Multi-User Diversity vs. Accurate Channel Feedback for MIMO Broadcast Channels

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***Abstract*— A multiple transmit antenna, single receive antenna (per receiver) downlink channel with limited channel feedback is considered. Given a constraint on the total system-wide channel feedback, the following question is considered: is it preferable to get low-rate feedback from a large number of receivers or to receive high-rate/high-quality feedback from a smaller number of (randomly selected) receivers? Acquiring feedback from many users allows multi-user diversity to be exploited, while high-rate feedback allows for very precise selection of beamforming directions. It is shown that systems in which a limited number of users feedback high-rate channel information significantly outperform low-rate/many user systems. While capacity increases only double logarithmically with the number of users, the marginal benefit of channel feedback is very significant up to the point where the CSI is essentially perfect.**

I. Introduction

Multiple antenna broadcast channels have been the subject of a tremendous amount of research since the seminal work of Caire and Shamai showed the sum-rate optimality of dirty-paper precoding (DPC) with Gaussian inputs [1]. If the transmitter is equipped with M antennas, then multi-user MIMO techniques (such as DPC or sub-optimal but low-complexity linear precoding) that allow simultaneous transmission to multiple users over the same time-frequency resource can achieve a multiplexing gain of M (as long as there are M or more receivers) even if each receiver has only one antenna. In contrast, orthogonal techniques (such as TDMA) that only serve one user achieve a multiplexing gain of only one.

Since the multiple antenna broadcast channel is a very natural model for many-to-one communication (e.g., a single cell in a cellular system), this line of work has been of great interest to both academia and industry. The multiple antenna broadcast channel with *limited channel feedback* has been of particular interest over the past few years because this accurately models the practical scenario where each receiver feeds back (imperfect) channel information to the transmitter. In a frequency-division duplexed system (or a time-division duplex system without accurate channel reciprocity) channel feedback is generally the only mechanism by which the transmitter can obtain channel state information (CSI). In the single receive antenna setting, most proposed feedback strategies either directly or indirectly involve each receiver quantizing its M-dimensional channel vector to the closest of a set of quantization vectors; finer quantization corresponds to a larger set of quantization vectors and thus higher rate channel feedback.

Within the literature on the MIMO broadcast with limited feedback, there has been a dichotomy between the extremes of systems with a small number of receivers (on the order of the number of transmit antennas) versus systems with an extremely large number of receivers.

- *Finite systems* have been shown to be *extremely* sensitive to the accuracy of the CSIT, and thus require *high-rate feedback*. This has been shown from a fundamental information theoretic perspective [2], as well as in terms of particular transmit strategies. In particular, zero-forcing beamforming has been shown to require CSIT quality that scales proportional to SNR [3][5].
- *Large systems* have been shown to be able to operate near capacity with extremely *low-rate channel feedback* in the asymptotic limit as the number of users is taken to infinity. In particular, *random beamforming* (RBF) [6] can operate with only $\log_2 M$ bits of feedback per user (plus one real number). The performance of this technique in the asymptotic limit is quite amazing: not only does the ratio of random beamforming throughput to perfect CSIT capacity converge to one as the number of users is taken to infinity, but the difference between these quantities actually has been shown to converge to zero [7].

Finite systems require high-rate feedback because imperfect CSIT leads to multi-user interference that cannot be resolved at each receiver. In order to prevent such a system from becoming interference-limited, the CSIT must be very accurate; in terms of channel quantization, this corresponds to using a very rich quantization codebook that allows the direction of each receiver's channel vector to be very accurately quantized. In large systems, on the other hand, *multi-user diversity* is exploited to allow the system to operate with extremely low levels of feedback. The RBF strategy involves a quantization codebook consisting of only M orthonormal vectors (e.g., the elementary basis vectors). If such a codebook is used with a small user population, each user's quantization will likely be quite poor due to the limited size of the quantization codebook. However, as the number of users increases, it becomes more and more likely that at least some of the users have channel vectors that lie very close to one of the M quantization vectors. This effect allows the system to get by with very low rate feedback. Although the RBF throughput does converge in the strong absolute sense to the perfect CSIT capacity, convergence is extremely slow, even for systems with a small number of transmit antennas.

Motivated by the apparent dichotomy between finite and asymptotically large MIMO broadcast systems with limited channel feedback, in this paper we ask the following simple question:
Is it preferable to have a system with a large number of receivers and low-rate feedback from each receiver (thereby exploiting multi-user diversity), or to have a system with a smaller number of receivers with high-rate feedback from each receiver (thereby exploiting the benefits of accurate CSIT)?

In order to fairly compare these systems, we equalize the total number of channel feedback bits (across users). Assuming that a total of T feedback bits are used, we compare the following:

- Random beamforming (RBF) is used with $\frac{T}{\log_2 M}$ receivers feeding back $\log_2 M$ bits each (in addition to one real number).
- $\frac{T}{B}$ receivers quantize their channel direction to B bits and feed back this information (plus one real number) to the transmitter, who uses a low-complexity user selection plus zero-forcing transmission strategy. The parameter B is varied within $\log_2 M \leq B \leq \frac{T}{M}$.

In performing this comparison, we assume the subset of users who feedback are selected according to some *channel-independent* criterion. For example, they could be completely randomly selected beforehand by the base station or the subset could be chosen as the users with the largest user weights in a weighted sum rate maximization setting.

Our main conclusion is simple but striking: for almost any number of antennas M and SNR level, **system throughput is maximized by choosing B (feedback bits per user) such that near-perfect CSIT is obtained for each of $\frac{T}{B}$ users that do feedback.** For example, in a 4 antenna ($M = 4$) system operating at 10 dB with $T = 100$ bits, the optimal is (approximately) achieved by having 4 users feedback 25 bits each, and the advantage relative to RBF (which involves 50 users feeding back $\log_2 M = 2$ bits each) is approximately 2.8 bps/Hz (9.6 vs. 6.8 bps/Hz). Note that $B = 25$ corresponds to CSIT at approximately 99.7% accuracy, which is orders of magnitude more accurate than current wireless systems. For larger values of T, the optimum is still achieved in the neighborhood of $B = 25$, i.e., a fraction of the user population feed back very accurate CSI, and the significant performance advantage is maintained even for very large values of T. For relatively small values of T, the optimal B is reduced because it is still desirable to have at least M users feedback, but high-rate quantization from a small number of users is still desirable (e.g., for $T = 40$ having 4 users feedback 10 bits gives a considerably larger throughput than RBF with 20 users). Multi-user diversity provides a throughput gain that is only double-logarithmic in the number of users (who feedback CSI), while the marginal benefit of increased channel feedback is much larger up to the point where essentially near-perfect CSIT (relative to the system SNR) is achieved (e.g., 25 bits when $M = 4$ and the system is at 10 dB).

II. Prior Work

Previous work [8][9][10][11] has studied situations where the individual receivers determine whether or not to feedback on the basis of their current channel conditions (i.e., channel norm and quantization error). If each receiver makes channel-dependent decisions then the base station (transmitter) does not *a priori* know who is going to feedback or how many users will feedback, which could potentially complicate system design (possible solutions include using random-access for feedback or somehow piggybacking the variable feedback load onto uplink data packets). From only a throughput maximization perspective, one would intuitively think that making channel-dependent feedback decisions would perform better than channel-independent decisions, because only users with strong channels and good quantization feed back. However, there are other scenarios where channel-independent selection of users would be preferable, e.g., when users have delay-sensitive traffic and are requested to feed back when their deadlines are approaching. There are many important differences between the approaches and both have their strengths and weaknesses. In this work, we consider only channel-independent approaches, although we expect to compare against channel-dependent approaches in the future.

Another recent work has studied the tradeoff between multi-user diversity and accurate channel feedback in the context of two-stage feedback [12]. In the first stage, all users feed back coarse estimates of their channel, based on which the transmitter runs a selection algorithm to select M users who feedback more accurate channel quantization during the second feedback stage. Our work differs in that we consider only a *single stage* approach, and more importantly in that we optimize the number of users (T/B randomly selected users) who feed back accurate information rather than limiting this number to M. Indeed, this optimization is precisely why our approach shows such large gains over naive RBF or un-optimized zero forcing.

III. System Model & Background

We consider a multi-input multi-output (MIMO) Gaussian broadcast channel in which the Base Station (BS) or transmitter has M antennas and each of the K users have 1 antenna each. The channel output y_k at user k is given by:

$$y_k = \mathbf{h}_k^{\mathsf{H}} \mathbf{x} + z_k, \quad k = 1, \ldots, K \tag{1}$$

where $z_k \sim \mathcal{CN}(0, 1)$ models Additive White Gaussian Noise (AWGN), $\mathbf{h}_k \in \mathbb{C}^M$ is the vector of channel coefficients from the k^{th} user antenna to the transmitter antenna array and $\mathbf{x}$ is the vector of channel input symbols transmitted by the base station. The channel input is subject to the average power constraint $\mathbb{E}[|\mathbf{x}|^2] \leq P$.

We assume that the channel *state*, given by the collection of all channel vectors $\mathbf{H} = [\mathbf{h}_1, \ldots, \mathbf{h}_K] \in \mathbb{C}^{M \times K}$, varies in time according to a block-fading model, where $\mathbf{H}$ is constant over each *frame*, and evolves from frame to frame according to an ergodic stationary spatially white jointly Gaussian process, where the entries of $\mathbf{H}$ are Gaussian i.i.d. with elements $\sim \mathcal{CN}(0,1)$.

Each user is assumed to know its own channel perfectly. At the beginning of each block, each user quantizes its channel to B bits and feeds back the bits perfectly and instantaneously to the access point. Vector quantization is performed using a codebook $\mathcal{C}$ that consists of 2^B M-dimensional unit norm vectors $\mathcal{C} \triangleq \{\mathbf{w}_1, \ldots, \mathbf{w}_{2^B}\}$. Each user quantizes its channel vector to the quantization vector that forms the minimum angle to it. Thus, user k quantizes its channel to $\widehat{\mathbf{h}}_k$, chosen according to:

$$\widehat{\mathbf{h}}_k = \arg\min_{\mathbf{w}\in\mathcal{C}} \sin^2(\angle(\mathbf{h}_k, \mathbf{w})). \quad (2)$$

and feeds the quantization index back to the transmitter. In addition to this, each user also feeds back a single real number, which can be the channel norm, or some other channel quality indicator.

We assume that a total of T bits are allocated for feedback, and that there are at least $\frac{T}{\log_2 M}$ users available to feedback CSI, if needed. The following feedback strategies are considered:

A. Random Beamforming

The Random beamforming scheme proposed in [6] is used, where each user feeds back $\log_2 M$ bits along with one real number. The number of users feeding back information is hence $\frac{T}{\log_2 M}$. In this case, $\mathcal{C}$ consists of M orthogonal unit vectors, and the codebook is common to all users. In addition to the quantization index, each user feeds back a real number representing its SINR. If $\mathbf{w}_m$ ($1 \le m \le 2^B = M$) is selected to be the 'best' quantization vector for user k, where $1 \le k \le \frac{T}{\log_2 M}$, the SINR for the user is:

$$\text{SINR}_{k,m} = \frac{|\mathbf{h}_k^{\mathsf{H}}\mathbf{w}_m|^2}{\frac{M}{P} + \sum\limits_{n\neq m} |\mathbf{h}_k^{\mathsf{H}}\mathbf{w}_n|^2}. \quad (3)$$

'Simple' user selection is used, i.e., the user with the highest SINR on each $\mathbf{w}_m$ is chosen, and $\mathbf{w}_1, \ldots, \mathbf{w}_M$ are used as the beamformers. This constitutes a simple and low-complexity user-selection algorithm.

B. PU2RC

A simplified version Per unitary basis stream user and rate control (PU2RC) is used, as described in [8]. Here, $\mathcal{C}$ consists of $2^{B-\log_2 M}$ 'sets' of orthogonal codebooks, where each orthogonal codebook consists of M randomly generated orthogonal unit vectors. This allows each user to specify a particular orthogonal set using $B - \log_2 M$ bits, and specify a particular beam within this set using $\log_2 M$ bits. This codebook is common to all users, and the information fed back is the same as RBF (i.e., quantization index as well as a real number representing the SINR).

User selection is performed as follows: for each of the orthogonal sets, the BS repeats the RBF user selection procedure, and then selects the orthogonal set which yields the highest rate. If $B = \log_2 M$, there is only a single orthogonal set, and this scheme reduces to ordinary RBF.

The parameter B is varied within $1 + \log_2 M \le B \le \frac{T}{M}$. In general, if $R_{\text{PU2RC}}(P, M, K, B)$represents the PU2RC rate for a system with M antennas at the transmitter, SNR P and K users, each feeding back B bits (in addition to the SINR value), the optimal B is found as follows:

$$B^{\text{OPT}}_{\text{PU2RC}} = \underset{1+\log_2 M \le B \le \frac{T}{M}}{\text{argmax}} \; R_{\text{PU2RC}}\left(P, M, \frac{T}{B}, B\right) \quad (4)$$

C. Random Vector Quantization

We consider the case when $\frac{T}{B}$ users quantize their channel direction to B bits and feed back this information to the transmitter, along with the channel norm $||\mathbf{h}_k||^2$. Here, $\mathcal{C}$ consists of random unit-vectors independently chosen from the isotropic distribution on the M-dimensional unit sphere [4] (random vector quantization or RVQ). Each user is assumed to use a different and independently generated codebook[1]. The transmitter uses low-complexity greedy user selection [13] along with zero-forcing transmission, where the quantized channel (i.e., the channel $||\mathbf{h}_k|| \cdot \widehat{\mathbf{h}}_k$) is treated as if it were the true channel, for user selection purposes. We consider only the case when the channel norm information $||\mathbf{h}_k||^2$ is fed back, as opposed to (the receiver's estimate of) the SINR, which may take quantization error into account [14].

The parameter B is varied within $1 + \log_2 M \le B \le \frac{T}{M}$. In general, if $R_{\text{ZF-RVQ}}(P, M, K, B)$ represents the ZF rate for a system with M antennas at the transmitter, SNR P and K users, each feeding back B bits (in addition to one real number), the optimal B is found as follows:

$$B^{\text{OPT}} = \underset{1+\log_2 M \le B \le \frac{T}{M}}{\text{argmax}} \; R_{\text{ZF-RVQ}}\left(P, M, \frac{T}{B}, B\right) \quad (5)$$

Random beamforming involves the maximum number of users $\left(\frac{T}{\log_2 M}\right)$ but the minimum number of feedback bits per user ($\log_2 M$), while the ZF and PU2RC strategies can vary from a large system with low-rate feedback ($B = 1 + \log_2 M$) all the way to a small system with very high-rate feedback (M users, $B = T/M$).

IV. Basic Results and Discussion

To gain an understanding of the optimal B, we propose the following approximate characterization for ZF with RVQ. It is assumed that M users are selected for beamforming and uniform (equal) power allocation is used. Let $R^k_{\text{ZF-RVQ}}$ be the rate for user k in a system with M antennas at the transmitter,

[1]Note that random vector quantization allows us to simulate large quantization codebooks using the statistics of the quantization error, allowing for Monte Carlo simulations to be used

SNR P and K users, each feeding back B bits. Then,

$$\begin{aligned} R^k_{\text{ZF-RVQ}} &= \mathbb{E}\log_2\left(1+\frac{|\mathbf{h}_k^{\mathsf{H}}\mathbf{w}_k|^2}{\frac{M}{P}+\sum\limits_{j\neq k}|\mathbf{h}_k^{\mathsf{H}}\mathbf{w}_j|^2}\right)\\ &\geq \mathbb{E}\log_2\left(\frac{P}{M}|\mathbf{h}_k^{\mathsf{H}}\mathbf{w}_k|^2\right)\\ &\quad -\mathbb{E}\log_2\left(1+\frac{P}{M}\sum_{j\neq k}|\mathbf{h}_k^{\mathsf{H}}\mathbf{w}_j|^2\right)\\ &\overset{(a)}{\approx} \mathbb{E}\log_2\left(\frac{P}{M}\log_2(K)\right)\\ &\quad -\mathbb{E}\log_2\left(1+\frac{P}{M}\log_2(K)\sum_{j\neq k}\frac{|\mathbf{h}_k^{\mathsf{H}}\mathbf{w}_j|^2}{||\mathbf{h}_k||^2}\right)\\ &\overset{(b)}{\geq} \log_2\left(\frac{P}{M}\log_2(K)\right)\\ &\quad -\log_2\left(1+\frac{P}{M}\log_2(K)\,2^{-\frac{B}{M-1}}\right) \end{aligned} \tag{6}$$

Here, (a) follows from the fact that $|\mathbf{h}_k^{\mathsf{H}}\mathbf{w}_k|^2$ and $||\mathbf{h}_k||^2$ grow as $\log_2(K)$ with user selection for large K [15], and (b) follows by Jensen's inequality and applying results from [3].

Hence, we model the rate expression in terms of the parameters P, M, B and T as follows:

$$\begin{aligned} R_{\text{ZF-Approx}}\left(P,M,\frac{T}{B},B\right) &= M\log_2\left(\frac{P}{M}\log_2\left(\frac{T}{B}\right)\right)\\ &\quad -M\log_2\left(1+\frac{P}{M}\log_2\left(\frac{T}{B}\right)2^{-\frac{B}{M-1}}\right) \end{aligned} \tag{7}$$

The $M\log_2\left(\frac{P}{M}\log_2\left(\frac{T}{B}\right)\right)$ term captures the effect of multiuser diversity due to $\frac{T}{B}$ users (as well as appropriate scaling with SNR and M) for ZF with perfect CSIT. This is asymptotically correct, to an $O(1)$ term [15]. The $M\log_2\left(1+\frac{P}{M}\log\left(\frac{T}{B}\right)2^{-\frac{B}{M-1}}\right)$ term serves to capture the throughput loss due to limited channel feedback, relative to perfect CSIT. The effect of finite rate feedback was quantified to be $\mathbb{E}\left[M\log_2\left(1+\frac{P}{M}|\mathbf{h}_k||^2 2^{-\frac{B}{M-1}}\right)\right]$ in [3], for a $K\leq M$ user system (i.e., without user selection). This is applied for a $K=\frac{T}{B}>M$ user system by noting that the quantization error remains unaffected in spite of $K>M$ users (as quantization error information is not fed back). However, we note that due to user selection, $\frac{P}{M}||\mathbf{h}_k||^2$ behaves as $\frac{P}{M}\log_2\left(\frac{T}{B}\right)$ when $\frac{T}{B}$ users are involved. This also captures the fact that keeping B fixed and taking T to ∞ (for a fixed P) will essentially nullify all multiuser diversity making the system interference limited, as described in [14].

Based on this expression, an approximate expression for B^{OPT} may be computed as:

$$\begin{aligned} \widehat{B}^{\text{OPT}} = \underset{B}{\operatorname{argmax}}\quad & \log_2\left(\log_2\left(\frac{T}{B}\right)\right) -\\ & \log_2\left(1+\frac{P}{M}\log_2\left(\frac{T}{B}\right)2^{-\frac{B}{M-1}}\right) \end{aligned} \tag{8}$$

The solution to this problem is obtained by solving:

$$\frac{M-1}{M}P2^{-\frac{\widehat{B}^{\text{OPT}}}{M-1}}\widehat{B}^{\text{OPT}}\left(\log_e\left(\frac{T}{\widehat{B}^{\text{OPT}}}\right)\right)^2=1 \tag{9}$$

This expression is obtained by equating the derivative of (8) to zero, and solving for B.

In Figure 1, the true throughput $R_{\text{ZF-RVQ}}\left(P,M,\frac{T}{B},B\right)$ and the approximation $R_{\text{ZF-Approx}}\left(P,M,\frac{T}{B},B\right)$ are plotted (versus B) for an $M=4$ system at 10 dB SNR with $T=150, 1000$ bits. For $T=150$, $B^{\text{OPT}}=\widehat{B}^{\text{OPT}}=19$ and for $T=1000$, $B^{\text{OPT}}=\widehat{B}^{\text{OPT}}=25$. In both cases, the approximation yields relatively accurate results. Also note that the throughput grows rapidly for smaller values of B, but falls relatively slowly after the optimal B has been attained, and there is not much difference in performance in this region.

Figure 2 depicts the behavior of B^{OPT} with T. $\widehat{B}^{\text{OPT}}$ is seen to reasonably capture the behavior of B^{OPT}, and this dependence is numerically found to be $B^{\text{OPT}}\sim O(\log(\log(T)))$. This intuitively makes sense, as this would mean that $2^{-\frac{B}{M-1}}\sim O(1/\log(T))$ which would compensate for the $\log_2\left(\frac{T}{B}\right)$ term in the interference portion of (8)[2]. Furthermore, this growth rate also implies that B^{OPT} grows extremely slowly for larger values of T, and one would prefer essentially the same feedback quality even if T is very large.

It is similarly observed that B^{OPT} scales linearly with the system SNR (as well as M), i.e., $B^{\text{OPT}}\sim O(M\log(P))$, which is seen in Figure 3. The approximate expression $\widehat{B}^{\text{OPT}}$ is seen to accurately model this behavior as well. Interestingly, this behavior of the number of feedback bits is the same as with an M-user system [3] (without user selection). Further, this also suggests that a smaller fraction of users should feedback as SNR grows, and at large SNR there would essentially be only M users feeding back with $\frac{T}{M}$ bits each.

V. Simulation Results

In a 4 antenna ($M=4$ system, Figure 4), $R_{\text{ZF-RVQ}}\left(P,M,\frac{T}{B},B\right)$ is plotted versus T for various values of B. For each choice of B, $\frac{T}{B}$ users feed back information. Random vector quantization with zero forcing and greedy selection are used, as described previously. This is compared with Random beamforming with a fixed codebook size of 2 bits. At an SNR of 10 dB with a total budget of $T=100$ bits for feedback, the optimal is (approximately) achieved when 4 users each feedback 25 bits worth of information.

For larger values of T, the optimum is still (approximately) achieved in the neighborhood of $B^{\text{OPT}}=25$, i.e., a fraction of the user population feed back very accurate CSI. It is seen that there is a significant performance advantage relative to RBF. This advantage is expected to diminish as T grows, but it is seen that the significant advantage is maintained even for very large values of T (5000 bits and above). The value of B^{OPT}

[2] It was observed in [14] that pure 'norm' information used for user selection (i.e., without taking the quantization error magnitude into account) would cause the system to become interference limited (as the number of users feeding back are taken to infinity). However, selection of an optimal B may be able to overcome this disadvantage.

grows very slowly beyond 25 as T increases, which agrees with the $O(\log(\log(T)))$ expression.

Figures 5, 6 and 7 compare the performance of optimized ZF with random vector quantization, optimized PU2RC and RBF for SNR values of 0, 5 and 10 dB respectively. While optimized ZF with RVQ performs better than RBF in all cases, optimized PU2RC emerges superior for an SNR of 0 dB, and is similar to ZF/RVQ at 10 dB. ZF with RVQ remains superior above 10 dB. It is to be noted that even at 0 dB, the optimal point for ZF with RVQ is achieved when each user feeds back 13 bits of information, which suggests that it is preferable to have highly accurate CSI even at low SNR.

Figure 8 depicts the variation of throughput with PU2RC as the number of feedback bits are varied. At $B = 2$ (for an $M = 4$ system), the system is identical to RBF. The optimal point for $T = 200$ bits is achieved at $B = 6$, with about 33 users in the system.

Finally, Figure 9 compares the performance of various strategies with varying M. It is noted that the advantage of ZF with RVQ is considerably enhanced for larger values of M. Hence, although for $M = 4$ (and below) PU2RC is superior at low SNR, this advantage diminishes as M increases.

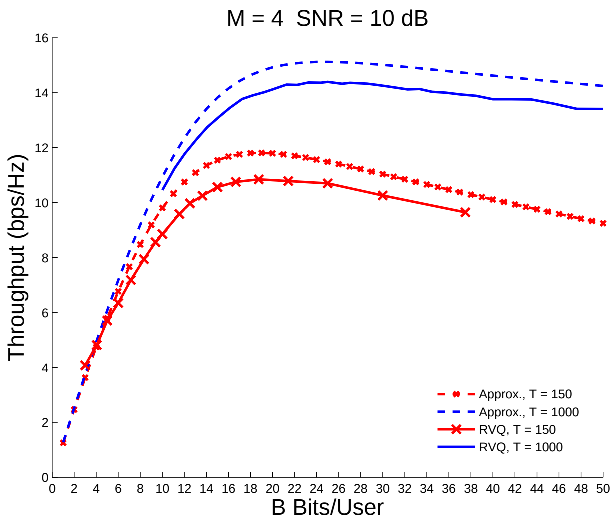

Fig. 1. Behavior of rate with B for RVQ

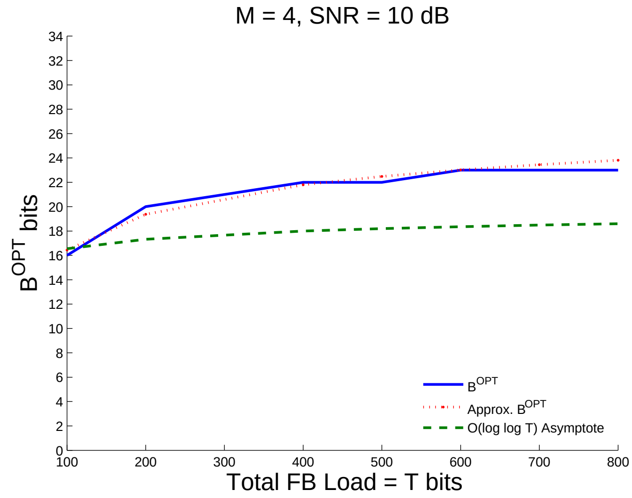

Fig. 2. Behavior of B^{OPT} with T

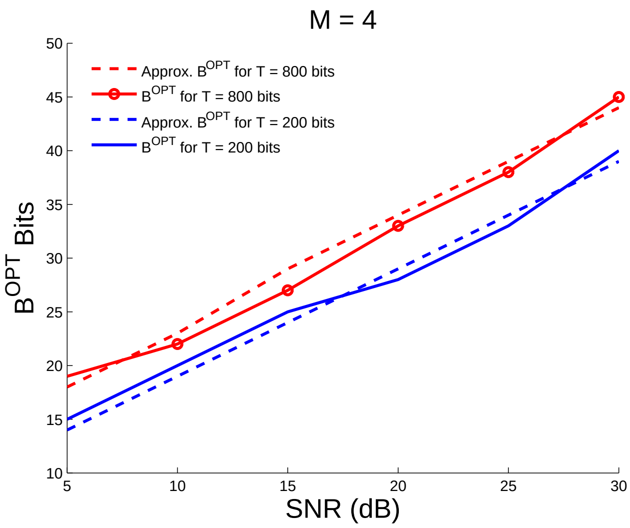

Fig. 3. Behavior of B^{OPT} with SNR P

VI. Conclusion

In this paper we have considered the very simple but apparently overlooked question of whether low-rate feedback/many user systems or high-rate feedback/limited user systems are preferable in the context of MIMO downlink channels. Answering this question essentially reduces to comparing the value of multi-user diversity (many users) versus channel information (high-rate feedback), and the surprising conclusion reached is that there is an extremely strong preference towards accurate channel information. Although there may be other issues that influence the design of channel feedback protocols, this work suggests that very high-rate channel feedback should receive serious consideration if multi-user MIMO techniques are employed on the downlink channel.

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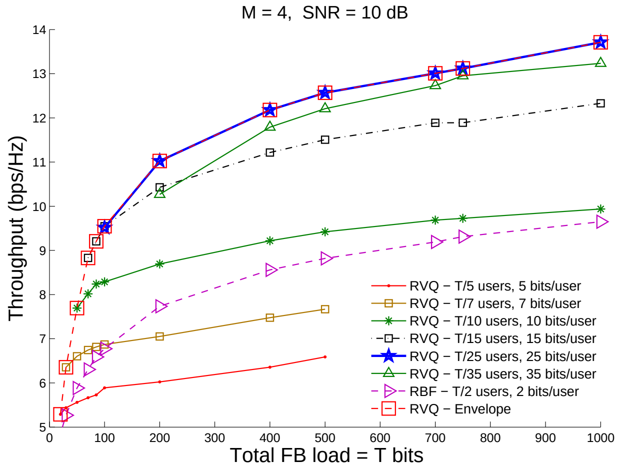

Fig. 4. RBF vs. Optimized number of feedback users, $M = 4$

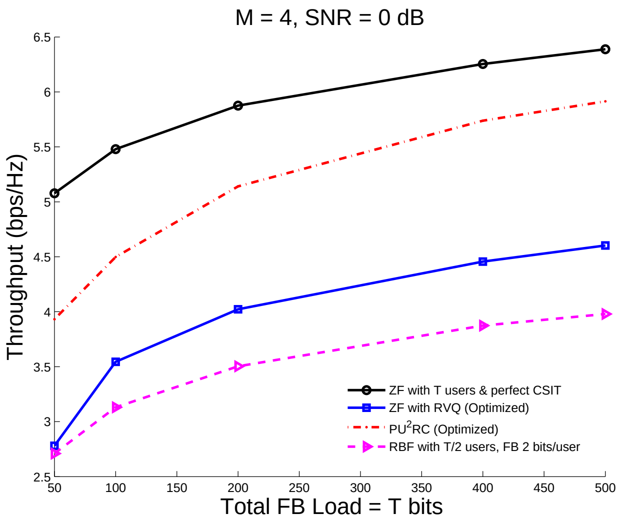

Fig. 5. RBF vs. Optimized number of feedback users, $M = 4, SNR = 0dB$

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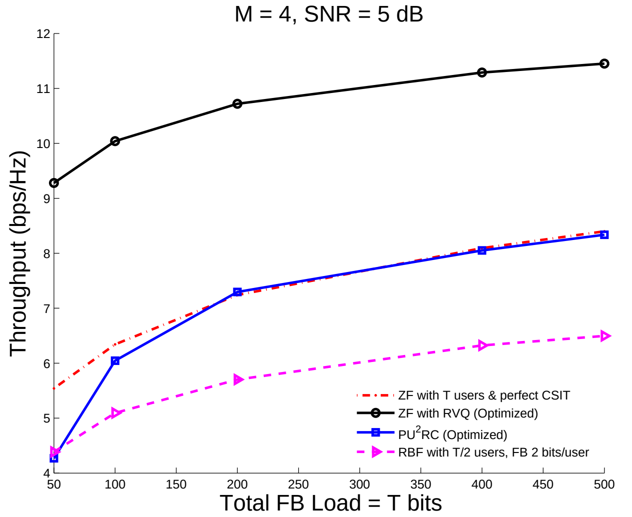

Fig. 6. RBF vs. Optimized number of feedback users, $M = 4, SNR = 5dB$

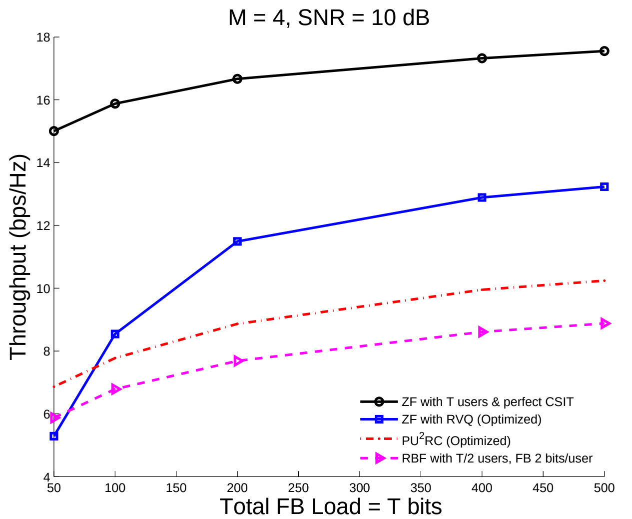

Fig. 7. RBF vs. Optimized number of feedback users, $M = 4, SNR = 10dB$

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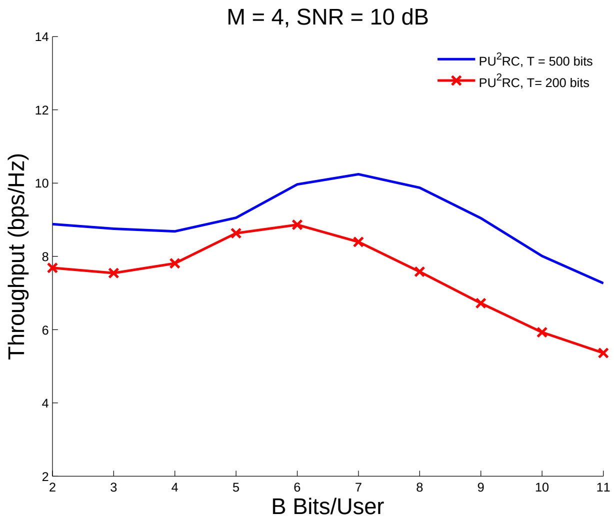

Fig. 8. Behavior of rate with B for PU^{2}RC

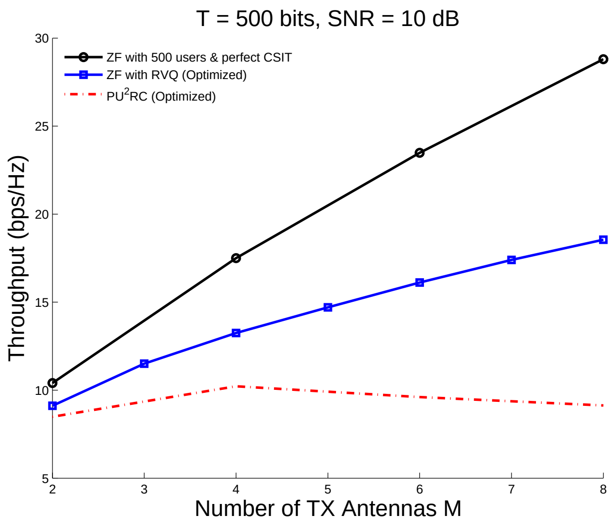

Fig. 9. RBF vs. Optimized number of feedback users - Variation with M, $T = 500bits, SNR = 10dB$