The Marginal Utility of Cooperation in Sensor Networks

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***Abstract*— We present arguments that a small number of sensors within the network provide most of the utility. That is, cooperation of more than a small number of nodes has little benefit. We present two scenarios. In the first scenario, all sensors provide identical utility, and their utilities are aggregated sequentially. The second scenario is sensor fusion with signal strength decreasing with distance. In that scenario the source is at the origin and the sensors are distributed, either uniformly or according to a planar standard normal distribution. We also vary the total number of sensors distributed in both scenarios to observe the utility/density trade off. Localization using the Fisher Information as the utility metric is used to demonstrate that few sensors are sufficient to derive most of the utility out of the sensor network. Simulation results back up an order statistics analysis of the behavior.**

The implication is that while co-operation is useful for some objectives such as combating fading and uncertainty of individual sensors, it is inefficient as a mean to increase the utility of a sensor network if the best sensor's utility is significantly short of the desired utility.

I. Introduction

The usual paradigm of sensor network research assumes there will be a large number of sensors present within the network. An important question for minimizing resource usage for either scalability or extension of sensor lifetime is how many of those sensors should be used at any given time. In many situations, after the first few sensors the utilities improvement rapidly decrease and each subsequent sensors added yield only marginal improvement [1], [2], [3], [4], [5], [6], [7], [8].

At one extreme of the spectrum, all the sensors have identical utility. Thus the overall utility grows linearly. Although the overall utility is unbounded, each additional sensor contributes less than sensors that were already considered. Therefore those sensors that are selected earlier have a larger impact on the overall utility.

On the other hand, typically sensors that are closer to the source will be weighted more heavily than those sensors that are farther away simply because of the higher signal strength that will be observed by the sensors that are closer. Suppose sensors are distributed uniformly in a unit disk with a source located at the origin. We will show a small number of 'good' sensors will provide most of the utility, and even in fading channels few are required.

Localization is a common task for sensor networks. Sensor networks can solve this type of problem effectively by incorporating multiple views of the source using different sensors and multiple types of observations (e.g. range, angle of arrival, time difference of arrival.) Sensor fusion for localization is straightforward and can be solved efficiently. Using Fisher Information as the utility function, we will show that for localization in a planar scenario, a small number of sensors (on the order of 10 sensors) will provide most of the utility.

In section II, we will assume each sensor has identical utility. We evaluate how each added sensor contributes relatively less than those added previously. However, the overall utilities can reach an arbitrary value. We show how this plays out for a localization problem in section III. A non-uniform sensor utility due to distance losses is considered in section IV. We observe that in a benign environment, the best sensor contributes significantly more than even the next best sensor, and cooperation is not critical when the network is sufficiently dense. This is driven by the order statistics of the sensors placement. The few 'good' sensors' utility is driven by the density of the deployment. The relative improvement of utility falls off quickly after the first few sensors. On the other hand, even in a fading environment, cooperation among the few 'good' neighbors is sufficient to avoid outage. In section V, we summarize the two types of sensor utility and their implications for the cooperation strategy in a sensor network.

II. Identical Utility

Suppose each sensor contributes an identical amount of utility, and the overall utility of the fused data will be the sum of the individual utilities, i.e. utility for n sensors will be simply n.

At the n-th iteration of data fusion, the existing sensor set provides $n-1$ units of utility, and the relative utility u_r of the existing set to the n-th iteration is

$$u_r(n) = \frac{n-1}{n} \tag{1}$$

The difference is

$$\Delta u_r(n) = \frac{2n-1}{n(n+1)} \tag{2}$$

To increase the utility n by factor k, it will require nk sensors, and the utility will approach nk in $O(1/n)$.

One simple model is coverage area. Let each sensor provide unit area coverage and there is no overlap in coverage when we place each sensor. Therefore n sensors provide utility n. We can cover an arbitrarily large area by deploying a sufficiently large number of sensors. However, the rate of each sensor's actual contribution to the overall utility decreases geometrically as the total utility increases, as seen in figure 1.

The implication is that in a very dense deployment, some overlap or otherwise a reduction in individual sensor utility will not be noticeable. Figure 2 shows that when individual sensor utilities are uniformly distributed between 0.5 to 1.5, as the number of sensors increase, new sensor contribution to the overall utility still diminishes geometrically. In addition, the variation in realization of this prior utility rate due to individual sensor utility variations diminishes as the number of sensors increases due to law of large numbers. Figure 2 displays the result of 10k trials, each with 25 sensors. The bar limit is the maximum and the minimum utility for a given number of sensors out of the 10k trials.

A more in-depth example can be seen in [2]. Even in their complex deployments the saturation effect is readily seen.

There are situations that the utility from sensors increases at more than a linear rate. They arise when the number of sensors used is less than necessary to provide the desired quality of service and the underlying utility has ambiguity. A typical example is localization

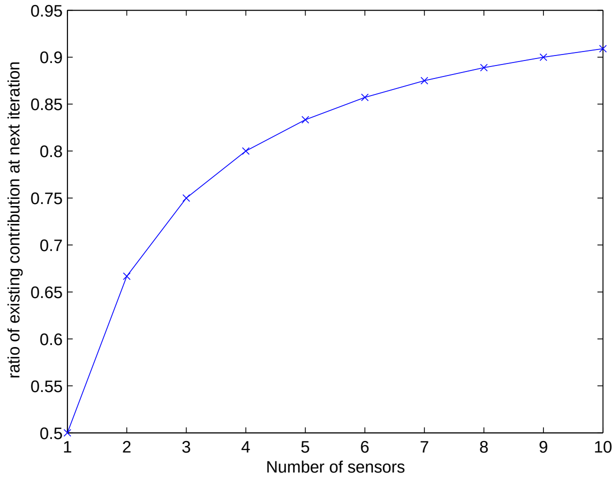


Fig. 1. Prior utility rate to the total utility

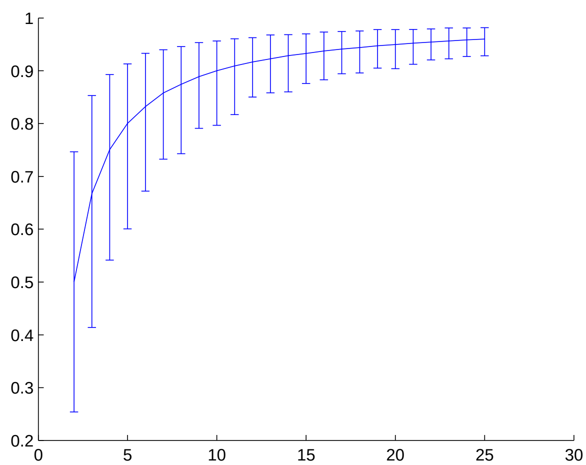


Fig. 2. Prior utility rate to the total utility with random sensor utility

when the observations available are less than required to form a unique solution. Such utility functions exhibit super modular behavior [9]. Under such utility functions, the benefit of cooperation among sensors are application specific. Suppose our goal is to meet a certain fixed amount of utility, and each additional sensor provides additional utility , i.e., $u_{n+1} = (1+\epsilon)u_n, \epsilon > 0$ where u_n is the utility of the n-th sensor. The overall utility U with N sensors is then simply

$$U = u_1 \sum_{i=1}^{N} (1+\epsilon)^{i-1} \tag{3}$$

This geometric series in eqn. 3 is also unbounded and can also reach any arbitrary utility for sufficiently large N. Even in this case, we can see that certain sensors provide the majority of the utility, but in this case it is the last few sensors as opposed to the first few sensors.

III. Localization case study

In this case study, we will consider the utility that can be derived from each sensor in a localization problem. Localization of sources can typically be achieved by three techniques: triangulation, scenes analysis and proximity sensing [10]. In this section, we will focus on a sub class of the triangulation technique: range/time of travel (RNG), angle of arrival (AOA) or time difference of arrival (TDOA) measurement.

A. Observation uncertainty model

We model the observation as a Gaussian distribution centered around the true reading. $\rho_i \sim \mathcal{N}(\bar{\rho}_i, \sigma_i)$ for i-th range sensors, $i = 1, \ldots, k_R$, $\theta_i \sim \mathcal{N}(\bar{\theta}_i, \sigma_i)$ for i-th AOA sensors, $i = 1, \ldots, k_A$ and $\tau_i \sim \mathcal{N}(\bar{\tau}_i, \sigma_i)$ for i-th TDOA sensors, $i = 1, \ldots, k_T$.

B. Sensor Utility

Due to the observation uncertainty, localization will have limited accuracy. A common criterion for accuracy of estimating parameters is the Cramer-Rao bound (CRB). The CRB matrix tells us the best we can expect from an unbiased estimator in terms of co-variance. A related quantity, the Fisher Information Matrix (FIM), which is the inverse of the CRB, will be used as the basis of the localization problem utility function. In particular, the trace of the FIM will be the utility measure. The overall FIM $\mathbf{F}$ of $\bar{S} \subseteq S$, where $\bar{S}$ is the set of sensors selected and S is the set of all available sensors, is:

$$\mathbf{F}_{\bar{S}} = \sum_{k \in \bar{S}} \mathbf{h}_k^* \mathbf{h}_k \tag{4}$$

From [11],[12],[13], based on the observation uncertainty model used in III-A, in a planar localization problem, $\mathbf{h}_k$ is

$$\mathbf{h}_k = \begin{cases} \frac{\mathbf{r}_s - \mathbf{r}_k}{\sigma_k \|\mathbf{r}_s - \mathbf{r}_k\|} & \text{RNG} \\ \frac{y_s - y_k, x_s - x_k}{\sigma_k \|\mathbf{r}_s - \mathbf{r}_k\|^2} & \text{AOA} \end{cases} \tag{5}$$

where $\mathbf{r}_s = [x_s, y_s]$ is source location and $\mathbf{r}_i = [x_k, y_k]$ is k-th sensors location. σ_k^2 is the k-th sensors' observation variance.

With time difference of arrival (TDOA) localization, in the case of unknown propagation velocity, the CRB matrix for location parameter and propagation velocity can be expressed as follows [14]:

$$\text{TDOA: } \mathbf{h} = \frac{1}{v} \left[\frac{1}{\sigma_i} \left(\frac{\mathbf{r}_s - \mathbf{r}_i}{\|\mathbf{r}_s - \mathbf{r}_i\|} - \frac{\mathbf{r}_s - \mathbf{r}_{\text{ref}}}{\|\mathbf{r}_s - \mathbf{r}_{\text{ref}}\|} \right), -\tau_i \right] \tag{6}$$

where $\mathbf{r}_{\text{ref}} = [x_{\text{ref}}, y_{\text{ref}}]$ is the reference sensor location, v is the propagation velocity and $\tau_{i\text{ref}}$ is the time difference between the i-th sensor and the reference sensor.

Note that the above derivation did *not* explicitly assume any distance loss in signal quality, i.e. σ_i in reality may also be a function of distance $\mathbf{r}$. This simplification has little impact when the source is 'far' away from the sensors group and most sensors actually observe similar signal strength.

However this simplification has considerable impact when the relative distances between the source and the sensors vary significantly among sensors, which will be the case when the source is 'near' the sensors group. This distance dependent interaction is affected by the distribution of sensors around the source and will be considered below in section IV.

C. Localization Utility Simulation

In the following simulation, the source was placed within the field of sensors. We ran 300 trials of the experiment. In each trial, there were 16 RNG and 16 AOA sensors, with .08 standard deviation on observation for both types of sensors. Therefore the d_{be} is 1. AOA sensors within radius 1 will be selected first, followed by the entire set of RNG sensors followed by the remaining AOA sensors. The sensors are distributed uniformly over a [-1, 1] × [-1, 1] square and the source is placed uniformly over a [-.1, .1]× [-.1, .1] box.

λ in Fig. 3 is the sum of the eigenvalues of the FIM and is the utility metric for this localization simulation. Note the rapid saturation of the utility after a few sensors, regardless of the selection algorithm used in selecting sensors. In this case the minimum number of sensors is three for the RNG sensors.

Several algorithms were used to select sensors in a sequential fashion among the entire set of sensors. The different selection algorithms show that the underlying localization problem render the sensor selection problem trivial when sensors are sufficiently dense in deployment. The algorithms we used are as follows:

1) Random: We simply pick sensors randomly. This is the simplest method; it requires no prior information. The density of the deployment determines the success of this method. In particular, this method will be successful in a dense deployment, and will fail easily in a sparse deployment.

2) Entropy Difference: From the observation model, each sensors' observation has a certain probability distribution. From [15], we may use a heuristic based on information theory to sort sensors according to their potential benefit in improving our accuracy in the localization problem. This method considers the problem in its entirety; both the sensor's observation variance and the geometric factors are considered.

This method requires two parts. First the entire observable space has to be discretized once to compute $\mathbf{H}_i^v$ ((9) in [15]) for each sensor to compute the a priori entropy of observation:

$$H_i^v = -\int p(z)\log p(z)dz$$

where z is the field of view of the sensors. This $\mathbf{H}_i^v$ only depends on sensors location and the geometry of the observable space.

At each iteration, based on the previously picked sensor, across the entire observable space, we need to compute $\mathbf{H}_i^s$ ((11) in [15]), where $\mathbf{H}_i^s$ represents the entropy of the sensor observation given that the source location is estimated based on knowledge available up to present.

$$H_i^s = -\int p(z|\hat{x})\log p(z|\hat{x})dz$$

where $\hat{x}$ is the latest maximum likelihood estimate of source location.

[15] detailed how $H_i^v - H_i^s$ approximates the mutual information comparison at each sensor and the difference from the actual mutual information. All of this is very computationally intensive. In fact, according to [15], it's $O\left(w^3\right)$, assuming the observable space is gridded into a $n \times w$ matrix. If sensors are to compute the entropy difference in a distributed fashion, each sensors requires the probability distribution of the source location. At the end of each iteration the probability distribution of the source location will be updated with the selected sensor's observation.

3) Nearest Sensor First: Another heuristic method is to sort the sensors according to their distance from the estimated source location, then pick the closest sensor first. Presenting the selection algorithm as an optimization problem, we want to select the i-th sensors that

$$\min_i \|\mathbf{r}_s - \mathbf{r}_i\|$$

All that is required is some kind of estimation of source location and all sensors locations.

A typical realization of utility progress, with a variety of sensor selection algorithms is shown in Fig. 3. Fig. 4 shows the number of sensors needed for a variety of sensor selection algorithms to achieve 90% of the utility achived by using all sensors.

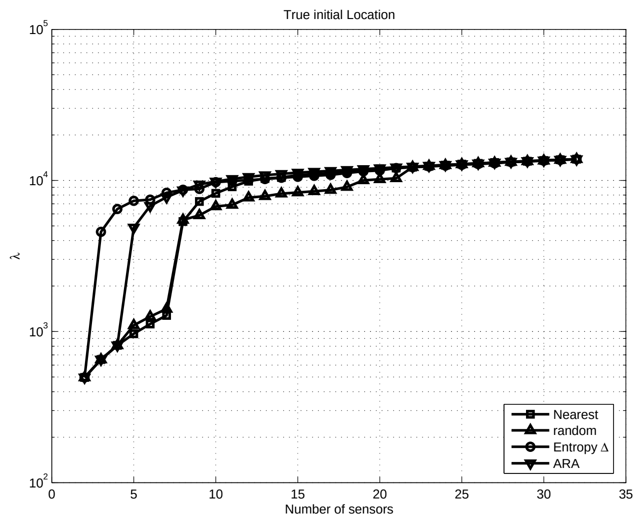


Fig. 3. One realization of of localization utility

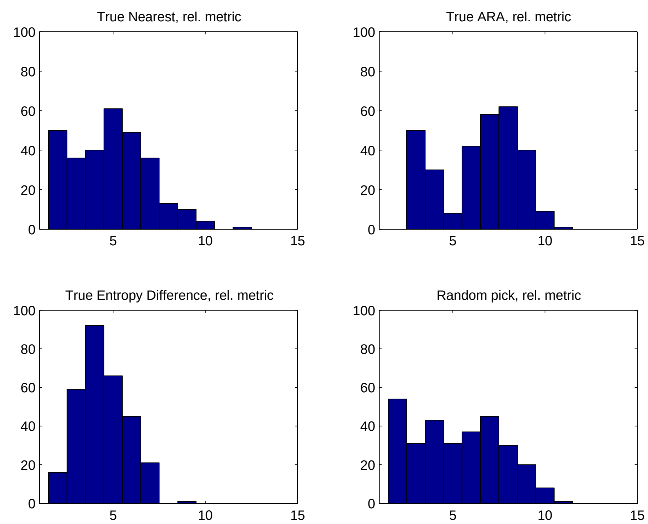


Fig. 4. Number of sensors to achieve 90% of total utility

4) Angle-Range-Angle: Yet another heuristic method is to select some AOA sensors first, then select all the RNG sensors and then the remaining AOA sensors. Suppose for all RNG sensors the observation variance is σ_{R} and for all AOA sensors the observation variance is σ_{A}. All RNG sensors are equivalent in terms of their utility toward the localization application, since $\mathbf{h}_{\mathrm{RNG}}$ are unit vector scaled by σ_{R}. On the other hand, AOA sensors that are closer to the source provide more utility than the farther away counterparts.

D. Localization Conclusion

Clearly the first few sensors contributed the majority of the utility of the data fusion, under this simplified utility function. If some sensors actually have a higher utility function than other sensors, even fewer sensors will contribute most of the utility. Consequently, while there are some small differences among the algorithms' performance, the utility function here penalizes the simple selection algorithm only slightly compared to more complex algorithms.

From the above exercise, we can see that a few sensors in addition to the minimum required to resolve the source location uniquely will be sufficient to localize the source, echoing a result observed in [16].

IV. Non-uniform sensor utility

Now we will consider non-uniform sensor utility that arises due to the sensors' geometric distribution and the resulting distance loss effect on the expected utility. Order statistics will play a key role to transform the sensors distribution to the expected utility.

In addition, we will observe that a small number of sensors cooperating help in overcoming outage due to fading.

A. Order Statistic

The distribution of the k-th statistic out of n IID drawn random variables can be written as follows [17]:

$$f_{X_{(k)}}(x) = n \begin{pmatrix} n-1 \\ k-1 \end{pmatrix} F(x)^{k-1}(1-F(x))^{n-k} f(x) \quad (7)$$

From eqn. 7, we can obtain the distribution for the nearest k-th sensor distance to the source given the number of sensors that are drawn, along with the sensor-source distance distribution, assuming all the sensors are drawn in an IID fashion. In the following we will consider two distributions: sensors are distributed in the unit disk uniformly, and sensors are distributed according to a normal distribution in a plane.

B. Uniform Disk

1) Layout and Assumptions: We will assume a source is at the origin and all sensors are distributed uniformly within the unit disk. The utility function for a given sensor is the distance of the sensor and the source, i.e. $r_i^{-\alpha}$ for the i-th sensor which is r_i away from the source and $2 \leq \alpha \leq 4$.

2) Theoretical distribution of distance: All the sensors are placed in an IID manner, and the distances are distributed according to $f_R(r) = 2r, r \in [0,1]$, a triangular distribution. The ordered statistic for the k-th closest sensor distance for n sensors total is as follows after putting the appropriate terms into eqn. 7.

$$f_{R_{(k)}}(r) = n \begin{pmatrix} n-1 \\ k-1 \end{pmatrix} r^{2(k-1)}(1-r^2)^{n-k} 2r \quad (8)$$

As fig. 5 shows, the first few distributions are similar and eqn. 8 matched well with the simulation. This similarity between the first few distributions is the key to understanding how to overcome fading as will be discussed in section IV-E.

C. 2D Gaussian

1) Layout and Assumptions: We continue to assume a source is at the origin but now all the sensors are distributed in a planar standard normal distribution. The utility function for a given sensor is the distance of the sensor and the source, i.e. $r_i^{-\alpha}$ for the i-th sensor which is r_i away from the source and $2 \leq \alpha \leq 4$.

2) Theoretical distribution of distance: All the sensors are placed in an IID manner, and the distance is distributed according to Rayleigh distribution, with $f_R(r) = re^{-r^2/2}$. The extreme ordered statistics are as follows.

$$f_{R_{(k)}}(r) = n \begin{pmatrix} n-1 \\ k-1 \end{pmatrix} (1-e^{-r^2/2})^{k-1}(e^{-r^2/2})^{n-k} re^{-r^2/2} \quad (9)$$

As fig. 6 shows, the first few distributions are also similar to each other. This distribution also shares a similar shape with the uniform disk distribution, with the exception that we no longer have a hard boundary limitation as in the disk model. However, our interest is in those that are close to origin. Thus the tail of the distribution has little impact.

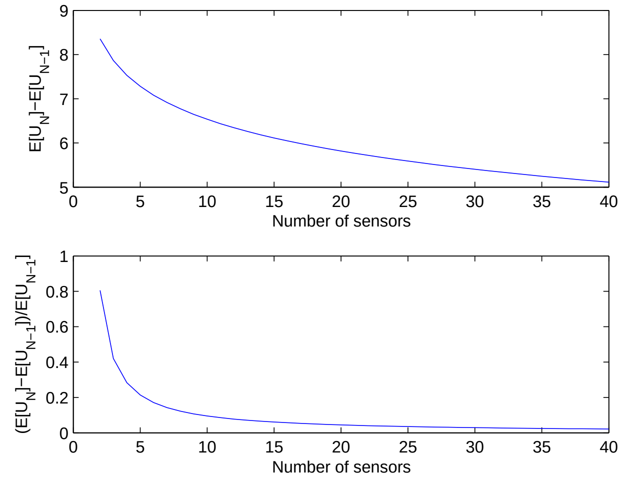


Fig. 7. Nearest sensor expected utility evolution from n to n+1 in a disk

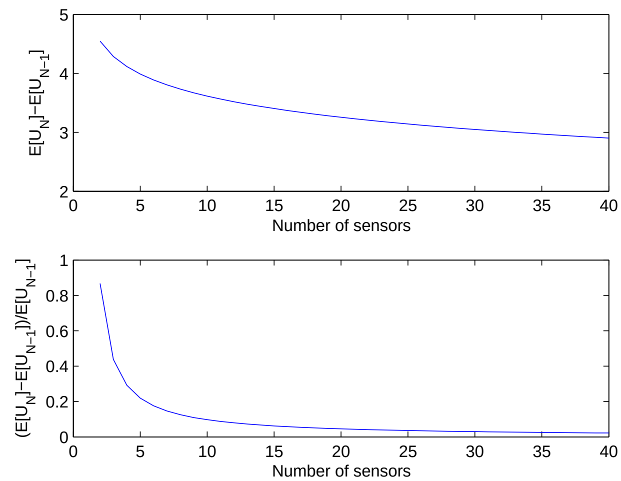


Fig. 8. Nearest sensor expected utility evolution from n to n+1 in a 2D normal distribution

D. Expected Utility by varying k, n

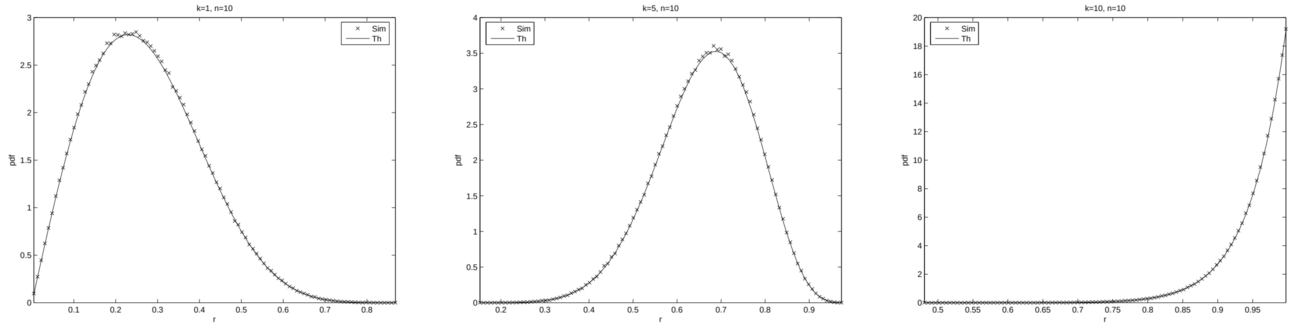
The expected utility of the k-th sensor is

$$E[u_k] = \int r^{-\alpha} f_{R(k)}(r) dr \quad (10)$$

From eqn. 10, and the respective order statistic distribution from eqn. 8 and 9, we obtain the following figures, illustrating evolution of utility derived from the nearest sensor as the total number of sensors increase in the respective environments.

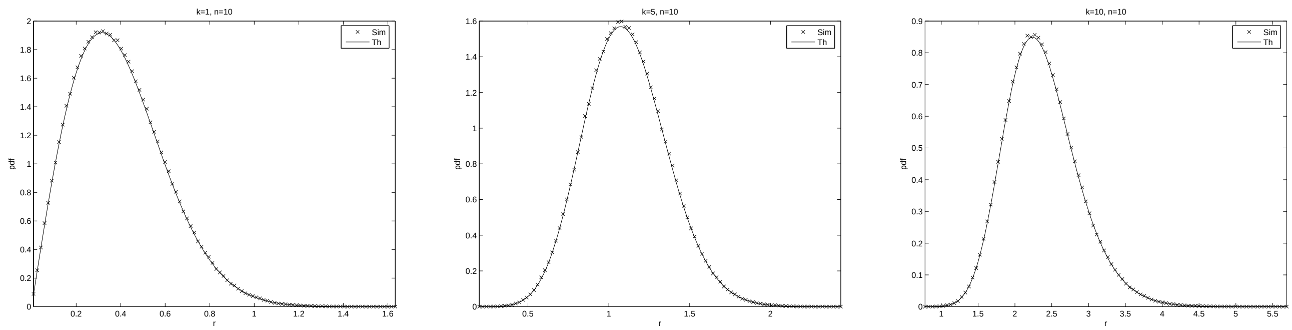
As shown in figures 7 and 8, both distributions behave similarly. Both experience a sharp increase in utility initially, and then the relative utility growth diminishes.

As seen from the figures 9 and 10, the utility is dominated by the nearest sensor. The relative utility is plotted, where the nearest sensor



(a) k=1, n=10 (b) k=5, n=10 (c) k=10, n=10

Fig. 5. Simulation and theoretical order statistic of distance to origin, uniform disk



(a) k=1, n=10 (b) k=5, n=10 (c) k=10, n=10

Fig. 6. Simulation and theoretical order statistic of distance to origin, planar normal

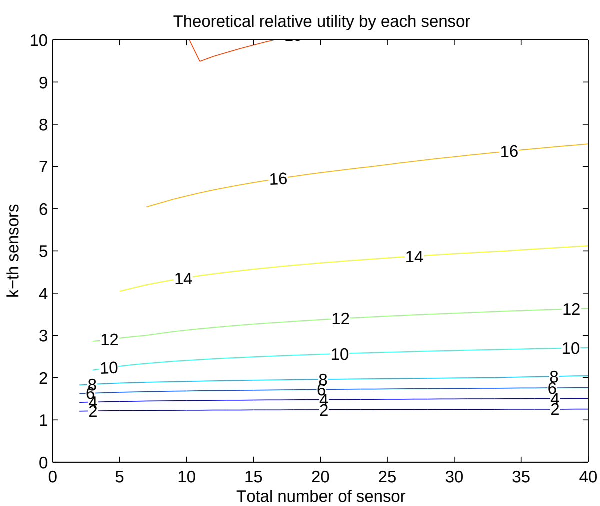


Fig. 9. Relative utility in disk

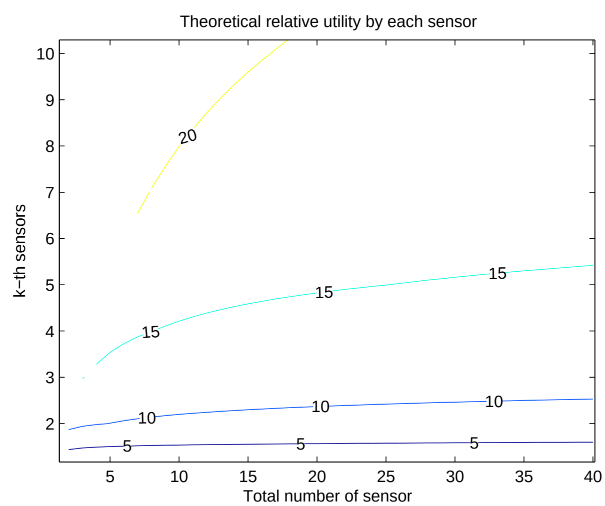


Fig. 10. Relative utility in 2D normal distribution

is the reference, and the level curve is the utility *below*, the reference, in dB.

The implication of the above result is that the one or two sensors that are closest to the source will generate most of the utility. This further implies that cooperation will not be an effective means to increase the utility of sensors. Thus cooperating beyond necessary, e.g. the minimum number of sensors required to uniquely localize a target, will not provide much increase in utility. However, some level of cooperation should be considered in a sensor network to defend against uncertain environments as discussed below.

E. Cooperation

One such infelicitous environmental factor is fading. Fading can occasionally cause significant degradation to signal strength. Here we consider the utility function $(r^{-\alpha})$ is multiplied by a fading factor g distributed according to the Rayleigh distribution, $f_G(g) = g/\sigma_f^2 \exp(-g^2/(2\sigma_f^2))$. For a given geometry, we will draw a set of

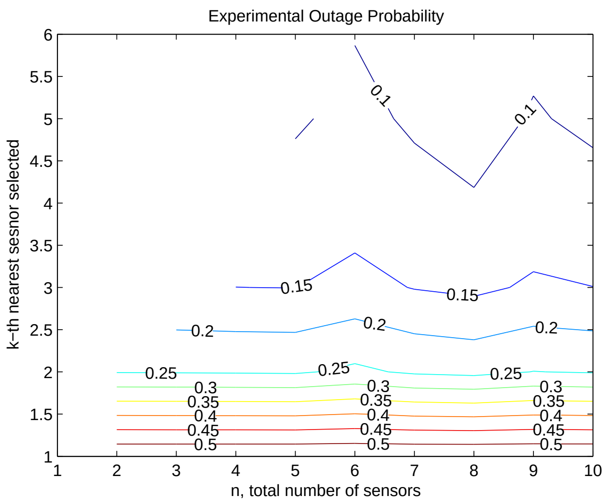


Fig. 11. Fading outage in a disk

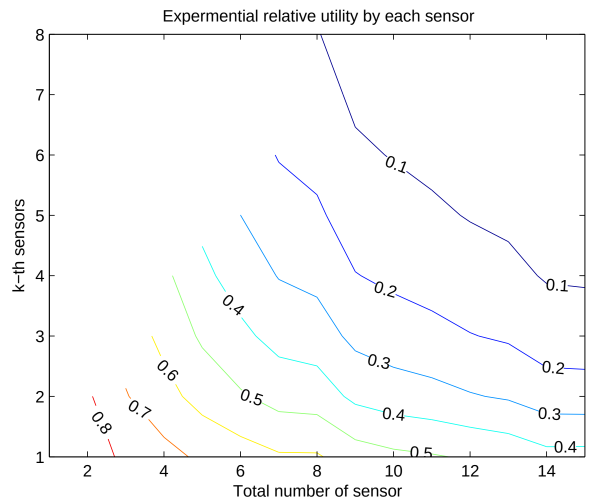


Fig. 13. Fading outage in a disk, fixed goal

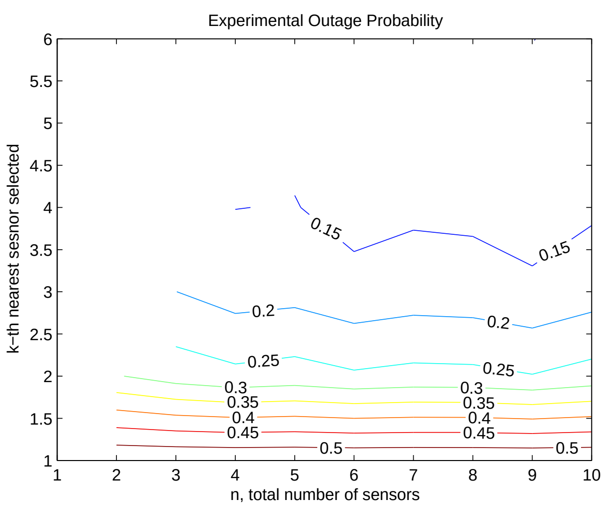


Fig. 12. Fading outage in 2D normal distribution

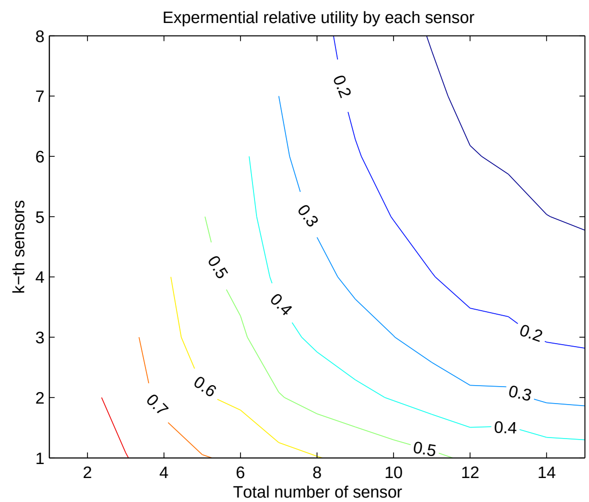


Fig. 14. Fading outage in 2D normal distribution, fixed goal

Rayleigh distributed random variables to simulate the fading effect and collect the statistics over multiple instances of the fading. We declare an outage if the sum of the faded utility is less than the nearest utility when there is no fading.

In figures 11 and 12, outage probability, as defined above, is shown with $\sigma_f = 0.8$. Both types of distributions experience improvement with a small number of collaborators and differ only in the tail region when outage is below 15%. As the number of sensors needed to mitigate fading increases, the difference between the two order statistics become apparent.

Not surprisingly, the outage is independent of the number of sensors in the entire deployment, as seen in figures 11 and 12 that the given probability of outage depends only on number of sensors used (k), and not on the total number of sensors (n). That is due to the outage definition above, where outage is related to the nearest sensor utility in a non-fading environment. For a given fading environment, a few sensors cooperating is necessary to provide acceptable performance. This is in contrast to the previous scenario. In the scenario where there is no fading, the nearest sensor alone is sufficient. Nonetheless, even in this case a small number of sensors suffice.

Suppose we define outage as a certain quality of service (QoS), in this case as the expected utility from two sensors under non-fading environment. The outage is shown in figures 13 and 14. With the fixed QoS, it is not surprising that as the total number of sensors or the number of sensors used in cooperation increases the outage decreases. Note also that as the total number of sensors increases, the number of sensors for actual cooperation can be reduced in order to reach the target QoS. That is achieved by sensors being closer to the source such that those sensors can provide the targeted QoS even in a fading environment.

V. Conclusion

Cooperation among sensors is not an effective means to increase sensor network utility from individual sensor utility or scaling of coverage. However, cooperation is an effective means to defend against fading and is necessary to provide coverage.

In the case of using cooperation against fading, a small number of sensors cooperating is sufficient. The number is mostly a function of the fading parameter, assuming the network is sufficiently dense such that the cooperating neighbors also have similar utility.

In the case of increased coverage, only the nearest few sensors are needed. This requires the network be deployed at sufficient density so that the few closest sensors will provide the desired quality of service. An example is the localization problem, where a certain minimum number of sensors are needed to produce a unique estimate. The required number based on geometry and a few additional sensors to mitigate poor geometry placement and/or fading will be sufficient to provide most of the utility. If the few closest sensors are not sufficient, a large number of sensors will not help, especially after considering the distance loss and the marginal utility provided by the later sensors.

These conclusions follow easily from consideration of the order statistics.

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