relay network. The more number of relays and the shorter LDPC code on senders, the more LDGM-like the mesh code becomes. We provided the BER simulation results of the mesh code in different types of channels such as AWGN, fast fading, and quasi-static fading channels. The LDPC-like mesh code out-performs the LDGM-like mesh code in AWGN and fast fading channels, whereas the LDGM-like mesh code out-performs the LDPC-like mesh code in quasi-static fading channels. Namely, we confirm that time-domain coding should be emphasized when the channel exhibits large time-diversity benefit while the spatial-domain coding should be emphasized when the channel exhibits large spatial diversity benefit.

. Our future work focuses on finding the distance spectrum properties of an ensemble of mesh codes and providing performance prediction based on union bound techniques. We envision that this analytic tool can serve as design guidance for finding the best network code for different multiple-access network.

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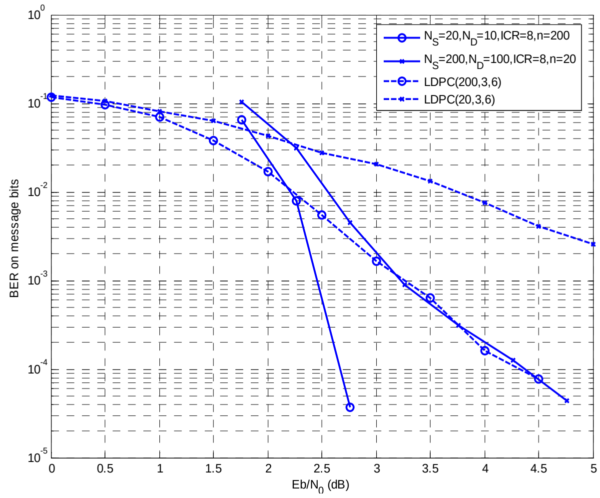

Figure 3: Performance comparisons among the mesh code and the single sender's (3,6) LDPC code.

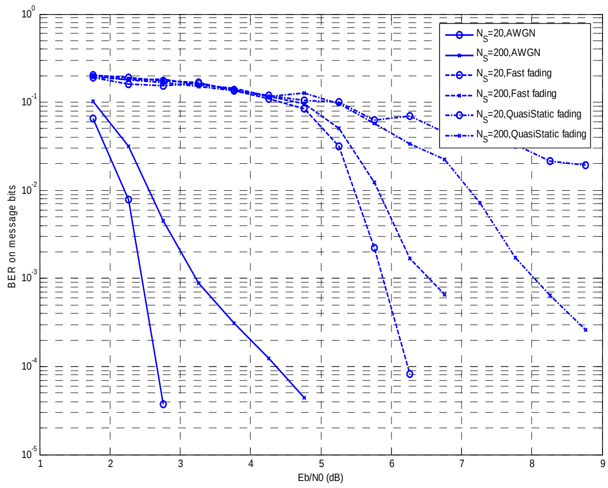

Figure 4. BER performance in AWGN, fast Rayleigh fading, and quasi-static Rayleigh fading.