Maximum Number of Active Links in Wireless Networks with Fading Channels

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***Abstract*—In this paper, the maximum number of active links supporting a minimum rate is asymptotically obtained in a single-hop wireless network with an arbitrary structure. It is assumed that each source-destination pair communicates through a fading channel; consequently, destinations receive interference from all other active source-destination pairs. Under the assumption of independent Rayleigh fading channels for different source-destination pairs, it is shown that the optimal number of active links is of the order $\log N$ with probability approaching one as the total number of nodes, N, tends to infinity. The achievable total throughput is also scales logarithmically with the total number of links/nodes in the network.**

I. Introduction

A wireless network consists of several transmitters and receivers. Specific cases are broadcast channels where there is only one transmitter sending information to several receivers and multiple-access channels where several transmitters send information to one receiver.

In [1], a power allocation scheme is considered for broadcast channels with a minimum rate constraint $R_{\min} > 0$. Since for a fixed $R_{\min}$, in a time-varying fading environment, it may not be always possible for all users to achieve the minimum rate simultaneously, a scheme is proposed to maximize the number of active receivers for each of which the minimum rate can be supported, while allocating no power to the other inactive receivers. As the number of supportable active users depends on the specific channel states, the asymptotic behavior is analyzed when the total number of users n is large. Under the assumption of independent Rayleigh fading channels for different users, we show that the maximum number of active receivers is very close to $\log\left(P \log n\right)/R_{\min}$ with probability approaching 1, where P denotes the total transmitted power. In [2], the same analysis is extended to multiple-access channels and it is shown that the maximum number of active transmitters also scales double logarithmically with the total number of transmitters in the system.

In this paper, the aforementioned idea is generalized to a wireless network with an arbitrary topology. To satisfy the requirements of delay-sensitive applications, it is assumed that each active link supports a minimum rate. Due to limited transmitted power and interference from other active source-destination pairs, it is not always possible for all nodes to keep this minimum rate. Hence, we allow nodes with good channel conditions to be active while others remain silent during each time slot. Thus, an on/off power allocation scheme can be exploited to maximize the number of active links while maintaining the minimum-rate constraint. It is asymptotically shown that the maximum number of simultaneous active links is of the order $\log n$, where n denotes the total number of links in the network. A tight bound on the number of active source-destination pairs and a lower bound on the total throughput are presented. Based on the strong law of large numbers (SLLN), our link activation method can be implemented in a distributed fashion. In [3], a multi-hop random wireless network is considered and throughput scaling laws are presented when link channels are drawn independently from any arbitrary distribution. For Rayleigh fading channels, [3] shows that the achievable and the optimal total throughput scale logarithmically with the number of nodes in the network; however, the constants for the lower and upper bounds are not the same. In [4], a rate-constrained single-hop wireless network is considered with Rayleigh fading channels. An upper bound is derived that shows the maximum number of active links scales with $\log n/R_{\min}$. Using threshold-based link activation strategies, lower bounds are achieved on the number of active links in the network; however, there is a gap between the lower and the upper bounds. In this paper, a tighter upper bound is obtained and it is shown that the gap between the lower bound and the upper bound asymptotically goes to zero.

II. Wireless Network Model

Consider a single-hop wireless network with N nodes located arbitrarily. It is assumed that source i is connected to destination i through a fading channel. Destinations are conventional receivers without multi-user detectors; in other words, no broadcast or multiple-access channel is embedded in the network. Every node has a receiver and a transmitter but it cannot transmit and receive signals simultaneously. Hence, the total number of links n (links herein mean source-destination pairs communicating through fading channels) equals $\lfloor N/2 \rfloor$, where $\lfloor x \rfloor$ denotes the largest integer no greater than x. Nodes transmit signals with maximum power of P or remain silent during each time slot. The received signal at node i, $Y_i(t)$, is

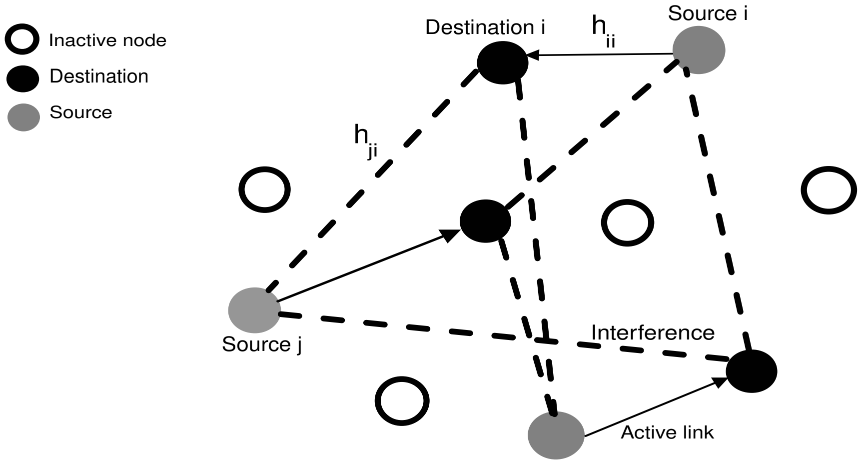

Fig. 1. A wireless network with active links $(-)$ and interference channels $(--)$

given by

$$Y_i(t) = h_{ii}(t)\,X_i(t) + \sum_{\substack{k=1\\k\neq i}}^{m} h_{ki}(t)\,X_k(t) + Z_i(t) \tag{1}$$

where $h_{ii}(t)$ denotes the active fading channel between transmitter i and receiver i, $h_{ki}(t)$ represents an interference channel for receiver i, m refers to the number of active links, and $Z_i(t) \sim \mathcal{CN}(0,\sigma^2)$ represents background noise at node i. Hence, the achievable rate of link i can be written as

$$R_i \quad \leq \quad \log\left(1 + \frac{P\,|h_{ii}|^2}{\sigma^2 + \sum_{\substack{k=1\\k\neq i}}^{m} P\,|h_{ki}|^2}\right) \tag{2}$$

III. Problem Motivation

In delay-sensitive applications, each active link needs to support a minimum rate. Due to limited transmitted power and interference from other active source-destination pairs, it is not always possible for all nodes to keep this minimum rate. Hence, we allow nodes with good channel conditions to be active while others remain silent during each time slot. Consider the wireless network (1) and assume $|h_{11}| \leq \cdots \leq |h_{nn}|$ without loss of generality. In this case, the maximum number of active links supporting the minimum rate is given by the following optimization problem.

$$\begin{cases} \max\{m\} & (3)\\ R_i \geq R_{\min}, \quad i = n-m+1,\ldots,n & (4)\end{cases}$$

where m denotes the maximum number of active links.

IV. Asymptotic Analysis

First of all, as standard notation, $o(\cdot)$, $O(\cdot)$, and $\omega(\cdot)$ have the following interpretations: for any positive infinite sequences $f(n)$ and $g(n)$, $n = 1,2,\ldots$, $f(n) = o(g(n))$ means $\lim_{n\to\infty}\left|\frac{f(n)}{g(n)}\right| = 0$; $f(n) = O(g(n))$ means $\limsup_{n\to\infty}\left|\frac{f(n)}{g(n)}\right| < \infty$; $f(n) = \omega(g(n))$ means $\lim_{n\to\infty}\left|\frac{f(n)}{g(n)}\right| = \infty$. Consider independent Rayleigh fading channels between different source-destination pairs, i.e., the channel gains $h_{ij};\ i,j = 1,\ldots,\lfloor N/2\rfloor$ are independent realizations of the complex Gaussian distribution; as a result, $|h_{ij}|^2;\ i,j = 1,\ldots,\lfloor N/2\rfloor$ are independent realizations of the exponential distribution.

Theorem 4.1: Under the assumption of independent Rayleigh fading channels for different source-destination pairs with channel gains $h_{ij} \sim \mathcal{CN}(0,1);\ i,j = 1,\ldots,\lfloor N/2\rfloor$, and for any $\epsilon > 0$ arbitrarily close to zero, the maximum number of active links, m determined by (3), is bounded as

$$\mathbb{P}\left\{\lfloor\beta_1(n)\rfloor \leq m \leq \beta_2(n)\right\} \to 1, \quad \text{as } n\to\infty, \tag{5}$$

where $n = \lfloor N/2\rfloor$ denotes the total number of links transmitting information in the network, and

$$\beta_1(n) \;=\; \frac{\log n}{(1+\epsilon)\,e^{R_{\min}} - (1-2\epsilon)} \tag{6}$$

$$\beta_2(n) \;=\; \frac{\log n}{(1-\epsilon)e^{R_{\min}} - (1+2\epsilon)}. \tag{7}$$

Proof: Consider the wireless network (1) with independent channel gains $h_{ij} \sim \mathcal{CN}(0,1)$, for $i,j = 1,\ldots,\lfloor N/2\rfloor$; as a result, $|h_{ij}|^2 \sim$ Exponential(1). Based on the weak law of large numbers (WLLN) [6],

$$\lim_{n\to\infty}\mathbb{P}\left(\left|\frac{1}{m}\sum_{k=1}^{m}|h_{ki}|^2 - E\left(|h_{11}|^2\right)\right| < \epsilon\right\} = 1 \tag{8}$$

For any fixed $h_0 > 0$, the number of "good" channels can be characterized with $|h_{ii}|^2$ greater than h_0 as follows. Let $p_0 = 1 - \mathbb{P}(|h_{ii}|^2 \leq h_0) = e^{-h_0}$. Consider a Bernoulli sequence:

$$x_i = \begin{cases} 1, & \text{with probability } p_0\\ 0, & \text{with probability } 1-p_0\end{cases} \tag{9}$$

for $i = 1,2,\ldots,n$. Then, the number of good channels has the same distribution as $X = \sum_{i=1}^{n} x_i$, which satisfies the Binomial distribution $B(n,p_0)$. For any integer $m \geq 1$, obviously,

$$\mathbb{P}(X \leq m-1) = \sum_{j=0}^{m-1}\binom{n}{j}p_0^j(1-p_0)^{n-j}$$

which is not easy to analyze. If $m-1 \leq np_0$, the Chernoff bound on the sum of independent Poisson trials can be used as [7, page 70]:

$$\mathbb{P}(X \leq m-1) \leq \exp\left(-\frac{1}{2p_0}\frac{(np_0-m+1)^2}{n}\right). \quad (10)$$

Hence,

$$\mathbb{P}(X \geq m) \geq 1-\exp\left(-\frac{1}{2p_0}\frac{(np_0-m+1)^2}{n}\right). \quad (11)$$

Let $h_0 = m\left((1+\epsilon)e^{R_{\min}}-(1-\epsilon)\right)$. We show that for any $m \leq \beta_1(n)$, $\min_{n-m+1\leq i\leq n}|h_{ii}|^2 \geq h_0$ holds asymptotically almost surely.

$$\begin{aligned}
p_0 &= 1-\mathbb{P}(|h_{ii}|^2 \leq h_0)\\
&= \exp(-h_0)\\
&= \exp\left(-m\left((1+\epsilon)e^{R_{\min}}-(1-\epsilon)\right)\right)\\
&\geq \exp\left(-\beta_1(n)\left((1+\epsilon)e^{R_{\min}}-(1-\epsilon)\right)\right)\\
&= \exp\left(-\frac{(1+\epsilon)e^{R_{\min}}-(1-\epsilon)}{(1+\epsilon)e^{R_{\min}}-(1-2\epsilon)}\log n\right)\\
&= \exp(-\lambda\log(n)) = n^{-\lambda} \quad (12)
\end{aligned}$$

where $\lambda = \frac{(1+\epsilon)e^{R_{\min}}-(1-\epsilon)}{(1+\epsilon)e^{R_{\min}}-(1-2\epsilon)} < 1$. It is obvious that as $n \to \infty$,

$$\frac{1}{2p_0}\frac{(np_0-m+1)^2}{n} \sim \frac{n^2p_0^2}{2np_0} = \frac{np_0}{2} \geq \frac{n^{1-\lambda}}{2} \to \infty. \quad (13)$$

Hence, according to (11), there are at least $m = \lfloor\beta_1(n)\rfloor$ good channels with $|h_{ii}|^2 \geq h_0$.

Next, we show that the minimum-rate constraint is satisfied for these m channel gains if $\min_{n-m+1\leq i\leq n}|h_{ii}|^2 \geq h_0$. As $n \to \infty$, $m \to \infty$; therefore, according to (8),

$$\begin{aligned}
&\log\left(1+\frac{P|h_{ii}|^2}{\sigma^2+\sum_{\substack{k=n-m+1\\k\neq i}}^{n}P|h_{ki}|^2}\right)\\
&\quad\geq \log\left(1+\frac{Ph_0}{\sigma^2+P(m-1)\left(E(|h_{11}|^2)+\epsilon\right)}\right)\\
&\quad= \log\left(1+\frac{Pm\left((1+\epsilon)e^{R_{\min}}-(1-\epsilon)\right)}{\sigma^2+P(m-1)(1+\epsilon)}\right)\\
&\quad\sim \log\left(1+e^{R_{\min}}-\frac{1-\epsilon}{1+\epsilon}\right) \geq R_{\min}
\end{aligned}$$

Obviously, based on (2), the minimum-rate constraint is satisfied for these m channel gains. Thus, we proved that as $n \to \infty$, with probability approaching 1, there are at least $m = \lfloor\beta_1(n)\rfloor$ good channels with $|h_{ii}|^2 \geq m\left((1+\epsilon)e^{R_{\min}}-(1-\epsilon)\right)$, for which the minimum rate constraint is satisfied.

Next, we prove $m \leq \beta_2(n)$ holds asymptotically almost surely. First, we show that the best active link should have channel gain $|h_{nn}|^2 \geq h_0^{'} = m\left((1-\epsilon)e^{R_{\min}}-(1+\epsilon)\right)$; otherwise, if $\max_{n-m+1\leq i\leq n}|h_{ii}|^2 < h_0^{'}$, according to (8),

$$\begin{aligned}
&\log\left(1+\frac{P|h_{ii}|^2}{\sigma^2+\sum_{\substack{k=n-m+1\\k\neq i}}^{n}P|h_{ki}|^2}\right)\\
&\quad< \log\left(1+\frac{Ph_0^{'}}{\sigma^2+P(m-1)\left(E(|h_{11}|^2)-\epsilon\right)}\right)\\
&\quad= \log\left(1+\frac{Pm\left((1-\epsilon)e^{R_{\min}}-(1+\epsilon)\right)}{\sigma^2+P(m-1)(1-\epsilon)}\right)\\
&\quad\sim \log\left(1+e^{R_{\min}}-\frac{1+\epsilon}{1-\epsilon}\right) < R_{\min}
\end{aligned}$$

which violates (2).

Hence, to show that

$$\mathbb{P}(m \leq \beta_2(n)) \to 1,$$

or

$$\mathbb{P}(m > \beta_2(n)) \to 0,$$

we only need to show that

$$\mathbb{P}\left(|h_{nn}|^2 \geq \beta_2(n)\left((1-\epsilon)e^{R_{\min}}-(1+\epsilon)\right)\right) \to 0.$$

Let $h_0^{'} = \beta_2(n)\left((1-\epsilon)e^{R_{\min}}-(1+\epsilon)\right)$ and define $p_1 = 1-\mathbb{P}(|h_{ii}|^2 \leq h_0^{'}) = \exp(-h_0^{'})$. The probability that all links have channel gains less than h_0 equals $(1-p_1)^n$. Hence,

$$\mathbb{P}\left(|h_{nn}|^2 \geq \beta_2(n)\left((1-\epsilon)e^{R_{\min}}-(1+\epsilon)\right)\right) = 1-(1-p_1)^n \quad (14)$$

which tends to 0 if and only if

$$\left(1-e^{-h_0^{'}}\right)^n = \left(1-\exp\left(-\beta_2(n)\left((1-\epsilon)e^{R_{\min}} -(1+\epsilon)\right)\right)\right)^n \to 1. \quad (15)$$

Since

$$\left(1-\exp(-h_0^{'})\right)^{\exp(h_0^{'})} \to e^{-1},$$

(15) holds if

$$\begin{aligned}
&n\cdot\exp\left(-\beta_2(n)\left((1-\epsilon)e^{R_{\min}}-(1+\epsilon)\right)\right)\\
&\quad= n\cdot\exp\left(-\frac{\left((1-\epsilon)e^{R_{\min}}-(1+\epsilon)\right)}{\left((1-\epsilon)e^{R_{\min}}-(1+2\epsilon)\right)}\log n\right)\\
&\quad= n^{1-\gamma} \to 0
\end{aligned}$$

which holds as $\gamma = \frac{\left((1-\epsilon)e^{R_{\min}}-(1+\epsilon)\right)}{\left((1-\epsilon)e^{R_{\min}}-(1+2\epsilon)\right)} > 1$. ■

Corollary 4.1: The probability in theorem 4.1 converges to 1 at the following rates:

$$\mathbb{P}(m < \lfloor\beta_1(n)\rfloor) = o\left(\exp\left(-\frac{n^{1-\lambda}}{2+\tilde{\sigma}}\right)\right) \quad (16)$$

and

$$\mathbb{P}(m > \beta_2) = O\left(n^{1-\gamma}\right) \quad (17)$$

where $\lambda = \frac{(1+\epsilon)e^{R_{\min}}-(1-\epsilon)}{(1+\epsilon)e^{R_{\min}}-(1-2\epsilon)} < 1$, $\tilde{\sigma} > 0$ can be arbitrarily small, and $\gamma = \frac{\left((1-\epsilon)e^{R_{\min}}-(1+\epsilon)\right)}{\left((1-\epsilon)e^{R_{\min}}-(1+2\epsilon)\right)} > 1$.

Proof: To prove (16), we only need to show that for $m = \lfloor \beta_1 \rfloor$,

$$\frac{1}{2p_0}\frac{(np_0 - m + 1)^2}{n} \geq \frac{n^{1-\lambda}}{2+\tilde{\sigma}} \tag{18}$$

We know that for sufficiently large n,

$$\frac{(np_0)^2}{(2+\tilde{\sigma})np_0} \leq \frac{1}{2p_0}\frac{(np_0 - m + 1)^2}{n} \leq \frac{(np_0)^2}{2np_0}$$

Hence, with the following modification, (18) is proved.

$$\frac{(np_0)^2}{(2+\tilde{\sigma})np_0} = \frac{n^{1-\lambda}}{2+\tilde{\sigma}}$$

To prove (17), noting (14), we have

$$\begin{aligned}\mathbb{P}(m > \beta_2(n)) &\leq \mathbb{P}\left(|h_{nn}|^2 \geq \beta_2(n)\left((1-\epsilon)e^{R_{\min}}\right.\right.\\ &\qquad\qquad \left.\left. -(1+\epsilon)\right)\right)\\ &= 1 - \left(1 - \exp\left(-\beta_2(n)\left((1-\epsilon)e^{R_{\min}}\right.\right.\right.\\ &\qquad\qquad \left.\left.\left. -(1+\epsilon)\right)\right)\right)^n\\ &= O\left(n \cdot \exp\left(-\beta_2(n)\left((1-\epsilon)e^{R_{\min}}\right.\right.\right.\\ &\qquad\qquad \left.\left.\left. -(1+\epsilon)\right)\right)\right)\\ &= O\left(n^{1-\gamma}\right)\end{aligned} \tag{19}$$

■

Remark 4.1: According to theorem 4.1, the total throughput of the wireless network is lower-bounded as

$$R_{sum} \geq \frac{R_{\min}}{e^{R_{\min}} - 1}\log n \tag{20}$$

In [4]-[5], a rate-constrained single-hop wireless network with Rayleigh fading channels is considered. An upper bound on the maximum number of active links is calculated as

$$m < \frac{\log n}{R_{\min}} \tag{21}$$

Based on the threshold-based link activation strategy (TBLAS) presented in [5], the maximum number of active links and the total throughput are given by

$$\begin{aligned} m_{TBLAS} &= \frac{\log n}{e^{R_{\min}} - 1}\\ R_{TBLAS} &= \frac{R_{\min}}{e^{R_{\min}} - 1}\log n \end{aligned} \tag{22}$$

Although the maximum number of active links is equal to the one obtained by theorem 4.1, it can be seen that the upper bound presented in [5] is not very tight. [5] also presents a centralized double threshold-based link activation strategy (DTBLAS) to reach the upper bound in (21); however, they cannot provide closed-form expressions for optimal thresholds and numerically show that DTBLAS reaches this upper bound at $R_{\min} = 0$ or ∞ which are not practical. Hence, this paper compared to [5] has two advantages: First of all, a tighter upper bound is provided. Second, the upper bound meets the lower bound asymptotically almost surely.

V. Implementation Issues

The proposed link activation strategy can be easily implemented in a distributed fashion. Consider a wireless network with a sufficiently large number of nodes. Suppose each source node knows its link channel gain by applying channel estimation algorithms. First of all, the maximum number of active links is calculated by

$$m = \frac{\log n}{e^{R_{\min}} - 1}$$

Then, each source node compares its channel gain with threshold h_0. If the channel gain is above the threshold, the corresponding link becomes active; otherwise, the node remains silent during the current time slot.

$$h_0 = m\left(e^{R_{\min}} - 1\right) = \log n$$

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