Information Rates for Multiantenna Systems with Unknown Fading

Oliver M. Collins, Krishnan Padmanabhan and Sundeep Venkatraman
Department of Electrical Engineering
University of Notre Dame, Notre Dame, Indiana 46556

Abstract—**This work first presents a general technique to compute tight upper and lower bounds on the information rate of a multiuser Rayleigh fading channel with no Channel State Information (CSI) at the transmitters or the receivers. The paper then presents analytical upper and lower bounds on the information rate which converge in the limit of large number of transmitting users where the channel bandwidth is large compared to the user data rates, e.g., when users can employ CDMA. In this limit, the capacity (per Hz) of the individual users is decreasing while the sum capacity (per Hz) is constant. The paper concludes with exact analytical expressions for the information rates of both the block fading and the continuous correlated fading channel models in this regime.**

1. Introduction

This paper considers a multi-user multi-receiver network where the channel between each transmitter receiver pair is an independent Rayleigh fading channel with no CSI at the transmitters or the receivers. It considers both a block fading model and a continuous correlated fading model. In the block fading case, the fading coefficients stay constant over a block and are independent over successive blocks. In the correlated fading case, the fading is modeled as an ergodic and stationary process defined by its power spectral density. The users each transmit independent signals which are also i.i.d in time and the receivers can cooperatively decode these transmitted signals. The paper presents upper and lower bounds on the information rate for this channel model under various scenarios. When the channel bandwidth is large compared to the users' data rates, we conjecture that i.i.d. signaling is capacity achieving.

Section III-A describes an optimal receiver scheme for the channel and signaling scheme described above. This receiver computes the exact a-posteriori probabilities (APPs)of the transmitted symbols by enumerating all future sequences. The achievable rate of this receiver is the information rate of the channel. However, this receiver has exponential complexity and hence its achievable rate cannot be computed exactly.

Section III-B modifies the optimal receiver to get upper and lower bounds on the information rate of the channel which are easy to compute. The lower bound (presented in section III-B) randomly prunes the set of strings of undecoded symbols to make the APP calculation easier. The achievable rate using this simplified APP is an obvious lower bound on constrained capacity just as any constructive mechanical solution would be. The upper bound in section III-B is based on a genie aided scheme which provides the estimator with additional side information about the undecoded symbols which eliminates the need for the exponentially complex enumerations. Since the availability of side information cannot decrease capacity, the mutual information obtained by using the genie aided APP estimates is an upper bound on the information rate of the fading channel. Plots of these bounds show that for very reasonable system parameters they are essentially coincident.

Section IV presents slightly different upper and lower bounds on the information rate and then shows that they converge in the limit where the system bandwidth and number of transmitting users increase proportionally while keeping each user's data rate (in bits/sec) constant. In this limit, each user is more and more spread and therefore, the information rate (in bits/sec/Hz) achieved by each user is vanishingly small. However, the aggregate spectral density (in W/Hz) remains constant and the aggregate information rate (in bits/sec/Hz) approaches a constant from below because the number of transmitting users is increased in proportion to the channel bandwidth. In other words, this limit represents a pure CDMA multiple access system. Section IV-A presents an analytical upper bound on the information rate of the multi-receiver network which is linear in the number of receivers. Section IV-B presents a lower bound on information rate which is also linear in the number of receivers when the system bandwidth is large. The section then proves that the single receiver bounds are tight in the limit, which proves that the upper and lower bounds coincide and the information rate of the multi-receiver network is linear with the number of receivers.

The section concludes with a closed form expression for the information rates of the block fading and brickwall fading channels.

The literature contains few results on capacity relevant to fading channels without Channel State Information(CSI) at moderate SNRs. Most papers that study the wideband regime consider either single-user systems or multi-user systems where the density of users is low enough that the users don't interfere with each other. The first rigorous study of the capacity of multipath fading channels in the wideband regime was done in [5] by Telatar and Tse. This paper studied the multipath channel with a fixed number of resolvable paths and presented results on the capacity and mutual information of such channels in the wideband regime. Verdu ([6]) studied the capacity of Rayleigh fading channels with known and unknown CSI and presented results on the tradeoff between spectral efficiency and energy per bit in the wideband regime. For the non-coherent Rayleigh flat fading channel, Subramaniam and Hajek[7] show that, under an average power constraint, the optimal input distribution for the wideband regime is bursty. However, such signaling is not possible in practical communication systems. Hence, Sethuraman and Hajek([8] and [9]) impose an additional peak power constraint and obtain bounds on the capacity which are asymptotically tight in the wideband regime. However, the wideband regime that Sethuraman and Hajek consider is very restrictive because the sum capacity is itself vanishingly small in the limit and so ideas of multiple access disappear. This paper preserves the essential character of the multi-user channel by increasing the number of transmitting users in proportion to the signaling bandwidth. As a result, even though the per user information rates in bits/sec/Hz are decreasing in the limit, the sum of the information rates of all users is a constant.

There are also a few results related to the numerical computation of information rate for different channels. Arnold et al. ([10]) present a simulation based technique to compute tight bounds on the information rates of finite state channels with memory. Extending this approach to non-finite state channels requires choosing a finite state channel model that approximates the channel of interest. The accuracy of the choice of finite state channel affects the tightness of the bounds. For the non-coherent case, Li and Collins([1] and [2]) consider the point to point Rayleigh flat fading channel and provide simple bounds on information rate which are tight at low SNR. Gopalan et al. ([3] and [4]) extend these bounds to the MIMO case.

2. MIMO CHANNEL MODEL

The model of the MIMO network considered in this paper consists of N independent users and a receiver array of K antennas, as shown in Figure 1.

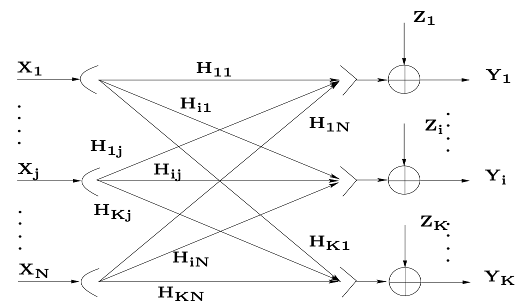

Fig. 1. A general MIMO fading network

Let $\mathbf{H_{ij}}$ $=[H_{ij}(1)\cdots H_{ij}(n)\cdots H_{ij}(T)]$ be the fading process between the j^{th} transmitter and the i^{th} receiver. $\mathbf{H_{ij}}$ is assumed to be a zero mean, unit power Gaussian random process, where $H_{ij}(n) \sim \mathcal{CN}(0,1)$, $E[|H_{ij}(n)|^2] = 1$ for $1 \leq j \leq N$, $1 \leq i \leq K$ and $1 \leq n \leq T$. The fading processes of all transmitter-receiver pairs are assumed to be mutually independent, so that H_{ij} is independent of H_{kl} for any $i \neq k$ or $j \neq l$.

The fading processes are correlated in time and this paper considers both block fading and continuous correlated fading models. In the block fading case, the channel coefficients are constant for a block of time Γ_0 sec, defined as the coherence time, and independent over successive blocks. In the case of continuous correlated fading, the process is ergodic and stationary with an arbitrary covariance matrix K_H. For the special case of a process with a brick-wall fading spectrum, the power spectral density is a rectangular function of bandwidth W_f Hz. The coherence time of the fading process, Γ_0 (in sec) is defined as $1/W_f$.

The N users, each equipped with a single transmitting antenna, transmit mutually independent signals, denoted by $\mathbf{X_1},\cdots,\mathbf{X_N}$ where $\mathbf{X_j} = [X_j(1)\ X_j(2)\ \cdots\ X_j(T)]$ is the length T transmitted signal of user j. Each transmitter transmits i.i.d. symbols from a constellation $\mathcal{A}$ with average power P_0. Let the signaling bandwidth be W symbols/sec.

The received signals at the K antennas are $\mathbf{Y_1},\cdots,\mathbf{Y_K}$ where $\mathbf{Y_i}$ $=[Y_i(1)\cdots Y_i(T)]$. The signal at the i^{th} receiver, shown in Figure 1, can be written as

$$Y_i(n) = \sum_{j=1}^{N} H_{ij}(n)X_j(n)+Z_i(n)\ \ \forall\, 1 \leq i \leq K, 1 \leq n \leq T \tag{2.1}$$

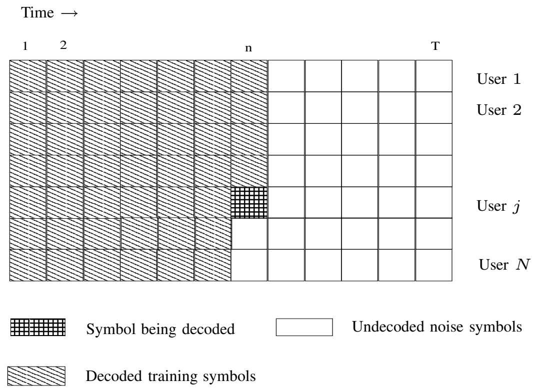

Fig. 2. Successive-decoding structure at each receiving antenna. $X_1^N(n) = [X_1(n), \cdots, X_N(n)]$ in (3.1) denotes the n^{th} column and $X_1^N(1, n-1) = [X_1^N(1), \cdots, X_1^N(n-1)]$ denotes the columns to the left of the n^{th} column.

where $\mathbf{Z_i}$ $=[Z_i(1)\cdots Z_i(n)\cdots Z_i(T)]$ is the AWGN channel noise at the i^{th} receiver with power $\sigma_z^2 = N_0 W$, or $E[|Z_i(n)|^2] = \sigma_z^2$ for $1 \leq n \leq T$.

The information rate of this channel is the maximum achievable rate of transmission with a fixed input distribution, denoted by $R^{K,N}$, as in

$$R^{K,N} = \lim_{T\to\infty} \frac{1}{T} I(\mathbf{X_1^N}; \mathbf{Y_1^K}) \tag{2.2}$$

where $\mathbf{X_1^j} \equiv [\mathbf{X_1}, \mathbf{X_2}, \ldots \mathbf{X_j}]$ and $\mathbf{Y_1^i} \equiv [\mathbf{Y_1}, \mathbf{Y_2}, \ldots \mathbf{Y_i}]$ for notational convenience.

3. Successive-Decoding Receiver for the Multiuser Channel

A. Optimal Receiver Scheme with Full Enumeration

This section presents a successive-decoding receiver that computes the exact APP of the transmitted symbol and whose achievable rate is the information rate of the channel. The successive-decoding receiver relies on a lossless expansion of the information rate using the chain rule of mutual information as given by (3.1).

$$\begin{aligned} &I(\mathbf{X_1^N}; \mathbf{Y_1^K}) \\ &= \sum_{n=1}^{T}\sum_{j=1}^{N} I(X_j(n); \mathbf{Y_1^K} | X_1^N(1, n-1), X_1^{j-1}(n)) \\ &= NTH(X) + \\ &\sum_{n=1}^{T}\sum_{j=1}^{N} E\left[\log(p(X_j(n)|\mathbf{Y_1^K}, X_1^N(1, n-1), X_1^{j-1}(n)))\right] \end{aligned} \tag{3.1}$$

The expansion of (3.1) suggests the decoding of the transmitted symbols instant by instant (column 1 to column T in Figure 2), and user by user (row 1 to row N) at each time instant. Calculating the individual terms in the expansion of (3.1) involves canceling interference from previously-decoded users and using the symbols from previous time instants as training for channel estimation. Each term in (3.1) represents the decoding of $X_j(n)$, which corresponds to the n^{th} time instant of the j^{th} user.

This decoding requires an exact computation of its APP, $\text{APP}_{jn}(d) = p\big(X_j(n) = d\big|\mathbf{Y_1^K}, X_1^N(1, n-1), X_1^{j-1}(n)\big)$. Using Bayes' rule, this APP can be expressed in terms of the likelihood function, $\text{L}_{jn}(d) = P\big(\mathbf{Y_1^K}\big|X_1^N(1, n-1), X_1^{j-1}(n), X_j(n) = d\big)$.

$$\begin{aligned} &\text{APP}_{jn}(d) \\ &= \frac{P\big(\mathbf{Y_1^K}\big|X_1^N(1, n-1), X_1^{j-1}(n), X_j(n) = d\big)}{\sum_{s\in\mathcal{A}} P\big(\mathbf{Y_1^K}\big|X_1^N(1, n-1), X_1^{j-1}(n), X_j(n) = s\big)} \end{aligned} \tag{3.2}$$

The exact computation of each likelihood term requires a complete enumeration of the undecoded symbols From Figure 2, we see that the undecoded symbols are $X_{j+1}^N(n)$ and $X_1^N(n+1 : T)$. The number of possible sequences of undecoded symbols is therefore $M^{(T-n)N+N-j}$. This is exponential in both blocklength and the number of users and is hence computationally infeasible. Therefore, upper and lower bounds on the information rates are derived which are computationally feasible.

B. Upper and Lower Bounds on Information Rate

A decoder that uses these exact APPs is optimal and its achievable rate is the information rate. However, the number of enumerations required in (3.2) is exponential in both the number of users and the length of the transmitted sequence and hence it is infeasible to implement this decoder directly. In practice, the decoder can be constructed by considering only a random subset, $\mathcal{X} \subset \mathcal{S}$, of all possible enumerations in (3.2)

$$\begin{aligned} &P\big(\mathbf{Y_1^K}\big|X_1^N(1, n-1), X_1^{j-1}(n), X_j(n) = d\big) = \\ &\sum_{\mathbf{e}\in\mathcal{X}} P\big(\mathbf{Y_1^K}, \mathbf{e}\big|X_1^N(1, n-1), X_1^{j-1}(n), X_j(n) = d\big) \end{aligned} \tag{3.3}$$

where $\mathbf{e}$ is a sequence of undecoded symbols $[X_{j+1}^N(n), X_1^N(n+1:T)]$.

Since such a suboptimal enumeration yields an inexact APP, the achievable rate of this decoder is less than information rate [12]. As the size of the random set, $|\mathcal{X}|$, increases, the performance of the decoder improves and it achieves capacity when $|\mathcal{X}| = |\mathcal{S}|$. To characterize the loss incurred by this practical decoder, the following paragraphs present a tight upper bound on the information rate.

The upper bound on the MIMO information rate is derived from the information rate of a genie aided decoder, where the genie provides side information to the decoder about the undecoded symbols in the block in Fig.2. The side information is in the form of a set of sequences, $\mathcal{Y} \subset \mathcal{S}$, out of which one is the actual transmitted sequence. In other words, the genie tells the decoder that none of the sequences outside $\mathcal{Y}$ were transmitted. Since the decoder is free to ignore this side information, the information rate of this genie-aided decoder is an obvious upper bound. The information rate of this genie aided decoder can be computed exactly by enumerating all possible sequences in the set $\mathcal{Y}$ using the total probability law.

$$P\big(\mathbf{Y_1^K}\big|X_1^N(1,n-1),X_1^{j-1}(n),X_j(n)=d\big) = \sum_{\mathbf{e}\in\mathcal{Y}} P\big(\mathbf{Y_1^K},\mathbf{e}\big|X_1^N(1,n-1),X_1^{j-1}(n),X_j(n)=d\big) \tag{3.4}$$

The rest of the decoding process is identical to the optimal decoder presented earlier. Note that this upper bound will converge to the exact information rate if there is no side information, i.e., the set $\mathcal{Y}$ consists of all possible sequences and is identical to $\mathcal{S}$. However, it would be computationally infeasible to compute it as the number of sequences grows exponentially with number of undecoded symbols. Hence the set should be chosen large enough to keep the bound tight, but small enough to be computationally feasible.

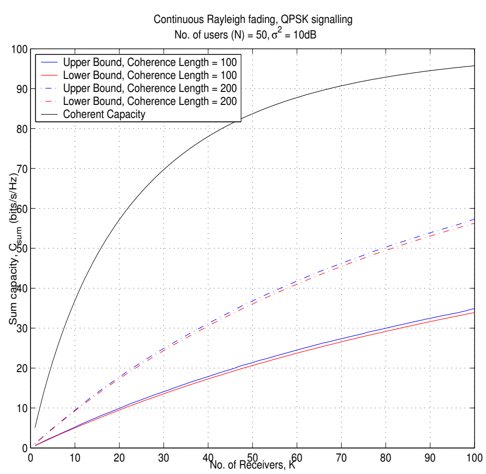

Fig. 3. Upper and lower bounds (based on enumerations) on the information rate for the continuous Rayleigh fading channel with two different coherence lengths along with the coherent capacity, for QPSK inputs. $N = 50, P_0 = 1$ and $\sigma_z^2 = 10$.

The upper and lower bounds are calculated using a Monte Carlo approach. Figure 3 plots the upper and lower bounds for $|\mathcal{X}| = |\mathcal{Y}| = 100$ for both block and continuous correlated Rayleigh fading with QPSK inputs. In the plots for continuous correlated fading, the fading spectrum is taken to be brickwall with coherence length defined to be the inverse of its bandwidth. With a large number of antennas, the use of QPSK signaling limits the achievable rates, and larger constellations would yield higher capacities. However, when the number of receivers is small, there is no significant gain in using a larger constellation [3].

The tightness of the bounds depends on the cardinality of the sets $|\mathcal{X}|$ and $|\mathcal{Y}|$. Hence these sets should be chosen large enough to keep the bounds tight, but small enough to be computationally feasible.

4. Information Rate for Large Populations

In this section, we compute upper and lower bounds on the information rate which converge in the limit where the system bandwidth and number of transmitting users increase proportionally while keeping each user's data rate (in bits/sec) constant. Although the information rate achieved by each user in bits/sec/Hz is vanishingly small in this limit, the aggregate information rate of the whole system approaches a constant because we also consider the number of transmitting users to increase proportionately with the bandwidth utilized by the system. In other words, the bandwidth and the number of users are increasing proportionately while keeping the signal power, noise power spectral density and the coherence time of the fading process constant. This limit represents a pure CDMA multiple access system.

A. Upper Bound

The information rate of a K receiver system is upper bounded by K times the information rate of a single receiver system. For the single receiver system, we can calculate an upper bound by expanding out the sum information rate using the chain rule of mutual information as the difference between the non-fading information rate $(I(\mathbf{H_1^N},\mathbf{X_1^N};\mathbf{Y})$ and the fading information rate $(I(\mathbf{H_1^N};\mathbf{Y}|\mathbf{X_1^N}))$ as

$$\begin{aligned} R^{1,N} &= \lim_{T\to\infty}\frac{1}{T}\left[I(\mathbf{H_1^N},\mathbf{X_1^N};\mathbf{Y}) - I(\mathbf{H_1^N};\mathbf{Y}|\mathbf{X_1^N})\right] \\ &\le \lim_{T\to\infty}\left(\log_2(1+\frac{NP_0}{\sigma_z^2}) - \frac{1}{T}\sum_{i=1}^{N} I(\mathbf{H_i};\mathbf{Y}|\mathbf{X_1^N},\mathbf{H_1^{i-1}})\right) \end{aligned} \tag{4.1}$$

We calculate a lower bound on the fading information rate, $I(\mathbf{H_i};\mathbf{Y}|\mathbf{X_1^N},\mathbf{H_1^{i-1}})$ from the variance of the MMSE channel estimates, $\sigma_{h_i}^2$. For the block fading channel, we get

$$R^{1,N} \leq \log_2(1+\frac{NP_0}{\sigma_z^2}) - \frac{1}{\Gamma_0 W}\sum_{i=1}^{N}\log(1/\sigma_{h_i}^2) \tag{4.2}$$
$$= U^{1,N}$$

The single receiver upper bound for the correlated fading case can also be calculated analogously. For the special case of brickwall fading, the single receiver upper bound turns out to be the same as (4.2).

Finally, the upper bound for the K receiver system is given by K times the single receiver upper bound as

$$R^{K,N} \leq KR^{1,N} \leq KU^{1,N} = U^{K,N} \tag{4.3}$$

In the limit where the number of users, N, grows in proportion to W, keeping the coherence time, Γ_0, the noise power spectral density, N_0, and the signal power, P_0, constant, we can explicitly calculate the upper bound, $U^{K,\infty}$ as

$$U^{K,\infty} = \lim_{\substack{N\to\infty \\ W\to\infty}} U^{K,N} \tag{4.4}$$

B. Lower Bound

The lower bound is calculated from a constructive decoding scheme. The decoding scheme converts the multiuser channel into a single user single receiver non-fading channel with i.i.d. BPSK inputs. In the wideband limit, the information rate of such a channel is lower bounded by considering the noise to be Gaussian [13] and can be calculated from the noise variance.

The decoding scheme is a mechanization of the chain rule of mutual information. For a single receiver network, we have

$$R^{1,N} = \lim_{T\to\infty}\frac{1}{T}\sum_{i=1}^{N}\sum_{t=1}^{T} I(X_i(t);\mathbf{Y}|\mathbf{X_1^{i-1}}, X_i(1:t-1)) \tag{4.5}$$

Each user transmits i.i.d. BPSK signals. The users are decoded successively and each user is decoded time-instant by time-instant. Each term in 4.5 corresponds to the decoding of the t^{th} bit of user i, with the previous users completely decoded and with the past bits of user i known. For each term in the above expansion, we compute the MMSE channel estimates of users 1 to $i-1$ based on the data $\mathbf{X}_1^{i-1}$. We also compute the MMSE channel estimate of the user i which has the past bits as training. Based on these channel estimates, we cancel out the estimated interference of the previously decoded users and compute the variance of the residual interference from the decoded users as well as the variance of the interference from the undecoded users. The SNR of this single user, single receiver system is given by

$$\text{SNR} = \frac{P_0(1-\sigma_{h_i}^2)}{(N-i)P_0 + (i-1)P_0\sigma_h^2 + \sigma_z^2} \tag{4.6}$$

where σ_h^2 is the MMSE channel estimate variance of users 1 to $i-1$ and $\sigma_{h_i}^2$ is the MMSE channel estimate variance of user i.

For a multireceiver system, we can show that the received signals at different receivers become uncorrelated in the wideband limit under consideration. Therefore, when we do maximal ratio combining across receivers, the signals add as voltages and the noises add as powers and the SNR of the combined signal grows linearly with the number of receivers.

In the wideband limit, the worst case noise is Gaussian ([13]) and so we get a lower bound on the information rate by considering the aggregate interference and noise to be Gaussian, as

$$R^{K,\infty} \geq \lim_{N\to\infty}\lim_{T\to\infty}\frac{1}{T}\sum_{i=1}^{N}\sum_{t=1}^{T}\left(\frac{KP_0(1-\sigma_{h_i}^2)}{(N-i)P_0+(i-1)P_0\sigma_h^2+\sigma_z^2}\right)$$
$$= L^{K,\infty} \tag{4.7}$$

We can explicitly evaluate the bounds on information rate for special cases like block fading and correlated fading with a brickwall spectrum. The information rate of these two cases (in bits/sec/Hz) in the limit turns out to be identical because of the way the coherence time is defined for the brickwall fading channel in Section II.

In the limit, the information rate (in bits/sec/Hz) of the multiuser block fading channel and the multiuser brickwall fading channel is given by

$$\begin{aligned} R^{K,\infty} &= L^{K,\infty} = U^{K,\infty} \\ &= \frac{K}{2\alpha\Gamma_0\rho\log 2}\Big[(\alpha+\alpha\Gamma_0\rho+\rho) - \sqrt{(\alpha+\alpha\Gamma_0\rho-\rho)^2+4\alpha\rho} \\ &\quad + 2\alpha\Gamma_0\rho\log\frac{2(\alpha+\rho)}{\alpha+\rho-\alpha\Gamma_0\rho+\sqrt{(\alpha+\alpha\Gamma_0\rho-\rho)^2+4\alpha\rho}} \\ &\quad - 2\rho\log\frac{2\rho}{-\alpha-\alpha\Gamma_0\rho+\rho+\sqrt{(\alpha+\alpha\Gamma_0\rho-\rho)^2+4\alpha\rho}}\Big] \end{aligned} \tag{4.8}$$

where, $\alpha = \frac{W}{N}$ Hz, $\rho = \frac{P_0}{N_0}$ Hz and Γ_0 is the coherence time in seconds. α is the bandwidth per user which is maintained constant as both the system bandwidth, W, and the number of users, N, increase proportionately. ρ is the ratio between the signal power, P_0, and noise power spectral density, N_0, which are both constants. As described in Section II, for the block fading channel, Γ_0 is the time for which the fading remains a constant, and for the brickwall fading channel, Γ_0 is the inverse of the fading bandwidth.

References

[1] T. Li and O. M. Collins, "A successive decoding strategy for channels with memory," *IEEE Trans. on Inform. Theory,* vol.52, no.2, pp.628-646, Feb. 2007.

[2] T. Li and O.M. Collins, "Capacity and coding for flat fading channels without channel state information," *Proc. Wireless Communications and Networking Conference*, pp. 1085-1090, vol. 2, New Orleans, USA, March 2005

[3] R. Gopalan, T. Li, S Ranganathan, K. Padmanabhan and O. M. Collins, "Capacity of MIMO systems without CSI," *Forty-Third annual allerton conference on communication, control, and computing*, Sep 2005.

[4] R. Gopalan, S. Ranganathan, K. Padmanabhan, T. Li and O. M. Collins, " Calculating and Achieving Capacity on the Unknown fading MIMO Channel," *Proc. IEEE International Symposium on Information Theory*, 2006.

[5] I. E. Telatar and D. N. C. Tse, "Capacity and Mutual Information of Wideband Multipath Fading Channels," *IEEE Trans. on Inform. Theory*, vol. 46, no.4, pp. 1384-1400, July 2000.

[6] S. Verdu, "Spectral Efficiency in the Wideband Regime," *IEEE Trans. on Inform. Theory*, vol. 48, no.6, pp. 1319-1343, June 2002.

[7] V. G. Subramanian and B. Hajek, "Broad-band fading channels: Signal burstiness and capacity," *IEEE Trans. on Inform. Theory,* vol. 48, no.4, pp. 809-827, Apr. 2002.

[8] V. Sethuraman and B. Hajek, "Low SNR Capacity of Fading Channels with Peak and Average Power Constraints," submitted to *IEEE Trans.on Inform. Theory*.

[9] V. Sethuraman and B. Hajek, "Capacity per Unit Energy of Fading Channels with a Peak Constraint," *IEEE Trans. on Inform. Theory*, vol. 51, no. 9, September 2005, pp 3102-3120.

[10] D. M. Arnold, H.-A. Loeliger, P. O. Vontobel, A. Kavcic and W. Zeng, "Simulation Based Computation of Information Rates for Channels with Memory," *IEEE Trans. Inform. Theory*, vol. 52, no. 8, pp. 3498-3508, Aug. 2006

[11] K. Padmanabhan, S. Venkatraman, O. M. Collins, "Optimal and Practicable Receivers and Coding for Non-Coherent MIMO Channels," *Submitted to IEEE Trans. Inform. Theory*

[12] T. R. M. Fisher, "Some remarks on the role of inaccuracy in Shannon's theory of information transmission," in *Trans. 8th Prag. Conf. Inform. Theory*, 1978, pp. 211-226.

[13] S. Shamai (Shitz), S. Verdu, "Worst-Case Power-Constrained Noise for Binary-Input Channels," in *IEEE Trans. on Inform. Theory*, vol. 38, no.5, pp. 1494-1511, Sep. 1992..