Orthogonalizing a Random Set of Beams

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***Abstract*—Recent work on wireless beamforming has focused on multi-user diversity effects, where terminals are chosen opportunistically based on their responses to test beams. Orthogonality between the test beams is typically chosen to aid subsequent interference-free transmission to the corresponding terminals. However, much of this work assumes that: (i) the pool of terminals on a given frequency-band is large enough that a subset of terminals can be found whose spatial signatures match the test beams; (ii) the responses to the test beams of the entire pool are known to the basestation; (iii) beamforming considerations can drive traffic scheduling and resource allocation. These conditions are not always met.**

Rather, we examine orthogonalizing a given set of beams for terminals that are chosen randomly or according to other resource considerations. Our orthogonalization is chosen to maximize the beamforming gain to the desired terminals, with the intent that other terminals may then "eavesdrop" on the beamformed signals, knowing that the beams are orthogonal. These orthogonal beams can be used for training channel state information even if the actual beams are not known. We distinguish between frequency-division duplex and time-division duplex systems and show that there are training advantages for each.

I. INTRODUCTION

The advantages of beamforming are well known and rely critically on accurate channel information at the transmitter. If the channel from M transmit antennas to a single receive antenna is some $1 \times M$ row-vector $\mathbf{h}^*$ (we use the superscript * to denote conjugate transpose), the transmitter may choose a beam $\mathbf{q}$ such that $\|\mathbf{q}\| = 1$ and this beam usually is modulated by a data signal d. The received signal is then

$$y = (\mathbf{h}^* \mathbf{q}) d + n. \tag{1}$$

The unit-power beam that maximizes channel power at the receiver is $\mathbf{q} = \mathbf{h}/\|\mathbf{h}\|$. The scalar $\mathbf{h}^*\mathbf{q}$ is the composition of the channel and the beam and is often referred to as the "equivalent channel". Usually, some beamformed pilot or training signals are needed so that the receiver can learn this equivalent channel. When in training, the receiver knows d.

In frequency-division duplex (FDD) systems, where the transmitter in one direction (say on the downlink from the basestation to a terminal) operates at one carrier frequency and the transmitter in the uplink uses another, learning which beam to use can be difficult, especially if there are many transmit antennas since $\mathbf{h}$ has M complex entries. The channel on the downlink and uplink generally differ if the carrier frequencies are sufficiently different. Therefore one possible way for the basestation to learn the downlink channel is to send training signals to the terminal and have the terminal feed back the learned channel information. Generally, the training signals involve orthogonal beams sent during specific times or on specific frequencies. We consider letting orthogonal beams serve dual-duty by letting these beams also carry data. We accomplish this by orthogonalizing the beams intended for desired terminals so that other terminals can eavesdrop on them.

In time-division duplex (TDD) systems, the transmitter in the downlink direction and uplink direction use the same carrier frequency and the channel is therefore reciprocal. Neglecting any time-variations in the channel, the downlink can use a beam learned during a training phase on the uplink. In fact, the uplink and downlink channels are simply transposes of each other:

$$\mathbf{h}^{*(\mathrm{d})} = \mathrm{trans}\,(\mathbf{h}^{*(\mathrm{u})}),$$

where $\mathrm{trans}\,(\cdot)$ indicates transpose (without conjugation). If the terminal transmits a single pilot signal then the basestation receives $\mathbf{h}^{*(\mathrm{u})}$ (plus noise) and can infer $\mathbf{h}^{*(\mathrm{d})}$ from it. Because of this reciprocity, there is little downlink beamforming training advantage to orthogonalizing the downlink beams for desired terminals. However, we show that there is an *uplink* advantage. Specifically, if the terminal wants to uplink beamform, we show that there is a training advantage to orthogonalizing the downlink data beams.

Recent work in FDD systems has concentrated on so-called opportunistic beamforming [1], [2], where "test-beams" are transmitted by the basestation and the terminals provide a "quality" response to these test beams. These test beams can be chosen to vary with time, thus providing time-variations to a channel that might otherwise be static. It is argued that with sufficiently fast feedback and a large number of terminals, that terminals can be found that respond well to one or more of these beams. If the multiple orthogonal test beams are transmitted and terminals are found that respond well to one (and only one) of these beams, then these terminals can be scheduled simultaneously with little interference between each other.

However, much of this work assumes that the pool of terminals on a given frequency-band is large enough that a subset of terminals can be found whose channels match the test beams. It is also generally assumed that the responses to the test beams of the large pool of terminals are known at the basestation and that beamforming considerations can drive traffic scheduling and resource allocation.

While orthogonality between beams is desirable, we take the view that it is generally difficult to schedule traffic simultaneously to terminals on strictly orthogonal beams, especially on the same band. Often, higher-level non-physical-layer considerations, such as delay-constraints, drive the decision to transmit to a given set of terminals in a given time slot. The challenge then is to orthogonalize a given set of data beams on *different* sub-bands. We may model the given set of data beams as "random" in the sense that the terminals corresponding to these beams are chosen according to resource considerations that do not include their beam characteristics.

Let the $M \times 1$ beams for K terminals be denoted $\mathbf{q}_1, \ldots, \mathbf{q}_K$ and arrange all the beams into an $M \times K$ matrix

$$Q = \begin{bmatrix} \mathbf{q}_1 & \mathbf{q}_2 & \ldots & \mathbf{q}_K \end{bmatrix}. \tag{2}$$

Assume for the moment that these beams are applied on separate time or subcarrier resources so we are not concerned about interference between terminals being served. If these beams are chosen to maximize the beamforming gain to the intended terminal, the matrix Q does not generally obey $Q^* Q = I_K$. Other terminals may then not readily eavesdrop and train their channel state information on these beams.

We examine orthogonalizing Q and analyzing the beamforming loss thereby incurred compared to the optimum beams. Not all orthogonalizations are equal in performance, and we show how to choose one according to an optimality criterion. We describe the problem now in greater detail.

II. PROBLEM DESCRIPTION

A basestation with M antennas serves K terminals with $K \leq M$. The basestation wishes to beamform to the K terminals. We assume that the K terminals do not occupy simultaneously the same subcarrier. By *subcarrier* we mean, broadly, a time-frequency resource such as might be obtained in an orthogonal frequency-division multiple-access (OFDMA) system. Although we generally discuss OFDMA systems, our conclusions apply readily to time-division (TDMA) or code-division (CDMA) systems by substituting "time-slot" or "spreading-code" for "subcarrier" in our discussions. We describe multiple-access issues in more detail in Section IV, and ignore these issues for the moment. We assume that each terminal has a single receive antenna, and generalize to multiple antennas in Section IV.

Let terminal k have $1 \times M$ channel $\mathbf{h}_k^*$. The entire channel between the M antennas and K single-antenna terminals is stored in the $K \times M$ matrix H defined to be

$$H = \begin{bmatrix} \mathbf{h}_1^* \\ \vdots \\ \mathbf{h}_K^* \end{bmatrix}. \tag{3}$$

Let $\mathbf{q}_k$ be the M-dimensional beam chosen for terminal k. We have that terminal k receives (on its assigned subcarrier)

$$y_k = \mathbf{h}_k^* \mathbf{q}_k d_k + n_k \tag{4}$$

where d_k represents a unit-energy data or pilot sequence and n_k is additive noise with zero mean and variance $\mathrm{E}\,|n_k|^2 = \sigma_{\mathrm{n}}^2$. In an OFDMA system, y_k is usually measured on some frequency f_k at some time t_k. We ignore this notational dependence in this section.

We describe a solution to the following problem. Let a basestation with M antennas serve K terminals with $K \leq M$. Let $\mathbf{q}_k$ be the M-dimensional beam chosen for terminal k and Q be the set of K beamforming vectors arranged into an $M \times K$ matrix (2). We seek a "good" choice of Q such that $Q^* Q = I_K$ (the $K \times K$ identity matrix).

We have two performance criteria: signal-to-noise ratio (SNR) and amplitude-to-noise ratio (ANR). The average (per-terminal) SNR is defined to be

$$\mathrm{SNR}(Q) = \frac{1}{K} \sum_{k=1}^{K} \frac{|\mathbf{h}_k^* \mathbf{q}_k|^2}{\sigma_{\mathrm{n}}^2} \tag{5}$$

and the average ANR is defined to be

$$\mathrm{ANR}(Q) = \frac{1}{K} \sum_{k=1}^{K} \frac{|\mathbf{h}_k^* \mathbf{q}_k|}{\sigma_{\mathrm{n}}} \tag{6}$$

We would like to solve for Q under two different constraints such that Q maximizes the ANR or SNR.

The SNR is generally considered to be more physically meaningful than the ANR, but we show that the maximum-ANR solution sheds light on the maximum-SNR solution when Q is constrained to be unitary; the maximum-ANR solution can be readily described, whereas the maximum-SNR solution cannot.

Because the terminals are chosen by a scheduler that does not choose terminals according to the directions of their beams, we assume that H has i.i.d.complex-Gaussian entries with zero mean and unit variance. We note that if $\mathbf{q}_1, \ldots, \mathbf{q}_K$ are chosen randomly without knowledge of H, then the expected values of these quantities are

$$\mathrm{E}\left(\mathrm{SNR}(Q_{\mathrm{rand}})\right) = \frac{1}{\sigma_{\mathrm{n}}^2}, \quad \mathrm{E}\left(\mathrm{ANR}(Q_{\mathrm{rand}})\right) = \frac{\sqrt{\pi}}{2} \frac{1}{\sigma_{\mathrm{n}}}. \tag{7}$$

III. SOLUTION FOR Q

A. ANR criterion

We wish to solve

$$Q_{\mathrm{unit}}^{\mathrm{ANR}} = \arg \max_{Q : Q^* Q = I} \mathrm{ANR}(Q).$$

We write

$$\mathrm{ANR}(Q) = \frac{1}{K\sigma_{\mathrm{n}}}\sum_{k=1}^{K}|\mathbf{h}_k^*\mathbf{q}_k| = \frac{1}{K\sigma_{\mathrm{n}}}\sum_{k=1}^{K}|(HQ)_{kk}|,$$

where we use the notation $(\cdot)_{kk}$ to indicate the kth diagonal entry of the matrix in parentheses. We use that $|(HQ)_{kk}| = p_k(HQ)_{kk}$ where $p_k = \mathbf{q}_k^*\mathbf{h}_k/|\mathbf{q}_k^*\mathbf{h}_k|$. Therefore

$$\mathrm{ANR}(Q) = \frac{1}{K\sigma_{\mathrm{n}}}\mathrm{tr}\,(HQP); \quad P = \begin{bmatrix} p_1 & 0 & \cdots & 0 \\ 0 & p_2 & 0 & \vdots \\ & & \ddots & \\ 0 & \cdots & & p_K \end{bmatrix}.$$

Clearly $P^*P = I$.

Define $Q' = QP$, and note that Q' also satisfies the constraint $Q'^*Q' = I$. We may therefore rename this Q' as Q, and solve

$$\begin{aligned} Q_{\mathrm{unit}}^{\mathrm{ANR}} &= \frac{1}{K\sigma_{\mathrm{n}}}\arg\max_{Q:Q^*Q=I}\sum_{k=1}^{K}(HQ)_{kk} \\ &= \frac{1}{K\sigma_{\mathrm{n}}}\arg\max_{Q:Q^*Q=I}\mathrm{tr}\,(HQ) \end{aligned}$$

where the trace is real. Using the singular value decomposition, we write $H = U\Sigma V^*$ where U is a $K \times K$ unitary matrix, Σ is a $K \times K$ nonnegative diagonal matrix, and V^* is a $K \times M$ matrix such that $V^*V = I_K$. This yields

$$\mathrm{tr}\,(HQ) = \mathrm{tr}\,(\Sigma Z) = \sum_{k=1}^{K}\sigma_k z_{kk}$$

where the $K \times K$ matrix $Z = V^*QU$ has real diagonal entries denoted as $\{z_{11}, \ldots, z_{KK}\}$. This trace is maximized by setting $Z = I$. To show this, let $W = QU$. Then $W^*W = U^*Q^*QU = I$ and therefore W has orthonormal columns, denoted $\{\mathbf{w}_1, \ldots, \mathbf{w}_K\}$. Similarly, V^* has orthonormal rows $\{\mathbf{v}_1^*, \ldots, \mathbf{v}_K^*\}$. Hence $z_{kk} = \mathbf{v}_k^*\mathbf{w}_k \leq 1$ with equality if and only if $\mathbf{w}_k = \mathbf{v}_k$. This is the same as setting $Z = V^*W = I$. Summarizing, we obtain

$$Q_{\mathrm{unit}}^{\mathrm{ANR}} = VU^*, \quad \mathrm{ANR}_{Q_{\mathrm{unit}}} = \frac{1}{K\sigma_{\mathrm{n}}}\mathrm{tr}\,\Sigma = \frac{1}{K\sigma_{\mathrm{n}}}\sum_{k=1}^{K}\sigma_k \tag{8}$$

The expected value of $\mathrm{ANR}_{Q_{\mathrm{unit}}}$ obeys

$$\lim_{M,K\to\infty}\frac{1}{\sqrt{M}}\mathrm{E}\,(\mathrm{ANR}_{Q_{\mathrm{unit}}}) = \frac{1}{\sigma_{\mathrm{n}}}\mu(\beta) \tag{9}$$

where

$$\mu(\beta) \triangleq \frac{2(1+1/\sqrt{\beta})}{3\pi}\left[(\beta+1)E(k^2) - (\sqrt{\beta}-1)^2K(k^2)\right] \tag{10}$$

and where $\beta = M/K$, and $K(k^2)$ and $E(k^2)$ are the so-called complete elliptic integrals of the first and second kind with "modulus" $k^2 = 4\sqrt{\beta}/(\sqrt{\beta}+1)^2$. These functions are defined as

$$K(k^2) = \int_0^{\pi/2}\frac{d\theta}{\sqrt{1-k^2\sin^2\theta}}$$

and

$$E(k^2) = \int_0^{\pi/2}d\theta\sqrt{1-k^2\sin^2\theta}.$$

The proof is omitted.

For $\beta = 1$ we obtain

$$\mu(1) = \frac{8}{3\pi}. \tag{11}$$

B. SNR criterion

We solve for

$$Q_{\mathrm{unit}}^{\mathrm{SNR}} = \arg\max_{Q:Q^*Q=I}\mathrm{SNR}(Q)$$

where

$$\mathrm{SNR}(Q) = \frac{1}{K\sigma_{\mathrm{n}}^2}\sum_{k=1}^{K}|(HQ)_{kk}|^2.$$

Unfortunately, the techniques of Section III-A that yield the closed-form solution $Q_{\mathrm{unit}}^{\mathrm{ANR}}$ do not apply directly. Thus, $Q_{\mathrm{unit}}^{\mathrm{SNR}} \neq Q_{\mathrm{unit}}^{\mathrm{ANR}}$. Nevertheless, the solutions are close and we now make this more precise.

Define

$$\tilde{Q}(\mathcal{E}) = Q(I-\mathcal{E})^{-1}(I+\mathcal{E}), \tag{12}$$

where Q is $M \times K$ such that $Q^*Q = I$ and $\mathcal{E}$ is $K \times K$ skew-Hermitian with complex entries $\varepsilon_{\ell m}$, $\ell, m = 1, \ldots, K$. (A skew-Hermitian matrix $\mathcal{E}$ has the property that $\mathcal{E} = -\mathcal{E}^*$.) Note that we are defining a matrix function of two arguments: Q and $\mathcal{E}$. By construction, $\tilde{Q}(\mathcal{E})$ is $M \times K$ and obeys $\tilde{Q}(\mathcal{E})^*\tilde{Q}(\mathcal{E}) = I$.

We wish to examine $\mathrm{SNR}(\tilde{Q}(\mathcal{E}))$ for $\tilde{Q}(\mathcal{E})$ in the neighborhood of $Q_{\mathrm{unit}}^{\mathrm{ANR}}$. The parameterization $\tilde{Q}(\mathcal{E})$ allows us to move along the manifold of matrices satisfying $Q^*Q = I$ in the neighborhood of $Q_{\mathrm{unit}}^{\mathrm{ANR}}$ by computing $\widetilde{Q_{\mathrm{unit}}^{\mathrm{ANR}}}(\mathcal{E}) = Q_{\mathrm{unit}}^{\mathrm{ANR}}(I-\mathcal{E})^{-1}(I+\mathcal{E})$ in the neighborhood of $\mathcal{E} = 0$. We begin by taking the derivative of $\mathrm{SNR}(\tilde{Q}(\mathcal{E}))$ with respect to $\mathcal{E}$.

Result 1:

$$\frac{d\mathrm{SNR}(\tilde{Q}(\mathcal{E}))}{d\mathcal{E}} = Q^*H^*D - D^*HQ. \tag{13}$$

where D is a diagonal matrix whose entries are

$$D = \begin{bmatrix} (HQ)_{11} & 0 & \cdots & 0 \\ 0 & (HQ)_{22} & 0 & \vdots \\ & & \ddots & \\ 0 & \cdots & & (HQ)_{KK} \end{bmatrix}. \tag{14}$$

We note that $\mathcal{E}$ is skew-Hermitian and hence this equation requires some explanation. We mean that the derivative is taken with respect to the real and imaginary parts of the distinct entries in $\mathcal{E}$ separately and then rearranged into matrix form. This form is useful, for example, for easily manipulating $\mathcal{E}$ in gradient-based searches for $Q^{\mathrm{SNR}}_{\mathrm{unit}}$, where we might choose updates of the form $\mathcal{E}_{t+1} = \mathcal{E}_t + \nu(Q_t^* H^* D_t - D_t^* H Q_t))$ where Q_t and D_t are computed using $\mathcal{E}_t$ in (12) and (14).

We now show that although $Q^{\mathrm{ANR}}_{\mathrm{unit}}$ was derived using the ANR metric, it also has an optimality property for the SNR.

Result 2: As $M,\ K \to \infty$ in some fixed ratio $\beta = M/K$,

$$\left.\frac{\partial \mathrm{SNR}(\widetilde{Q^{\mathrm{ANR}}_{\mathrm{unit}}(\mathcal{E})})}{\partial \varepsilon_{\ell m}}\right|_{\mathcal{E}=0} \to 0 \tag{15}$$

for any fixed ℓ, $m = 1, \ldots, K$, where $Q^{\mathrm{ANR}}_{\mathrm{unit}} = VU^*$ as given in (8).

Thus, although generally $Q^{\mathrm{SNR}}_{\mathrm{unit}} \neq Q^{\mathrm{ANR}}_{\mathrm{unit}}$, asymptotically $Q^{\mathrm{ANR}}_{\mathrm{unit}}$ is a local maximum of the function $\mathrm{SNR}(Q)$. We now compute the limiting value of the expected value of $\mathrm{SNR}(Q^{\mathrm{ANR}}_{\mathrm{unit}})$.

Result 3: As $M,\ K \to \infty$ in some fixed ratio $\beta = M/K$,

$$\lim_{M,K\to\infty} \frac{1}{M} \mathrm{E}\left(\mathrm{SNR}(Q^{\mathrm{ANR}}_{\mathrm{unit}})\right) = \frac{1}{\sigma_{\mathrm{n}}^2}\mu^2(\beta) \tag{16}$$

where $\mu(\beta)$ is defined in (10).

This result says that for large M, the average SNR per terminal when using the beams $Q^{\mathrm{ANR}}_{\mathrm{unit}}$ is

$$\mathrm{E}\left(\mathrm{SNR}(Q^{\mathrm{ANR}}_{\mathrm{unit}})\right) \approx \frac{M}{\sigma_{\mathrm{n}}^2}\mu^2(\beta) \tag{17}$$

The beamforming gain is therefore

$$\mathrm{Gain}_{Q^{\mathrm{ANR}}_{\mathrm{unit}},\mathrm{dB}} \approx 10\log_{10}(M\mu^2(\beta)). \tag{18}$$

These results may be compared to the case where Q is not constrained to be unitary and the corresponding average SNR per terminal is M/σ_{n}^2 and beamforming gain is $\log(M)$. We denote the matrix of unconstrained beamforming vectors as Q_{arb}. The loss is then

$$\mathrm{Gain}_{Q^{\mathrm{ANR}}_{\mathrm{unit}},\mathrm{dB}} - \mathrm{Gain}_{Q_{\mathrm{arb}},\mathrm{dB}} \approx 10\log_{10}\mu^2(\beta). \tag{19}$$

We observe that this loss is independent of M. Figure 1 displays (19) as a function of β. When $\beta = 1$ the loss is $10\log_{10}\mu^2(1) = 10\log_{10}(8/3\pi)^2 = -1.4$ dB; see (11). When $\beta = 1$ the basestation is simultaneously serving as many terminals as it has antennas and hence the orthogonality constraint is most restrictive. As β grows the orthogonality constraint relaxes and the loss therefore decreases.

We may conclude that the SNR loss of imposing orthogonality is on average at most 1.4 dB, and depends only on the ratio of antennas to number of terminals being served.

IV. MULTIPLE-ACCESS ISSUES AND EAVESDROPPING

A. Frequency-division duplex systems

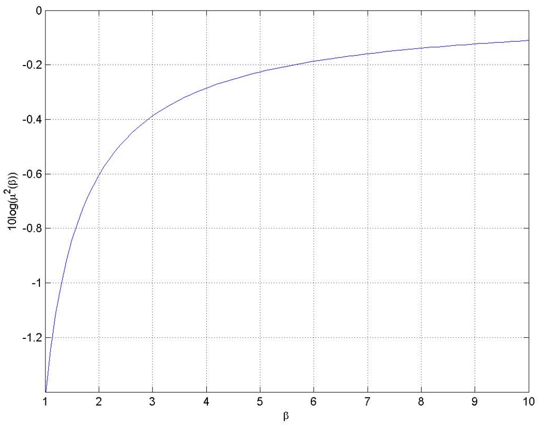

Fig. 1. SNR loss of using beams that are constrained to be orthogonal versus beams that are unconstrained. Plotted is equation (19) as a function of β. The loss is worst (approximately -1.4 dB) when $\beta = 1$; at this point the number of terminals served simultaneously equals the number of basestation antennas.

In frequency-division duplex systems, the downlink (basestation to terminal) and uplink (terminal to basestation) transmissions occupy distinct well-separated frequency bands. Hence, channel or beam information on one band cannot directly be used on the other. We show how an eavesdropper can use a unitary set of downlink beams to train downlink channel state information.

Suppose that we have an OFDMA system where terminal k is assigned subcarriers $f_{k1}, \ldots, f_{kN}$. One simple implementation of beamforming has the basestation applying beam $\mathbf{q}_k$ for the kth terminal on all these subcarriers. Any other terminal is generally assigned a different set of subcarriers $f_{\ell 1}, \ldots, f_{\ell N}$ such that $f_{\ell i} \neq f_{kj}$ for all $i, j = 1, \ldots, N$.

Nevertheless, it often holds that the subcarriers assigned to distinct terminals are interlaced in such a way that any eavesdropper receiving a consecutive contiguous set of K subcarriers (say $f = 1, \ldots, K$) can observe transmissions intended for the K distinct intended terminals, one on each frequency. We assume that the eavesdropper has a frequency-flat channel across these K contiguous subcarriers and that these subcarriers contain pilot symbols known to all the terminals. This assumption is not restrictive because subcarriers containing pilot symbols are traditionally included to train the intended terminals [3].

The signal received by the eavesdropper on frequency k then becomes

$$y_k = (\mathbf{h}^* \mathbf{q}_k) d_k + n_k,$$

where $\mathbf{h}^*$ is the channel of the eavesdropper and $d_k = 1$ is the known pilot. Arranging the K measurements taken by the eavesdropper into a row-vector, we obtain

$$\mathbf{y} = \mathbf{h}^* Q + \mathbf{n},$$

where Q is as in (2). Because we have ensured that Q is unitary, the eavesdropper may relay all or part of $\mathbf{y}$ to the basestation and allow a beam to be directed to the eavesdropper on the next scheduled transmission.

B. Time-division duplex systems

In time-division duplex systems, the downlink and uplink occupy the same frequency bands, and hence channel information on the downlink can be used on the uplink. We show how a unitary set of beams can allow an eavesdropper to beamform optimally on the uplink.

We assume a similar interlaced-frequency OFDMA system as described in the previous section, where the eavesdropper has access to a contiguous set of K subcarriers. We also suppose that $K = M$ (the number of terminals served equals the number of basestation antennas). Let the terminal have N antennas that it can use to receive and transmit.

Let the uplink channel be denoted by the $M \times N$ matrix G. The considerations in Section IV-A allow us to conclude that the eavesdropper receives the equivalent channel $G' = Q^* G$, where Q is the unitary matrix of downlink beams. (The matrix Q^* appears on the left in this equation rather than on the right as in Section IV-A because the uplink channel is the transpose of the downlink.) The eavesdropper performs the singular value decomposition

$$G' = U \Sigma V^*.$$

This singular value decomposition is related to the singular value of the actual channel

$$G = (QU) \Sigma V^*.$$

where QU is unitary. Optimum beamforming on the uplink requires the terminal to apply the matrix V on transmission. Because the basestation knows G, it may apply $(QU)^*$ to the received signal. This decomposes the original channel G into $\min(M, N)$ parallel non-interfering subchannels as is required in optimum beamforming.

C. Parting comments

The price of imposing orthogonality on M transmitted beams is on average approximately 1.4 dB. Hence, in four-antenna systems where the unconstrained beamforming gain is approximately 6 dB per terminal, the orthogonality-constrained beamforming gain is approximately 4.6 dB. This is the price to pay for being able to eavesdrop on the resulting beams, knowing that they are orthogonal by design. Such eavesdropping may reduce training overhead in multi-access systems, since dedicated training symbols are very costly in power and time. In an unloaded system where the basestation is largely idle, training symbols do not generally lower the downlink data rates. In fact, such training symbols are needed to permit beamforming. But as soon as the system load increases, the resources occupied by training become more precious. Fortunately, as the system load increases, if the basestation imposes orthogonality on its downlink beams, the need for dedicated training symbols decreases since we may eavesdrop on the beamformed signals. We may therefore reduce the amount of dedicated training.

In deriving our result, we assumed that the beams were chosen randomly or without consideration for choices that might make the process of orthogonalization easier. Any ability by the basestation to schedule terminals based on near-orthogonality of their beampatterns would clearly make the orthogonalization process easier.

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