# Cellular Systems with Multicell Processing and Conferencing Links between Mobile Stations

O. Simeone$^{(1)}$, O. Somekh$^{(2)}$, G. Kramer$^{(3)}$, H. V. Poor$^{(2)}$ and S. Shamai (Shitz)$^{(4)}$
$^{(1)}$ CWCSPR, ECE Dept., NJIT, Newark, NJ 07102, osvaldo.simeone@njit.edu
$^{(2)}$ Dept. of Electrical Engineering, Princeton University, Princeton, NJ 08544, {orens, poor}@princeton.edu
$^{(3)}$ Bell Labs, Alcatel-Lucent, Murray Hill, NJ 07974, gkr@research.bell-labs.com
$^{(4)}$ Department of Electrical Engineering, Technion, Haifa, 32000, Israel, sshlomo@ee.technion.ac.il

***Abstract*— In this paper, multi-cell processing for the uplink of a cellular system is studied in the presence of orthogonal channels of fixed capacity between mobile stations in adjacent cells (conferencing). It is shown that a rate-splitting transmission strategy, where part of the message is exchanged on the conferencing channels and then transmitted cooperatively to the base stations, is capacity-achieving for an asymptotically large conferencing capacity. This strategy can be regarded as able to perform convolutional pre-equalization of the signal encoding the common messages in the spatial domain, where the number of taps of the finite-impulse response equalizer depends on the number of conferencing rounds. Analysis in the low signal-to-noise ratio regime and numerical results validate the advantages of conferencing as a complementary technology to multi-cell processing.**

## I. INTRODUCTION

The solution of choice for providing broadband communications is by now considered to include the support of an infrastructure made of base stations (BSs, or access points) connected by a high-capacity backbone. This class of solutions includes: conventional cellular systems, where BSs are regularly placed in the area of interest; distributed antenna systems, which are characterized by a less regular (e.g., random) deployment; and hybrid networks, where infrastructure nodes coexist with multi-hopping. In all these networks, a solution that promises to greatly improve the overall throughput and that is gaining increasing interest in the community is *multicell processing*. This refers to the class of transmission/ reception technologies that exploit the high-capacity backbone among the BSs to perform joint encoding/ decoding at different cell-sites (see [1] [2] for a list of references).

In addition to the quickly growing body of work on multicell processing for cellular systems [1] [2] [3], there has recently been some activity around the basic idea of complementing and comparing the advantages of cooperation between BSs with some form of collaboration at the mobile stations (MS) level . In references [4] [5] [6], the basic linear Wyner model for cellular networks [3] was extended by including a layer of dedicated relay terminals, one for each cell, that forward traffic from MSs to BSs (uplink). Another related work is [10], where the linear Wyner model with intra-cell TDMA and single-cell processing was modified by assuming that the active MS in a given cell knows (non-causally) the messages to be sent by a number of its neighbors (as might be the case in some implementations of the principle of cognitive radio).

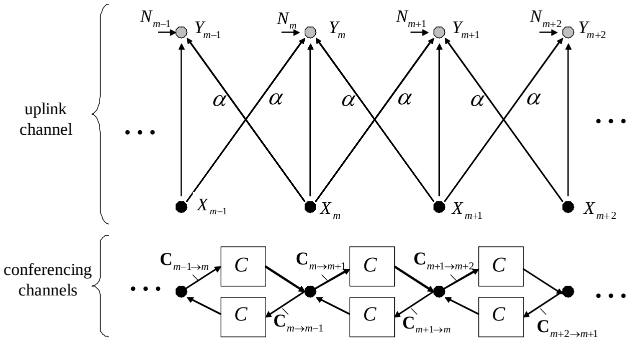

Fig. 1. Linear Wyner model with inter-cell conferencing and intra-cell TDMA.

In this paper, we focus on the uplink of a cellular system, abstracted according to the linear Wyner model [3], and investigate a scenario where cooperation between MSs takes place via additional spectral resources that allow nearby MSs to exchange signals over finite-capacity channels, which are orthogonal to the main uplink channel. These links can be realized when the MSs are equipped with an orthogonal wireless interface (say, Bluetooth or Wi-Fi) that is not available at the BSs. In modelling the interaction among the terminals, we follow the framework of conferencing encoders first studied in [8] in the context of a two-user multiple access channel and then extended in a number of recent works to other scenarios (see, e.g., [9] and references therein). Under such assumptions, we consider the case where only one MS is active in each cell at any given time (*intra-cell Time Division Multiple Access, TDMA*) and conferencing channels exist between MSs belonging to adjacent cells (*inter-cell conferencing*), as it is conceivable in a network with small cells. An achievable rate is presented that is based on rate splitting at the MSs, where part of the message (the "common" message) is exchanged during the conference phase among neighboring MSs and transmitted cooperatively to the BSs. The scheme is proved to be optimal as long as the conferencing capacity is large enough. Finally, an approximate analysis in the low signal-to-noise ratio regime is presented that enables to gain further insight into the advantages of conferencing. Numerical results validate the performance analysis. Proofs of the results herein, and analysis of a scenario where *intra-cell* conferencing is enabled can be found in [11].

## II. SYSTEM MODEL AND DEFINITIONS

We consider the uplink of a cellular system, abstracted according to the linear Wyner model, as sketched in the upper part of Fig. 1. $M$ cells are arranged into a linear array, where each cell contains $J$ MSs ($J = 1$ in the figure). Following [3], the signal transmitted by each MS is received only by the local BS, with unitary gain, and by the two adjacent BSs with inter-cell gain $\alpha$. As mentioned above, we consider the case where only one MS transmits in each cell at any give time in a TDMA fashion (intra-cell TDMA). It should be remarked that this choice does not entail any loss of optimality in a basic Wyner model with no conferencing and Gaussian channels (no fading), as shown in [3]. Overall, defining as $X_m$ the input symbol of the MS active in the $m$th cell, the signal received by the $m$th BS reads ($X_m = 0$ for $m > M$ and $m < 1$)

$$Y_m = X_m + \alpha(X_{m-1} + X_{m+1}) + N_m,\ m = 1, ..., M, \quad (1)$$

with $N_m \sim \mathcal{CN}(0, 1)$ being an i.i.d. sequence of complex Gaussian noise samples. Notice that we assume full (symbol and codeword) synchronization among the cells. We focus on multicell processing, that is we assume that the signals received by the BSs, $\{Y_m\}_{m=1}^{M}$, are jointly processed by a central unit that detects the transmitted signals. Finally, each MS has an average power constraint of $P$ so that the available power per cell is $\tilde{P} = JP$. With intra-cell TDMA, each MS is active for a fraction $1/J$ of the time, wherein it can transmit with power $\tilde{P}$, still satisfying the average power constraint. The power constraint then is given by $E[|X_m|^2] = \tilde{P}$ (over the active period of the $m$th user), which can be interpreted as the signal-to-noise-ratio (SNR) for the system at hand.

We now extend the basic linear Wyner model discussed above to include conferencing among the active MSs in adjacent cells (inter-cell conferencing). A different variation of the Wyner model where intra-cell conferencing is enabled is discussed in [11]. As shown in the lower part of Fig. 1, with inter-cell conferencing, $2M-2$ orthogonal channels with capacity $C$ (bits/symbol) are assumed to exist, each linking the MS currently active in any $m$th cell to the active MS in an adjacent cell (i.e., the $m + 1$ or $m - 1$th cell, unless $m = 1$ or $m = M$). We assume block-transmission. Within any $t$th block and in any $m$th cell, the MS currently active generates a message $W_m(t) \in \mathcal{W} = \{1, 2, ..., 2^{NR/J}\}$ meant to be decoded by the central processor connecting the BSs, where $N$ is the number of channel uses per block and $R$ is the *per-cell rate* (bits/ channel use). According to standard information-theoretic assumption, we will consider large block length $N \to \infty$. Transmission of a given set of messages $\{W_m(t)\}_{m=1}^{M}$ takes place in two successive phases (or slots). In the first phase (*conferencing phase*), during the $t$th block, the MSs exchange information over the conferencing channels during $K$ rounds. This information collected during the conferencing phase by each active MS is then leveraged by the latter to encode the local message $W_m(t)$ for transmission to the BSs in the $(t + 1)$th block (*transmission phase*). Notice that the conferencing phase corresponding to $\{W_m(t)\}_{m=1}^{M}$ can be carried out at the same time as the transmission phase for messages $\{W_m(t-1)\}_{m=1}^{M}$ given the orthogonality between conferencing channels and uplink channel. We use standard definitions for conferencing channels and achievable rates [8] (see [11] for details).

## III. REFERENCE RESULTS

In this section, we discuss lower and upper bounds on the per-cell achievable rate $R$ in the presence of inter-cell conferencing. The first result is due to [3] and does not assume a priori intra-cell TDMA.

*Proposition 1* (lower bound, no conferencing) [3]: *The per-cell capacity (i.e., maximum achievable per-cell rate) in a basic linear Wyner model with no conferencing ($C = 0$) and $M \to \infty$ is achieved with intra-cell TDMA and is given by*

$$R_{lower} = \int_0^1 \log\left(1 + \tilde{P} \cdot H(f)^2\right) df, \quad (2)$$

*with*

$$H(f) = 1 + 2\alpha\cos(2\pi f). \quad (3)$$

It should be noted that the rate (2) can be intuitively explained by regarding the Wyner model of Fig. 1 as an inter-symbol interference (ISI) channel in the spatial domain, characterized by the channel impulse response $h_m = \delta_m + \alpha\delta_{m-1} + \alpha\delta_{m+1}$ ($\delta_m$ denotes the Kronecker delta function) and corresponding transfer function $H(f)$ in (3). Moreover, we emphasize that the rate (2) clearly sets a lower bound on the performance achievable with inter-cell conferencing since it assumes $C = 0$.

The following proposition defines a useful upper bound on the performance attainable with inter-cell conferencing and intra-cell TDMA.

*Proposition 2* (upper bound, perfect conferencing): *An upper bound on the rate achievable with inter-cell conferencing and intra-cell TDMA in the linear Wyner model (with $M \to \infty$) is given by*

$$R_{upper} = \int_0^1 \log\left(1 + \tilde{P} \cdot H(f)^2 S(f)\right) df \quad (4)$$

*with*

$$S(f) = \left(\mu - \frac{1}{\tilde{P}H(f)^2}\right)^+ \quad (5a)$$

$$\text{s.t.} \int_0^1 S(f)df = 1. \quad (5b)$$

This results follows by considering the cut-set bound [16] applied to the cut that divides MSs and BSs, or equivalently by assuming a perfect conferencing phase ($C \to \infty$) where each $m$th active MS is able to exchange the local message $W_m$ with all the other active MSs in other cells. In fact, in such an asymptotic regime, joint encoding of the set of messages $\{W_m\}_{m=1}^{M}$ by all the $M$ MSs is feasible, and, recalling the equivalence of (1) with an ISI channel, we can conclude that the optimal transmission strategy is defined by the waterfilling solution (5) [12]. Notice that the waterfilling solution is obtained for a sum-power constraint over the MSs but, given the symmetry of our setting, it also applies to the considered per-MS power constraint. It should also be remarked that this result shows that, in the limit $C \to \infty$, a stationary input in the spatial domain with power spectral density $S(f)$ is capacity-achieving. Finally, a closed-form solution of (4) is derived in [11] in a certain regime of interest.

## IV. AN ACHIEVABLE RATE

In this section, we derive an achievable rate for the Wyner model with inter-cell conferencing and intra-cell TDMA and discuss some of the implications of this result.

*Proposition 3* (achievable rate): *The following per-cell rate is achievable for the linear Wyner model with inter-cell conferencing and intra-cell TDMA for $M \to \infty$ and any $K \geq 1$:*

$$\begin{aligned} R \;=\; \max_{P_c,\, P_p,\, \mathbf{h}_c} \min \Bigg\{ & \int_0^1 \log(1 + P_p H(f)^2 \\ & + P_c H(f)^2 |H_c(f)|^2) df, \\ & \int_0^1 \log\left(1 + P_p H(f)^2\right) df + \frac{C}{K} \Bigg\}, \end{aligned} \tag{6}$$

*with constraints*

$$P_c + P_p \;=\; \tilde{P} \tag{7a}$$

$$\|\mathbf{h}_c\|_2^2 \;=\; 1, \tag{7b}$$

*definitions:* $\mathbf{h}_c = [h_{c,-K} \cdots h_{c,K}]^T \in \mathbb{C}^{2K+1}$, *and*

$$H_c(f) = \sum_{m=-K}^{K} h_{c,m} \exp(-j2\pi f m). \tag{8}$$

Here we would like to briefly discuss the transmission scheme that attains the rate (6) and to point out some implications of this result, leaving the details of the proof of achievability to [11]. Again, to fix the ideas, consider the set of $M$ active MSs at a given time, one per each cell, which employ a fraction of time $1/J$ of both the uplink and the conferencing channels. The proposed scheme works as follows. In the *conference phase*, each $m$th MS first splits its message $W_m$ into two parts, say *private* ($W_{p,m}$) and *common* ($W_{c,m}$). Then, it shares the common part $W_{c,m}$ with the $2K$ neighboring MSs in cells $m+i$ with $i = -K, -K+1, ..., -1, 1, ..., K$, during $K$ conferencing rounds. More precisely, in the first round, the $m$th MS transmits its local common information $W_{c,m}$ to the two adjacent MSs $m-1$ and $m+1$, which then propagate the information towards the two edges of the network, and so on. Notice that, after the conference phase, each $m$th MS is aware of the $2K+1$ common messages $\{W_{c,m+k}\}_{k=-K}^{K}$. During the *transmission phase*, each common message $W_{c,m}$ can be then transmitted cooperatively by all the $2K+1$ MSs that have acquired the information on $W_{c,m}$ in the conferencing phase. On top of the cooperative signal encoding common information, each MS jointly encodes the private message $W_{p,m}$. Gaussian codebooks are employed and the total power $\tilde{P}$ is divided as (7a) between the common ($P_c$) and private ($P_p$) parts.

As shown by Proposition 3, the impact of inter-cell conferencing, according to the scheme discussed above, is equivalent to that of allowing *precoding* (*pre-equalization*) of the common information by a $2K \times 1$ FIR filter $\mathbf{h}_c$ with frequency response $H_c(f)$ (8). We emphasize that, while the number of taps increases with the number of conference rounds, the overall achievable rate may suffer according to (6). We further explore this trade-off in Sec. VII with a numerical example.

## V. ASYMPTOTIC OPTIMALITY OF THE CONSIDERED SCHEME

From Proposition 3, it is easy to see that the proposed scheme is optimal under a specific asymptotic regime, as stated in the following Proposition.

*Proposition 4* (asymptotic optimality): *The transmission scheme achieving the rate (6) is optimal for $C \to \infty$, $K \to \infty$ and $\frac{C}{K} \geq R_{upper}$.*

*Proof*: It is enough to prove that the rate (6) equals the upper bound (4) under the conditions in the proposition above. This follows easily by setting $P_c = \tilde{P}$ (and $P_p = 0$) and recalling that the optimal power spectral density $S(f)$ (5) can be approximated arbitrarily well by the frequency response $|H_c(f)|^2$ in (8) as the number of taps $2K+1$ increases unboundedly [14] (which corresponds to perfect cooperation among the MSs).

*Remark 1*: The argument in the proof above shows that, under the asymptotic conditions stated in Proposition 4, it is optimal to allocate all the power to the common messages, $P_c = \tilde{P}$ (and $P_p = 0$), and to select the filter $\mathbf{h}_c$ so that $|H_c(f)|^2 = S(f)$.

## VI. DISCUSSION: THE LOW-SNR REGIME

Here and in the next section, we elaborate on the performance of the considered scheme that exploits inter-cell conferencing. Here, this goal is pursued via an (approximate) analytical approach that focuses on the low-SNR regime according to the framework in [13], whereas in the next section we resort to numerical simulations to study the case of arbitrary SNR. The attention to the low-SNR regime is justified by the fact that, as discussed above, the advantages of inter-cell conferencing are (asymptotically) related to the opportunity of performing waterfilling power allocation, which is known to provide relevant gains only for low to moderate SNRs (see, e.g., [15]). In the following, we focus for simplicity on the minimum energy per bit $E_b/N_0|_{\min}$, and use this criterion to compare the performance of inter-cell conferencing with the lower and upper bounds (2) and (4) in the low-SNR regime. Starting with the bounds, the minimum energy per bit is given by :

$$\left.\frac{E_b}{N_0}\right|_{\min,lower} = \frac{\ln 2}{1 + 2\alpha^2} \tag{9}$$

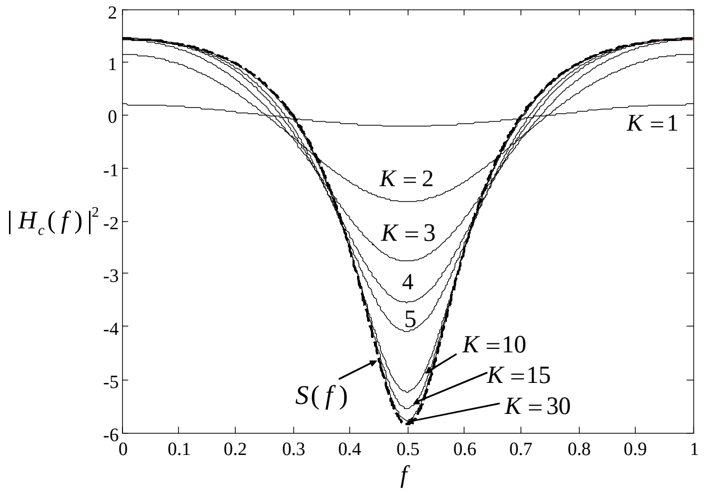

Fig. 2. Optimal waterfilling solution (5) and approximation obtained by the FIR pre-equalizer (8) for $\alpha = 0.2$ and $\tilde{P} = 3dB$.

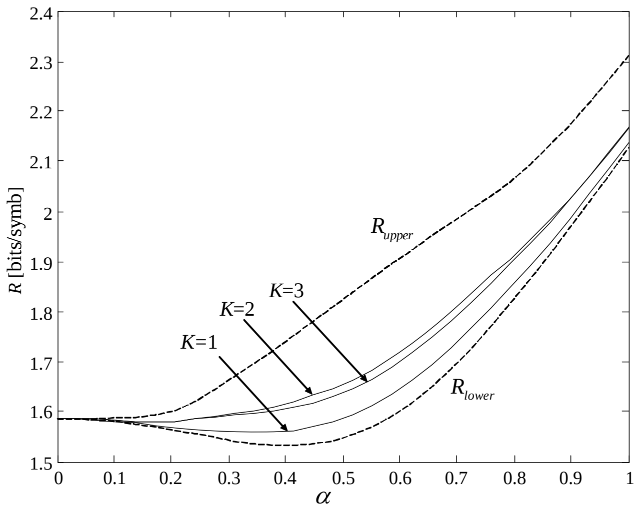

Fig. 3. Achievable rate (6) with inter-cell conferencing and intra-cell TDMA versus the inter-cell gain $\alpha$. The lower bound (2) and upper bound (4) are also shown for reference ($\tilde{P} = 3dB$, $C = 1$, $J = 1$).

for the lower bound (2) (see [2]) and

$$\left.\frac{E_b}{N_0}\right|_{\min,upper} = \frac{\ln 2}{(1+2\alpha^2)^2} \tag{10}$$

for the upper bound (4). The latter can be proved by noticing, similarly to [13], that when the SNR tends to zero ($\tilde{P} \to 0$), it is optimal to allocate all the available power around the maximum value of the channel transfer function, $\max_f H(f)^2 = (1+2\alpha)^2$, which occurs at $f = 0$. In other words, the optimal waterfilling power allocation is $S(f) = \delta(f)$, where $\delta(f)$ is a Dirac delta function. Plugging $S(f) = \delta(f)$ into (4) and using tools from [13], equality (10) is easily shown.

Let us now consider the rate (6) achievable by inter-cell conferencing. It is shown in [11] that, under the low-SNR conditions and assuming large $K$, the rate (6) is approximated by

$$R \simeq 2\int_0^{1/K} \log\left(1 + \frac{1}{2}K\tilde{P}H(f)^2\right) df, \tag{11}$$

so that the minimum energy can be calculated following [13] and after some algebra, as

$$\left.\frac{E_b}{N_0}\right|_{\min} \simeq \frac{\ln 2}{(1+2\alpha)^2\left(1 - \frac{8\alpha\pi^2}{3(1+2\alpha)K^2}\right)}. \tag{12}$$

From the previous equation, it is clear that the minimum energy per bit of inter-cell conferencing (12) is a decreasing function of the number of conferencing rounds $K$ and, as expected from Proposition 4, tends to the optimal performance (10) for $K \to \infty$.

## VII. NUMERICAL RESULTS

In this section, we present some numerical examples in order to assess the performance of the discussed inter-cell conferencing scheme. Since the optimization problem (6) that yields the considered achievable rate $R$ is generally non-convex, here we focus on a simple feasible solution that is asymptotically (in the sense of Proposition 4) optimal and allows to gain interesting insight into the system performance. As discussed in Remark 1, for $C \to \infty$, $K \to \infty$ and $C/K \geq R_{upper}$, the (global) optimal power allocation is $P_c = \tilde{P}$ (and $P_p = 0$) and the optimal frequency response $|H_c(f)|^2$ satisfies $|H_c(f)|^2 = S(f)$. Based on this result, for any choice of the parameters, first the $2K+1$ taps of filter $\mathbf{h}_c$ are generated according to the frequency-sampling method with target amplitude of the frequency response given by the waterfilling solution $\sqrt{S(f)}$ [14] (the filter is scaled to satisfy the constraint (7b)). Then, for fixed filter $\mathbf{h}_c$, the optimization problem (6) is convex in the powers $(P_c, P_p)$, and can be solved efficiently by using standard numerical methods. Illustration of the performance of the frequency-sampling filter design for different values of $K$ is shown in Fig. 2 for $\tilde{P} = 3dB$ and $\alpha = 0.2$. It can be seen that with $K$ large enough the FIR filter $H_c(f)$ in (8) is able to approximate closely the (asymptotically) optimal waterfilling solution $S(f)$.

As discussed above, increasing $K$ is always beneficial to obtain a better approximation of the waterfilling strategy (5). However, due to the finite conferencing capacity $C$, it is not necessarily advantageous in terms of the achievable rate (6). To show this, Figs. 3 and 4 present the achievable rate (6) versus the inter-cell gain $\alpha$ along with the lower bound (2) and upper bound (4) for $J = 1$, and $C = 1$ and $C = 10$, respectively. Fig. 3 shows that, with $C = 1$, while increasing the conferencing rounds from $K = 1$ to 2 increases the achievable rate, further increments of the conferencing capacity $C$ are disadvantageous, according to the trade-off mentioned above. With a larger capacity $C = 10$, Fig. 4 shows that very relevant performance gains can be harnessed by

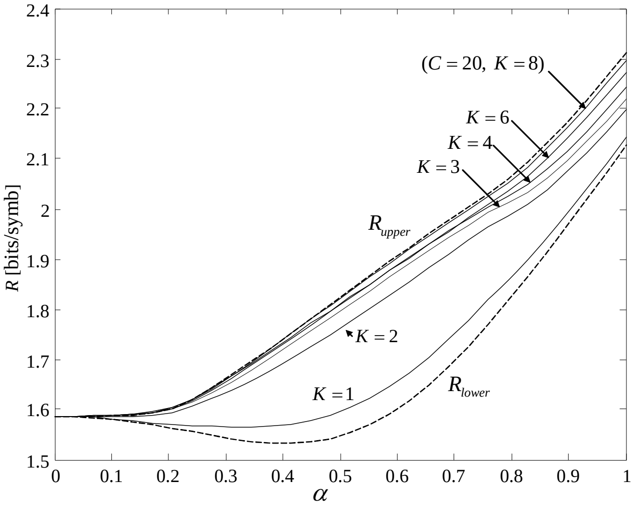

Fig. 4. Achievable rate (6) with inter-cell conferencing and intra-cell TDMA versus the inter-cell gain $\alpha$. The lower bound (2) and upper bound (4) are also shown for reference ($\tilde{P} = 3dB,\ C = 10,\ J = 1$).

increasing the number of conference rounds, especially from $K = 1$ to $K = 2$. Moreover, as expected from Proposition 4, having sufficient large conferencing capacity $C$ and number of conference rounds $K$ (with $C/K \geq R_{upper}$) enables the upper bound (4) to be approached.

## VIII. CONCLUDING REMARKS

An interesting issue left open by this work is the establishment of capacity-achieving schemes for any value of the conferencing capacity and finite number of cell-sites. The main challenge in this regard appears to be the extension of the converse result in [8] to the scenario at hand. In particular, it remains to be determined whether, unlike the simpler model in [8], interactive communications among the MSs during the conferencing phase is necessary to achieve capacity. The results of this paper have shown that this is not the case in the regime of high conferencing capacity.

## IX. ACKNOWLEDGMENT

The research was supported in part by a Marie Curie Outgoing International Fellowship and the NEWCOM++ network of excellence both within the 6th European Community Framework Programme, by the REMON Consortium and by the U.S. National Science Foundation under Grants CNS-06-25637, CNS-06-26611, and ANI-03-38807.

## REFERENCES

[1] S. Shamai (Shitz), O. Somekh, and B. M. Zaidel, "Multi-cell communications: An information theoretic perspective," in *Proc. Joint Workshop on Communications and Coding (JWCC 2004)*, Donnini, Italy, Oct. 14-17 2004.

[2] O. Somekh, O. Simeone, Y. Bar-Ness, A. Haimovich, U. Spagnolini, and S. Shamai (Shitz), “An information theoretic view of distributed antenna processing in cellular systems,” in *Distributed Antenna Systems: Open Architecture for Future Wireless Communications*, Auerbach Publications, CRC Press, New York, NY, 2007.

[3] A. D. Wyner, "Shannon-theoretic approach to a Gaussian cellular multiple-access channel," *IEEE Trans. Inform. Theory*, vol. 40, pp. 1713-1727, Nov. 1994.

[4] O. Simeone, O. Somekh, Y. Bar-Ness, and U. Spagnolini, “Uplink throughput of TDMA cellular systems with multicell processing and Amplify-and-Forward cooperation between mobiles,” *IEEE Trans. Wireless Commun*, vol. 6, no. 8, pp. 2942-2951, Aug. 2007.

[5] O. Simeone, O. Somekh, Y. Bar-Ness, and U. Spagnolini, “Low-SNR analysis of cellular systems with cooperative base stations and mobiles,” in *Proc. Asilomar Conference on Signals, Systems and Computers*, Pacific Grove, CA, 2006.

[6] O. Somekh, O. Simeone, H. V. Poor, and S. Shamai, “Cellular systems with full-duplex Amplify-and-Forward relaying and cooperative base stations,” in *Proc. International Symposium on Inform. Theory (ISIT 2007)*, Nice, France, June 24-29, 2007.

[7] A. Sanderovich, O. Somekh, and S. Shamai (Shitz), "Uplink macro-diversity with limited backhaul capacity," in *Proc. International Symposium on Inform. Theory (ISIT 2007)*, Nice, France, June 24-29, 2007.

[8] F. M. J. Willems, *Informationtheoretical Results for the Discrete Memoryless Multiple Access Channel*, Ph.D. thesis, Katholieke Universiteit Leuven, 1982.

[9] C. T. K. Ng, I. Maric, A. J. Goldsmith, S. Shamai, and R. D. Yates, “Iterative and one-shot conferencing in relay channels,” in *Proc. IEEE Information Theory Workshop (ITW 2006)*, Punta del Este, Uruguay, Mar. 13-17, 2006.

[10] A. Lapidoth, S. Shamai (Shitz), and M. Wigger, "A linear interference network with local side-information," in *Proc. International Symposium on Inform. Theory (ISIT 2007)*, Nice, France, June 24-29, 2007.

[11] O. Simeone, O. Somekh, G. Kramer, H. V. Poor, and S. Shamai (Shitz), "Throughput of cellular systems with conferencing mobiles and cooperative base stations," submitted to *EURASIP Journal on Wireless Communications and Networking*, special issue on "Theory and Applications in Multiuser/Multiterminal Communications".

[12] W. Hirt and J. L. Massey, “Capacity of the discrete-time Gaussian channel with intersymbol interference,” *IEEE Trans. Inform. Theory*, vol. 34, no. 3, pp. 380-388, May 1988.

[13] S. Verdù, "Spectral efficiency in the wideband regime," *IEEE Trans. Inform. Theory*, no. 6, pp. 1319-1343, June 2002.

[14] A. V. Oppenheim, R. W. Schafer, and J. R. Buck, *Discrete-Time Signal Processing*, Prentice Hall, Upper Saddle River, NJ, 1999.

[15] E. Telatar, “Capacity of multi-antenna Gaussian channels,” *Europ. Trans. Telecommun.*, vol. 10, pp. 585-596, Nov.-Dec. 1999.

[16] T. M. Cover and J. A. Thomas, *Elements of information theory*, Wiley, New York, NY, 2006.

[17] D. Slepian and J.K. Wolf, "A coding theorem for multiple access channels with correlated sources," *Bell Systems Tech. J.*, vol. 52, pp. 1037-1076, Sept. 1973.