Subspace Beamforming for Near-Capacity MIMO Performance

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***Abstract*—A subspace beamforming method is presented that decomposes a MIMO channel into multiple pairs of subchannels. The pairing is done based on singular values such that similar channel capacity is obtained between different subchannel pairs. This new capacity balancing concept is key to achieving high performance with low complexity. We apply the subspace idea to geometric mean decomposition (GMD) and maximum likelihood (ML) detection. The proposed subspace GMD scheme requires only two layers of detection/decoding, regardless of the total number of subchannels, thus alleviating the latency issue associated with conventional GMD. We also show how the subspace concept makes the optimization of ML beamforming and ML detection itself feasible for any $K \times K$ MIMO system. Simulation results show that subspace beamforming performs nearly as well as optimum GMD performance, and to within only a few dB of the Shannon bound.**

***Index Terms*—MIMO, SVD, GMD, subspace, maximum likelihood**

I. Introduction

Multiple-input, multiple-output (MIMO) transmission is a key technology to achieve high spectrum efficiency in wireless systems [1] [2]. Methods for MIMO detection include those based on joint detection, e.g., maximum likelihood (ML) detection [3] [4], and those involving channel decomposition and per-stream detection, e.g., interference nulling [5], successive interference cancellation (SIC) [6], and transmitter beamforming [2] [7]. A common challenge among various approaches is to achieve good tradeoffs between complexity and performance as the number of antennas/data streams grows. Another important aspect of MIMO implementation is the concept of capacity summation. Methods such as singular value decomposition (SVD) and SIC are known to be 'optimum' in the sense that the sum of capacities of individual decomposed subchannels is equal to the Shannon MIMO channel capacity [2] [8]. However, actual system performance depends greatly on how this capacity summation is realized. In general, capacity summation is possible through: (i) bit loading based on channel feedback [9], (ii) coding across subchannels with a single error-correcting codeword [10], (iii) spectrum factorization [11], and (iv) capacity balancing, such as geometric mean decomposition (GMD) [7] and the singular value pairing method described in this paper. For example, SVD may produce subchannels with highly unequal signal-to-noise ratios (SNRs) and suffer a significant performance loss when low rate coding and/or a broad range of modulation levels (e.g., 256QAM) are not available. In contrast, GMD, which requires both beamforming and SIC, produces subchannels with equal SNRs and has been shown to be the first known benchmark that achieves the MIMO channel capacity to within the single-input, single-output (SISO) gap for any code rate and without bit loading [7]. In order to avoid error propagation, a GMD scheme should use decisions after decoding to subtract out interference, therefore it requires the detection/decoding of different subchannels to be done sequentially. This may impose a serious latency issue as the number of subchannels increases, especially when long code blocks are used for improved coding gain (e.g., in turbo coding [12]).

This paper presents a subspace beamforming method [13] based on singular value pairing and block diagonalization of the effective channel matrix. These are done such that similar channel capacity is obtained between different pairs of subchannels (i.e., the capacity balancing concept) and a subspace GMD scheme requires only two layers of detection/decoding (alleviating the latency issue). The proposed subspace beamforming method can also be used to minimize complexity of ML detection and ML beamforming.

II. Background

A. System Model

Consider a MIMO system with N transmit antennas and M receive antennas. The receive signal is given by

$$\mathbf{r} = \mathbf{HFx} + \mathbf{n} \tag{1}$$

where $\mathbf{H}$ is the channel matrix of size $M \times N$, $\mathbf{F}$ is the beamforming (or precoding) matrix of size $N \times K$, $\mathbf{x}$ is the transmitted signal vector of size K (i.e., K multiplexed data streams), and $\mathbf{r}$ and $\mathbf{n}$ are the received signal and noise vectors, both of size M. In general, $K \leq \rho$, where ρ is the rank of $\mathbf{H}$. Denote the SVD of $\mathbf{H}$ as $\mathbf{H} = \hat{\mathbf{U}}\hat{\mathbf{S}}\hat{\mathbf{V}}^H$ (^{H} denotes Hermitian transpose), where $\hat{\mathbf{U}}$ and $\hat{\mathbf{V}}$ contain the left and the right singular vectors, respectively, and $\hat{\mathbf{S}}$ is a diagonal matrix whose diagonal elements $s_1 \geq s_2 \geq \cdots \geq s_\rho$ are the sorted non-zero singular values of $\mathbf{H}$. Without loss of generality, the optimum beamforming matrix $\mathbf{F}$ is written as $\mathbf{F} = \mathbf{VP}$, where $\mathbf{V}$ contains only the first K columns of $\hat{\mathbf{V}}$ in the case of rank reduction, and $\mathbf{P}$ is a second beamforming matrix of size $K \times K$, assumed to be an orthogonal (unitary and real) matrix in this paper. We can then rewrite (1) as

$$\mathbf{r} = \mathbf{HVPx} + \mathbf{n} = \mathbf{USPx} + \mathbf{n} \tag{2}$$

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where $\mathbf{U}$ and $\mathbf{S}$ are the possible rank-reduced versions of $\hat{\mathbf{U}}$ and $\hat{\mathbf{S}}$.

The Shannon capacity of the above MIMO system is

$$C = \log_2 \left| \mathbf{I} + \frac{1}{N_0}\mathbf{S}^2 \right| = \log_2 [\prod_{i=1}^{K} [1 + \frac{s_i^2}{N_0}]] \quad (3)$$

where N_0 is the noise power (assumed to be the same for all receive antennas).

B. Review of GMD

There are two types of GMD beamforming: (i) zero-forcing (ZF) GMD, and (ii) minimum mean-square error (MMSE) GMD (also known as “uniform channel decomposition (UCD)” [7]).

As with any system involving SIC, it is easy to understand GMD by first considering the ZF condition. The beamforming matrix $\mathbf{P}$ for ZF-GMD is obtained by decomposing the singular value matrix $\mathbf{S}$ into $\mathbf{S} = \mathbf{QLP}^T$ (^{T} denotes transpose), where all matrices are real matrices of size $K \times K$, $\mathbf{Q}$ and $\mathbf{P}$ are orthogonal matrices, and $\mathbf{L}$ is a lower triangular matrix with equal diagonal elements

$$l_{kk} = l = (\prod_{i=1}^{K} s_i)^{\frac{1}{K}} \quad (4)$$

Using the above $\mathbf{P}$ in (2) and mutiplying $(\mathbf{UQ})^H$ to both sides yields

$$\mathbf{y} = (\mathbf{UQ})^H \mathbf{r} = \mathbf{Lx} + (\mathbf{UQ})^H \mathbf{n} \quad (5)$$

Thus, the effective channel matrix is given by the lower-triangular matrix $\mathbf{L}$. With SIC operation (all elements below the diagonal elements of $\mathbf{L}$ are subtracted out), the effective channel matrix is diagonalized, and the K subchannels have equal output SNRs.

The above receiver operation is equivalent to ZF-SIC, therefore it does not achieve the channel capacity given by (3). Meanwhile, we can use MMSE-SIC instead; however, this will not yield equal subchannel SNRs. In order to both achieve the channel capacity and yield equal subchannel SNRs, the beamforming matrix $\mathbf{P}$ must be computed in a MMSE fashion:

$$(\mathbf{S}^2 + N_0\mathbf{I})^{\frac{1}{2}} = \mathbf{QLP}^T \quad (6)$$

Using $\mathbf{P}$ from the above decomposition in (2) and assuming a MMSE-SIC receiver, all subchannels will have equal output SNRs and the system achieves the channel capacity.

III. Subspace Beamforming

A. Subspace GMD

The subspace method involves block diagonalization of the effective channel matrix such that there are pairs of data streams that are detected independently of one another. For simplicity of notation, we only describe the case where the number of data streams K is even. When K is odd, the only difference is that there is one unpaired data stream left over, which is to be detected by itself with a simple linear MMSE receiver.

Again we explain how the method works under the ZF condition first, then describe extension to MMSE later. We begin by rewriting the received signal in (2) as

$$\mathbf{r} = \mathbf{H}\tilde{\mathbf{V}}\mathbf{Px} + \mathbf{n} = \tilde{\mathbf{U}}\tilde{\mathbf{S}}\mathbf{Px} + \mathbf{n} \quad (7)$$

where $\tilde{\mathbf{S}}$ is the singular value matrix whose diagonal elements are reordered as $\tilde{\mathbf{S}} = diag(s_1, s_K, s_2, s_{K-1}, \cdots, s_{K/2}, s_{K/2+1})$ and $\tilde{\mathbf{U}}$ and $\tilde{\mathbf{V}}$ are obtained by reordering the columns of $\mathbf{U}$ and $\mathbf{V}$ accordingly. We now rewrite $\tilde{\mathbf{S}}$ as a block diagonal matrix whose diagonal elements are submatrices of size 2: $\tilde{\mathbf{S}} = diag(\tilde{\mathbf{S}}_1, \tilde{\mathbf{S}}_2, \cdots, \tilde{\mathbf{S}}_{\frac{K}{2}})$; then perform GMD of each submatrix $\tilde{\mathbf{S}}_i = \tilde{\mathbf{Q}}_i\tilde{\mathbf{L}}_i\tilde{\mathbf{P}}_i^T$ (i.e., pairwise ZF-GMD). Accordingly,

$$\tilde{\mathbf{S}} = diag(\tilde{\mathbf{Q}}_1\tilde{\mathbf{L}}_1\tilde{\mathbf{P}}_1^T, \cdots, \tilde{\mathbf{Q}}_{\frac{K}{2}}\tilde{\mathbf{L}}_{\frac{K}{2}}\tilde{\mathbf{P}}_{\frac{K}{2}}^T) = \tilde{\mathbf{Q}}\tilde{\mathbf{L}}\tilde{\mathbf{P}}^T \quad (8)$$

Substituting (8) into (7), letting $\mathbf{P} = \tilde{\mathbf{P}}$, and multiplying $(\tilde{\mathbf{U}}\tilde{\mathbf{Q}})^H$ to both sides yields

$$\mathbf{y} = diag(\tilde{\mathbf{L}}_1, \tilde{\mathbf{L}}_2, \cdots, \tilde{\mathbf{L}}_{\frac{K}{2}})\mathbf{x} + (\tilde{\mathbf{U}}\tilde{\mathbf{Q}})^H\mathbf{n} \quad (9)$$

The effective channel matrix is block diagonal, therefore only pairwise SIC is needed.

The above subspace GMD only guarantees pairwise equal subchannel SNRs. The output SNRs between pairs depends on the geometric mean of the singular values that are paired together. The way we pair the i^{th} largest singular value with the i^{th} smallest singular value in each submatrix $\tilde{\mathbf{S}}_i$ is the best possible way to balance those output SNRs.

The MMSE version of subspace GMD is derived by replacing the pairwise ZF-GMD in the above scheme by the following pairwise MMSE-GMD, and using pairwise MMSE-SIC instead of pairwise ZF-SIC:

$$(\tilde{\mathbf{S}}_i^2 + N_0\mathbf{I})^{\frac{1}{2}} = \tilde{\mathbf{Q}}_i\tilde{\mathbf{L}}_i\tilde{\mathbf{P}}_i^T, i = 1, 2, \cdots, K/2 \quad (10)$$

The output SNR for the i^{th} pair of subchannels is given by

$$\tilde{\Gamma}_i = \frac{\sqrt{(s_i^2 + N_0)(s_{K-i+1}^2 + N_0)}}{N_0} - 1, i = 1, 2, \cdots, K/2 \quad (11)$$

B. Subspace ML

Optimum beamforming for ML detection is known to be an NP problem that requires exhaustive search and becomes prohibitively complex as K increases [14]. Furthermore, the complexity of ML detection itself grows exponentially with K. With subspace beamforming, however, the number of data streams that need to be jointly detected is limited to 2, therefore making ML problems more feasible to solve.

We again refer to the received signal in (7) and use the same block diagonal representations of the reordered singular value matrix $\tilde{\mathbf{S}}$ and the beamforming matrix $\mathbf{P}$ as before. Multiplying the received signal by $\tilde{\mathbf{U}}^H$ yields

$$\mathbf{y} = diag(\tilde{\mathbf{S}}_1\mathbf{P}_1, \tilde{\mathbf{S}}_2\mathbf{P}_2, \cdots, \tilde{\mathbf{S}}_{\frac{K}{2}}\mathbf{P}_{\frac{K}{2}})\mathbf{x} + \tilde{\mathbf{U}}^H\mathbf{n} \quad (12)$$

Here we can let $\mathbf{P}_i$ $(i = 1, \cdots, \frac{K}{2})$ be a Givens rotation $\mathbf{G}(\theta_i)$ whose angle θ_i is to be optimized with respect to $\tilde{\mathbf{S}}_i = diag(s_i, s_{K-i+1})$.

In this paper, we consider two methods for optimizing the 2×2 ML beamforming angle θ_i. The first method [14], called the "$\mathrm{d_{min}}$" method, is based on maximizing the minimum distance between received vectors:

$$\max_{\theta_i} \min_{\mathbf{x}_j, \mathbf{x}_k} d(\mathbf{x}_j, \mathbf{x}_k) \tag{13}$$

where $\mathbf{x}_j$ and $\mathbf{x}_k$ are two possible different transmitted symbol vectors, and $d(\mathbf{x}_j, \mathbf{x}_k) = \left\| \tilde{\mathbf{S}}_i \tilde{\mathbf{P}}_i (\mathbf{x}_j - \mathbf{x}_k) \right\|$. The second method, called the "BER" method, is based on minimizing the uncoded bit error rate (BER) of ML detection for a given channel $\tilde{\mathbf{S}}_i$. The $\mathrm{d_{min}}$ method is relatively simple to compute and can be stored in a set of 1-dimension look-up table with respect to the condition number $\tilde{s}_1/\tilde{s}_2$, where $\tilde{s}_1 \geq \tilde{s}_2$ here represent the two diagonal elements of any 2×2 singular value matrix $\tilde{\mathbf{S}}_i$. The BER method requires exhaustive simulation trials (with random Gaussian noise samples) and a 2-dimension look-up table to store the optimum angle value as a function of $\tilde{s}_1/\tilde{s}_2$ and the SNR $(\tilde{s}_1^2 + \tilde{s}_2^2)/N_0$. In terms of performance, the $\mathrm{d_{min}}$ method is expected to be inferior to the BER method, as the minimum distance criterion is based on approximation at high SNR.

Figs. 1(a) and (b) show examples of the optimum beamforming angle θ (representing θ_i) based on the two methods for 16QAM. Note that only angles in the range $[0, \pi/4]$ need to be considered, due to the symmetry of the received vectors for QAM. In Fig. 1(a), we also plot the Givens rotation angle $\theta = tan^{-1}\sqrt{\tilde{s}_1/\tilde{s}_2}$ for 2×2 ZF-GMD. We will show later that, despite the simplicity of ZF-GMD beamforming and the fact that it is optimized for a different receiver (i.e., ZF-SIC), this scheme actually performs quite well with subspace ML and 2×2 ML in general.

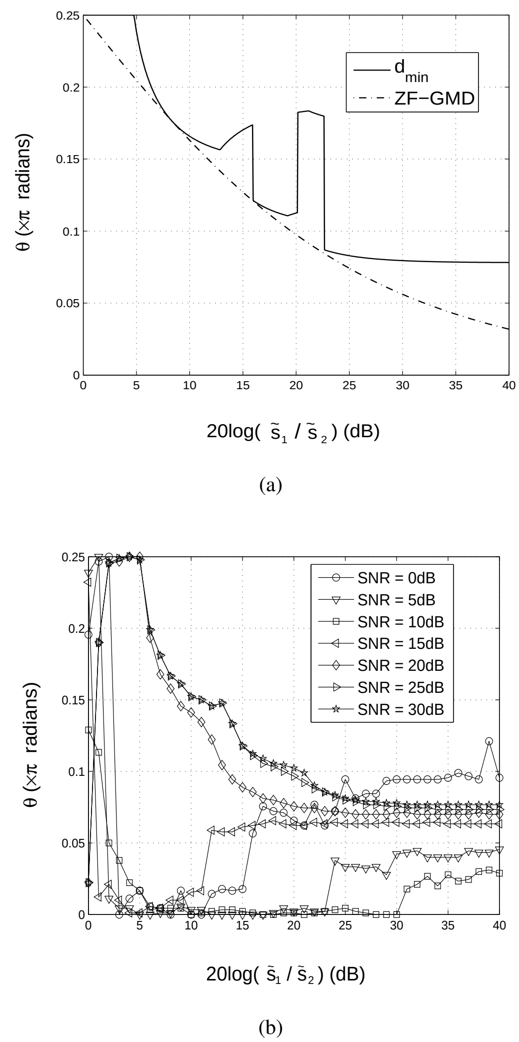

Fig. 1. Optimum 2×2 ML beamforming angle θ based on (a) the $\mathrm{d_{min}}$ method and GMD, and (b) the BER method. 16QAM

IV. Performance Examples

A. Subspace GMD Results

We present simulation results to demonstrate the performance of subspace beamforming. Our simulation model is based on the orthogonal frequency-division multiplexing (OFDM) system for the 3rd Generation Partnership Project-Long Term Evolution (3GPP-LTE) [15]. 16QAM with rate 1/2 turbo coding is assumed. Figs. 2 and 3 show the packet error rate (PER) as a function of the average SNR for 3×3 and 4×4 MIMO systems with i.i.d. flat Rayleigh fading. No rank reduction is assumed. Each packet occupies 7 OFDM symbols, with 72 assigned subcarriers per OFDM symbol. The number of information bits in each packet is on the order of 3000 for the 3×3 , and 4000 for the 4×4 system. In each figure, we plot the performances of (i) full GMD with parallel encoding (K codewords for K subchannels); (ii) subspace GMD with parallel encoding; (iii) subspace GMD with 2 codewords, where multiple data streams belonging to the same SIC layer are coded together together (i.e., all the odd streams in (9) belong to the first layer and they are coded and interleaved as one codeword; all the remaining streams belong to the second layer and are coded and interleaved as another codeword); (iv) the Shannon outage bound—the probability that the specified spectral efficiency (6 b/s/Hz for the 3×3 , and 8 b/s/Hz for the 4×4 system) is not supported by the Shannon MIMO channel capacity in (3); and (v) the subspace GMD bound—the probability that the specified spectral efficiency per data stream (2 b/s/Hz) is not supported by the capacity of any of the subspace GMD's subchannel (this is a lower bound for the PER of subspace GMD with parallel encoding). We see in Figs. 2 and 3 that full GMD performs to within about 2 dB of the Shannon bound (i.e., the SISO gap). Similarly, subspace GMD with parallel encoding performs to within about 2 dB of the subspace GMD bound. The gap between subspace GMD with parallel encoding and full GMD (or the gap between the two bounds) indicates the degree to which capacity balancing through singular value pairing deviates from perfect balance. The 3×3 system (with one unpaired data stream) shows a bigger gap than the 4×4 system. Subspace GMD with 2 codewords performs better than subspace GMD with parallel encoding because it benefits from coding across multiple data

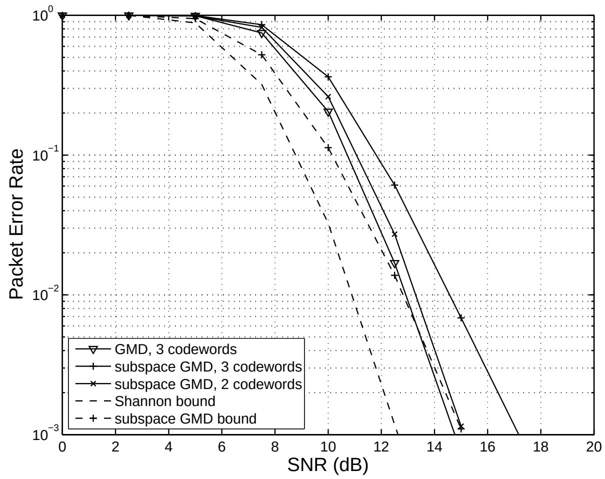

Fig. 2. PER of subspace GMD beamforming for a 3x3 MIMO system. Flat fading. 16QAM with rate 1/2 turbo coding

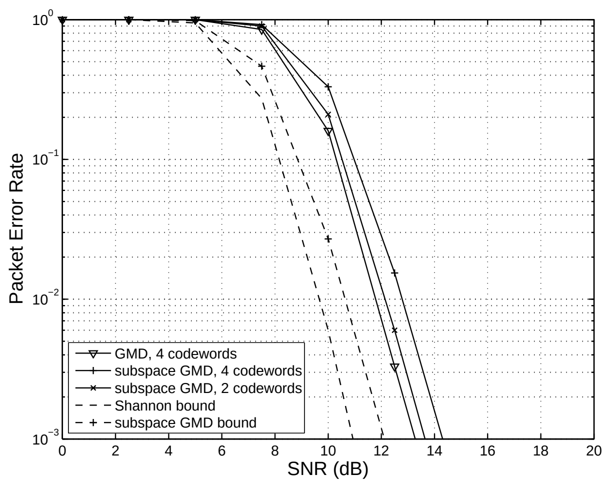

Fig. 3. PER of subspace GMD beamforming for a 4x4 MIMO system. Flat fading. 16QAM with rate 1/2 turbo coding

streams (it also benefits somewhat from the resulting longer block size for turbo decoding). Therefore, not only does this scheme alleviate the latency issue, it also gives superior performance.

B. Subspace ML Results

Before presenting results for the subspace ML approach, we first compare performance of the different 2×2 ML beamforming schemes described in Section III-B. Fig. 4 shows PER performance for a 2×2 MIMO system with ML detection. All ML results assume all substreams to be coded and interleaved together as one codeword. For reference, we provide benchmark performances, the corresponding Shannon outage bound and the GMD performance, as we did previously; again GMD performance is within the SISO gap of about 2 dB from the Shannon bound. We also plot the PER for "open-loop" ML, i.e., ML detection without transmitter beamforming. The open-loop ML shows a gap as large as 6.5 dB from the Shannon bound at 1% PER. Transmitter beamforming can significantly reduce this gap. Among the three schemes, "ML

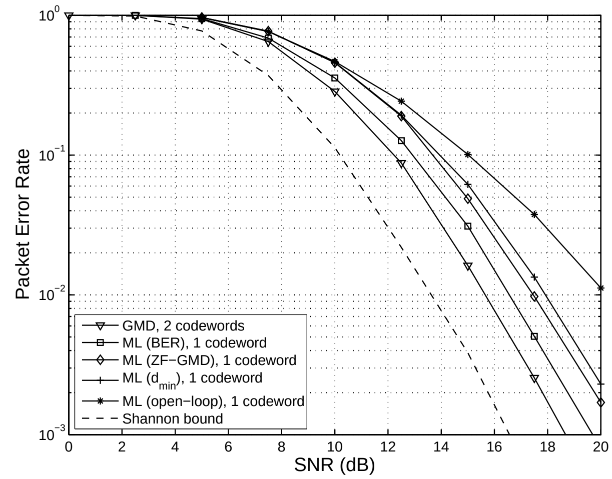

Fig. 4. PER of ML beamforming for a 2×2 MIMO system. Flat fading. 16QAM with rate 1/2 turbo coding

(BER)" using the BER method gives the best performance and it performs to within only 3 dB of the Shannon bound. "ML (ZF-GMD)" based on ZF-GMD beamforming also performs quite well despite its ad hoc nature—it even outperforms "ML ($\mathrm{d_{min}}$)" which uses the $\mathrm{d_{min}}$ method. As for the sizes of the lookup tables used for the BER and the $\mathrm{d_{min}}$ methods in this simulation, the optimum angle values were computed and stored as a function of $\tilde{s_1}/\tilde{s_2}$ in 1 dB steps from 0 to 35 dB, and (for the BER method only) as a function of the SNR $(\tilde{s_1}^2 + \tilde{s_2}^2)/N_0$ in 1 dB steps from -2 to 33 dB.

Figs. 5 and 6 show performance of subspace ML beamforming for 3×3 and 4×4 MIMO systems. Again for reference, we provide the Shannon outage bound and the GMD performance (these are the same curves as shown in Figs. 2 and 3), and also the open-loop performance with full ML detection. The subspace ML schemes use the same three 2×2 beamforming methods as shown in Fig. 4. Overall, we see similar trends of performance as before: a gap of 6 dB or more between open-loop full ML and the Shannon bound; the BER method being the best among the three ML beamforming methods, with the ZF-GMD method being second, and the $\mathrm{d_{min}}$ method being last. Furthermore, the performance with the BER method continues to be within 3 dB of the Shannon bound, similar to results in the 2×2 case. This may seem as though the proposed subspace approach is *not* suboptimum after all. In reality, we know that the subspace approach is suboptimum and the fact that the gap from the Shannon bound remains the same (or even becomes smaller) is rather due to the higher diversity order as we go from a 2×2 system to 3×3 and 4×4 systems. We also note that subspace ML using the BER method has essentially the same performance as subspace GMD with 2 codewords in Figs. 2 and 3, even though subspace ML does not have the latency issue associated with SIC that subspace GMD does.

C. Performance with a Weaker Code

Finally we provide results for a higher code rate to demonstrate that the performance of subspace beamforming is not

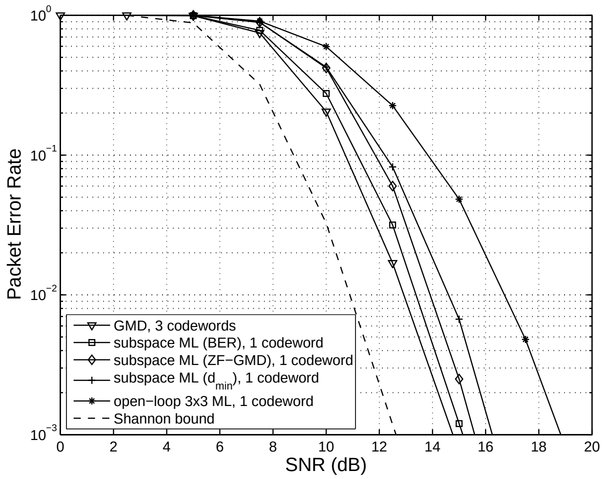

Fig. 5. PER of subspace ML beamforming for a 3×3 MIMO system. Flat fading. 16QAM with rate 1/2 turbo coding

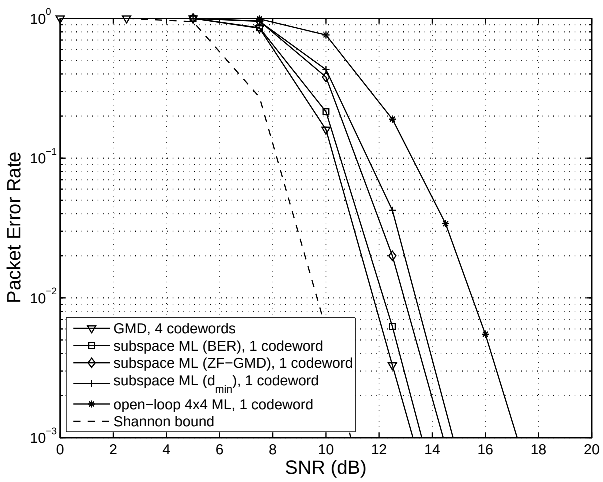

Fig. 6. PER of subspace ML beamforming for a 4×4 MIMO system. Flat fading. 16QAM with rate 1/2 turbo coding

greatly affected by weaker codes. So far we have assumed the use of a rate 1/2 turbo code and we know for a fact that subspace GMD with 2 codewords achieves a performance benefit from coding across subchannels. Most likely subspace ML must also gain some performance benefit from coding across subchannels because it uses a single codeword structure. Fig. 7 shows performance of these subspace schemes for a 3×3 MIMO system with rate 3/4 turbo coding. We see that performance for subspace GMD with 2 codewords and subspace ML using the BER method indeed show a bigger gap from the Shannon bound than results with rate 1/2 coding in Figs. 2 and 5. Still both schemes continue to perform to within 3 dB of the Shannon bound and have similar performance to one another. This is an indication that the singular value pairing to balance capacity is in effect and all subchannels have quite (but not completely) uniform detection performance.

V. Conclusions

We have presented a subspace beamforming method that decomposes a MIMO channel into multiple pairs of sub-

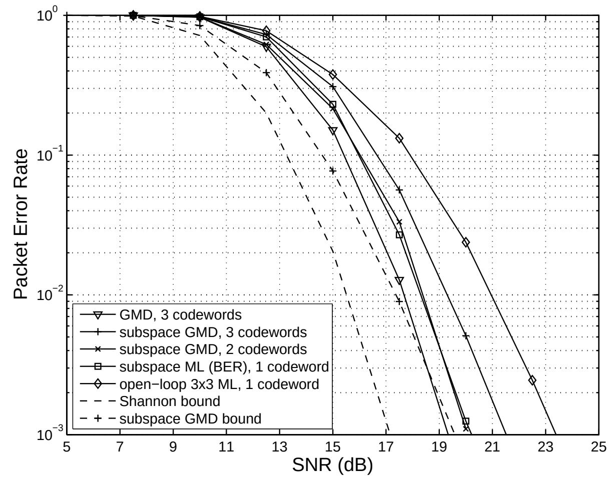

Fig. 7. PER of subspace GMD beamforming and subspace ML beamforming for a 3×3 MIMO system. Flat fading. 16QAM with rate 3/4 turbo coding

channels. The purpose is to avoid the complexity of joint detection and/or the latency of having many SIC layers. We have introduced a new capacity balancing concept, where the pairing of subchannels is done based on singular values such that similar channel capacity is obtained between different subchannel pairs. This concept is key to achieving high performance with low complexity. We have described a subspace GMD scheme with 2 codewords, which not only alleviates the latency issue, but also gives superior performance compared to subspace GMD with parallel encoding. We have also applied the subspace idea to ML detection and showed how it makes the optimization of ML beamforming and ML detection itself feasible for any $K \times K$ MIMO system. Simulation results show that subspace GMD with 2 codewords and subspace ML based on a proposed 2×2 ML beamforming method perform nearly as well as optimum GMD performance, and to within only a few dB of the Shannon bound.

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