Near-Optimum STBC/SFBC using 1-Bit Feedback for the 4-Transmit Antenna system

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***Abstract*—In this paper, new space time/frequency block coding (STBC/SFBC) scheme with angle feedback for 4-transmit antennas is proposed to achieve the full diversity gain with the full code rate. This scheme can be generalized with multiple receive antennas. With the proposed scheme, we show the channel characteristics can be much improved with as small as 1-bit feedback overhead. Simulation shows that the proposed scheme with 1-bit feedback achieves near-optimum performance.**

***Index Terms*—MIMO, STBC, SFBC, Feedback**

I. Introduction

Diversity transmission systems enable blocks of data to be transmitted via multiple transmitting antennas. By utilizing multiple transmit antennas, each of the transmitting signals may follow a distinct propagation path. A process of generating signals for transmission in a diversity transmission system is referred to as a diversity coding.

Space time/frequency block coding (STBC/SFBC) is a popular method as a diversity coding utilized in the field of wireless communication recently. The appeal of STBC/SFBC is that it seeks to enable wireless communication systems to utilize advantages of diversity transmission at a transmitting station without knowledge of the channel, while allowing simplified decoding techniques at a receiving station. The first STBC/SFBC scheme proposed by Alamouti [1] achieves full code rate and full diversity for two transmit antennas. It is known, however, that similar complex orthogonal code design does not exist for more than two transmit antennas [2]; therefore, STBC/SFBC schemes for higher number of antennas either have lower code rates (e.g., [2], [3]) or are based on quasi-orthogonal (QO) codes with reduced diversity (e.g., [4], [5]). However, they suffer from loss of diversity gain due to the coupling between symbols in the codewords. In order to overcome its non-orthogonality, QO-STBC/SFBC techniques can be combined with Maximum-Likelihood (ML) [6] but the complexity of the receiver design is significantly huge. Another way to improve the diversity gain is constellation expansion and symbol rotation [7]. Nevertheless, this often causes bigger transmit Error Vector Magnitude (EVM). There have also been many closed-loop methods for STBC/SFBC proposed to attain full diversity gain and unity code rate ([8], [9], [10], [11], [12]). Such a scheme, however, would rely on a sufficient number of feedback bits to achieve full diversity, mostly $3 \sim 5$ bits for the feedback, due to an inefficient quantization method.

In this paper, we propose a 4-antenna transmit diversity scheme based on QO-STBC/SFBC using different codewords with only 1-bit feedback. The 1-bit feedback information is used to minimize the non-orthogonality of QO-STBC/SFBC by selecting the smaller between the real and imaginary parts of the most dominant interference among all receive antennas, i.e., the 1-bit is not simply the quantized version of exact angle feedback. We also show our 1-bit feedback scheme can be extended for the system with multiple receive antennas. As a result, the proposed scheme achieves near-optimum performance with a significantly reduced amount of feedback compared to previous QO-STBC/SFBC schemes with quantized angle feedback ([8], [10], [11]).

The paper is organized as follows. Section II describes our new STBC/SFBC scheme with symbol rotations. For the feedback, we introduce 1-bit quantization method by selecting the smaller between the real and imaginary parts of the most dominant interference among all receive antennas. This proposed scheme can be generalized with multiple receive antennas as in Section III. Section IV reveals the characteristic of the effective channel with the angle feedback is much improved. In Section V, we compare the proposed scheme with conventional QO-STBC/SFBC methods and show that it outperforms with 2 to 3 dB gain. Conclusions follow in Section VI.

II. QO-STBC/SFBC with rotation for One Recieve Antenna

While complex STBC/SFBC with full code rate and full diversity gain does not exist for more than two transmit antennas, we show it is possible to obtain this with an aid of feedback from the receiver. The proposed scheme in this paper can be applied to popular forms of QO-STBC/SFBC schemes, such as what is known as ABBA scheme [4] or Jafarkhani's scheme [5]. In following sections, we show a simple feedback of angle to rorate symbols in QO-STBC/SFBC codewords can achieve full diversity gain without loss of code rates.

A. With ABBA Scheme

We consider the following 4×4 QO-STBC/SFBC code matrix similar to the ABBA code [4] with constellation rotation:

$$S = \begin{bmatrix} x_1 & x_2 & x_3 & x_4 \\ -x_2^* & x_1^* & -x_4^* & x_3^* \\ c \cdot x_3 & x_4 & x_1 & c \cdot x_2 \\ -x_4^* & c^* \cdot x_3^* & -c^* \cdot x_2^* & x_1^* \end{bmatrix}, \qquad (1)$$

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where row represents an antenna index and column represents either a time instance for STBC or frequency tone for SFBC. Superscript * denotes complex conjugate. x_i is the original input quadrature amplitude modulation (QAM) symbol in one-stream sequence and S is a 4×4 block codeword to be transmitted over 4 transmit antennas. Note four symbols in the codeword S in (1) are multipied by c which is equivalent to rotation by an angle θ, where $c = e^{j\theta}$. In [6], a scheme with $\theta = \pi/2$ was shown to give the best open-loop performance. In this paper, we assume that θ is a variable to be fed back in the general angle feedback scheme.

Assuming the channel is not varying over the size of the codewords, the received signal equation for the new STBC/SFBC codewords in (1) is given by

$$\begin{bmatrix} r_1 \\ r_2^* \\ r_3 \\ r_4^* \end{bmatrix} = \begin{bmatrix} h_1 & h_2 & h_3 & h_4 \\ h_2^* & -h_1^* & h_4^* & -h_3^* \\ h_3 & c \cdot h_4 & c \cdot h_1 & h_2 \\ h_4^* & -c \cdot h_3^* & c \cdot h_2^* & -h_1^* \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} n_1 \\ n_2 \\ n_3 \\ n_4 \end{bmatrix} = H\underline{\mathrm{x}} + \underline{\mathrm{n}}, \tag{2}$$

where r_k and n_k is the received signal and noise on the kth time instance or frequency tone, h_i is the channel from i^{th} transmit antenna $(i = 1, \cdots, 4)$ to the receiver and H is the effective channel matrix with a group of 4 time instance or frequency tones.

If H were orthogonal, i.e., $H^H H$ were a diagonal matrix (^{H} denotes Hermitian transpose), this code could be optimally detected by left-multiplying the receive signal vector in (2) by H^H (what is known as the matched filtering approach). However, $H^H H$ is actually given by

$$\begin{bmatrix} \sum_{i=1}^4 |h_i|^2 & 0 & \delta & 0 \\ 0 & \sum_{i=1}^4 |h_i|^2 & 0 & \delta^* \\ \delta^* & 0 & \sum_{i=1}^4 |h_i|^2 & 0 \\ 0 & \delta & 0 & \sum_{i=1}^4 |h_i|^2 \end{bmatrix}, \tag{3}$$

where

$$\delta = \alpha - c^* \alpha^* \tag{4}$$

and

$$\alpha = h_1^* h_3 + h_2 h_4^* \tag{5}$$

Note that all the non-zero off-diagonal terms of the matrix in (3) can be represented by δ and its conjugate. We see in (4) that $\delta = 0$ when $c = e^{j\theta}$ and $\theta = -2 \times \angle(\alpha)$, where $\angle(\cdot)$ denotes the argument of a complex number. With full information of such angles by feedback, this new STBC/SFBC codewords can be orthogonalized and achieves full diversity.

However, in practice with a digital communication system, the phase information θ to be fed back needs to be quantized. While such information can be obtain by quantizing the angles within $[0, 2\pi]$ range, the simplest feedback with minimal overhead could be 1-bit feedback on θ with two choices, either $\theta = 0$ or π. For this 1-bit feedback, we propose the following criterion instead of calculating the exact angles to quantize:

$$c = \begin{cases} 1 & \text{if } |Re(\alpha)| \geq |Im(\alpha)|, \\ -1 & \text{otherwise} \end{cases} \tag{6}$$

With this 1-bit feedback scheme, the interference term δ may not be zero, but it will have the magnitude of the smaller between the real and imaginary parts of 2α. As shown later, this alone can significantly improve the performance compared to the conventional QO-STBC/SFBC.

B. With Jafarkhani's Scheme

The proposed scheme can also be applied to another QO-STBC/SFBC code such as $AB(-B)^* A^*$, what is often called Jafarkhani's codes [5]. Similarly, we can add constellation rotation as follows:

$$S = \begin{bmatrix} x_1 & x_2 & x_3 & x_4 \\ -x_2^* & x_1^* & -x_4^* & x_3^* \\ -c^* \cdot x_3^* & -x_4^* & c^* \cdot x_1^* & x_2^* \\ c^* \cdot x_4 & -x_3 & -c^* \cdot x_2 & x_1 \end{bmatrix}, \tag{7}$$

Note that constellation rotation is applied to QAM symbols at different location compared to (1).

The received signal equation for the new STBC/SFBC codewords in (7) is given by

$$\begin{bmatrix} r_1 \\ r_2^* \\ r_3^* \\ r_4 \end{bmatrix} = \begin{bmatrix} h_1 & h_2 & h_3 & h_4 \\ h_2^* & -h_1^* & h_4^* & -h_3^* \\ ch_3^* & h_4^* & -ch_1^* & -h_2^* \\ h_4 & -c^* h_3 & -h_2 & c^* h_1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} n_1 \\ n_2 \\ n_3 \\ n_4 \end{bmatrix} = H\underline{\mathrm{x}} + \underline{\mathrm{n}} \tag{8}$$

Then, $H^H H$ is given by

$$\begin{bmatrix} \sum_{i=1}^4 |h_i|^2 & 0 & 0 & \eta \\ 0 & \sum_{i=1}^4 |h_i|^2 & -\eta^* & 0 \\ 0 & -\eta^* & \sum_{i=1}^4 |h_i|^2 & 0 \\ \eta & 0 & 0 & \sum_{i=1}^4 |h_i|^2 \end{bmatrix}, \tag{9}$$

where

$$\eta = \beta + c^* \beta^* \tag{10}$$

and

$$\beta = h_1^* h_4 - h_2 h_3^* \tag{11}$$

Note that all the non-zero off-diagonal terms of the matrix in (9) are placed at different locations compared to (3). However, we still observe that such non-zero terms can be represented by one variable η and its conjugate. Similarly, we obtain $\eta = 0$ when $c = e^{j\theta}$ with $\theta = -2 \times \angle(\beta) + \pi$ is applied. Therefore, this new STBC/SFBC codewords can also be orthogonalized and achieves full diversity with angle feedback.

For the angle feedback, θ can be quantized similarily. Especially for the 1-bit feedback scheme, θ can be chosen to be 0 or π, according to the following criterion:

$$c = \begin{cases} 1 & \text{if } |Re(\beta)| \leq |Im(\beta)|, \\ -1 & \text{otherwise} \end{cases} \tag{12}$$

Effectively, the performance is the same as the one of $ABBA$ scheme in II-A.

III. Generalized for Multiple Receive Antennas

The proposed scheme can be generalized to the system with multiple receive antennas. With the same STBC/SFBC codewords in (1), the effective channel H_j from the transmitter to the jth receive antenna is

$$H_j = \begin{bmatrix} h_{1j} & h_{2j} & h_{3j} & h_{4j} \\ h_{2j}^* & -h_{1j}^* & h_{4j}^* & -h_{3j}^* \\ h_{3j} & c \cdot h_{4j} & c \cdot h_{1j} & h_{2j} \\ h_{4j}^* & -c \cdot h_{3j}^* & c \cdot h_{2j}^* & -h_{1j}^* \end{bmatrix} \tag{13}$$

where h_{ij} is the channel response from the ith transmit antenna to the jth receive antenna.

With N_r receive antennas, the received signal equation for the new STBC/SFBC codewords in (1) can be written as

$$\begin{aligned} \begin{bmatrix} R_1 \\ R_2 \\ \vdots \\ R_{N_r} \end{bmatrix} &= \begin{bmatrix} H_1 \\ H_2 \\ \vdots \\ H_{N_r} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} N_1 \\ N_2 \\ \vdots \\ N_{N_r} \end{bmatrix} \\ &= \mathbf{H}\underline{\mathbf{x}} + \underline{\mathbf{n}}, \end{aligned} \tag{14}$$

where

$$\begin{aligned} R_j &= \begin{bmatrix} r_{1j} & r_{2j}^* & r_{3j} & r_{4j}^* \end{bmatrix}^T \\ N_j &= \begin{bmatrix} n_{1j} & n_{2j} & n_{3j} & n_{4j} \end{bmatrix}^T. \end{aligned} \tag{15}$$

R_j and N_j are the 4×1 received signal vector and the 4×1 noise vector, respectively, at the jth receive antenna. The new effective channel $\mathbf{H}$ is a $4N_r \times 4$ tall matrix with a group of 4 adjacent time instance or frequency tones. Then, with this new effective channel $\mathbf{H}$, $\mathbf{H}^H\mathbf{H}$ is given by

$$\begin{bmatrix} \Sigma & 0 & \delta & 0 \\ 0 & \Sigma & 0 & \delta^* \\ \delta^* & 0 & \Sigma & 0 \\ 0 & \delta & 0 & \Sigma \end{bmatrix}, \tag{16}$$

where

$$\begin{aligned} \Sigma &= \sum_{i=1}^{4} \sum_{j=1}^{N_r} |h_{ij}|^2 \\ \delta &= \alpha - c^* \alpha^* \end{aligned} \tag{17}$$

and

$$\alpha = \sum_{j=1}^{N_r} (h_{1j}^* h_{3j} + h_{2j} h_{4j}^*) \tag{18}$$

Similarly, c can be chosen such that $c = e^{j\theta}$ with $\theta = -2 \times \angle(\alpha)$ for the optimum solution. In practice with quantization with 1-bit feedback, the same criterion as in (6) can be applied with new α in (18).

IV. The Condition Number of the Effective Channel

As seen in (3) and (9), QO-STBC/SFBC does not obtain full orthogonality which results in interference between symbols within STBC/SFBC codewords. In order to suppress such interference, it is required for the receiver to employ zero-forcing (ZF) or minimum mean-square error (MMSE), or, for better performance, more complicated receivers, such as maximum likelihood (ML) or successive interference cancellation (SIC).

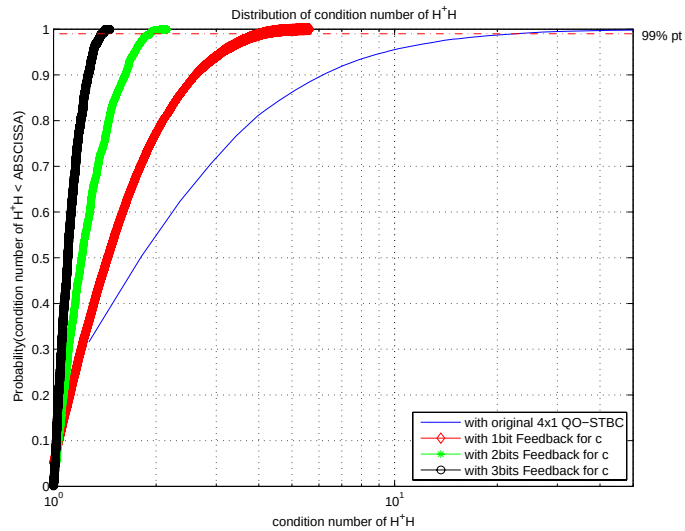

Fig. 1. Cumulative distribution of the condition numbers for $H^H H$ for QO-STBC/SFBC with 4 transmit and one receive antennas

In general, the performance of these interference suppression at receivers depends on the condition number of the effective channel [13].

In order to demonstrate the effectiveness of the proposed scheme, we ran a simple test for a 4-transmit, 1-receive antenna (4×1) system described in Section II-A. For simplicity, h_i is assumed to be an i.i.d. complex Gaussian variable. Figure 1 shows the accumulated distribution of condition number of $H^H H$ with and without 1 or more number of bits feedback for the angle. In the plots, we can see the significant improvement of the channel condition even with as small as 1-bit feedback. For an example, at the 99th percentile, the condition number is reduced from 25 to less than 4 with 1-bit feedback. With such a well-conditioned channel with 1 bit feedback, it is not necessary to use complicated techniques such as ML or SIC; ZF or MMSE should be sufficient, to suppress the interference as shown by our simulation results next.

V. Simulation and Comparison

We simulated the performance of the proposed scheme with 1-bit and exact (full resolution) angle feedback for a 4×1 system and a 4×2 system based on the orthogonal frequency-division multiplexing (OFDM) link model for the 3rd Generation Partnership Project - Long Term Evolution (3GPP-LTE) [14]. One packet, referred to one sub-frame with 7 OFDM symbols in 3GPP-LTE standards, is 0.5 msec long. For the bandwidth, 5 MHz is chosen to have 512 FFT size with 301 data tones. Each user has 6 physical resource block (PRB) equally spaced over the bandwidth where each PRB has 12 adjacent tones. Within a sub-frame, first 3 OFDM symbols have 36 cyclic-prefix tones and next 4 OFDM symbols have 37 cyclic-prefix tones, so the sampling frequency is 7.68 MHz ($i.e., 7.68E6 = (3 \times 36 + 4 \times 37 + 7 \times 512)/0.5E-3$). Channels are assumed to have Doppler shift of 5 Hz. The MMSE receiver is employed with an assumption of perfect channel estimation. For coding, turbo codes defined in [14] are considered. With 3GPP-LTE working assumption of constant

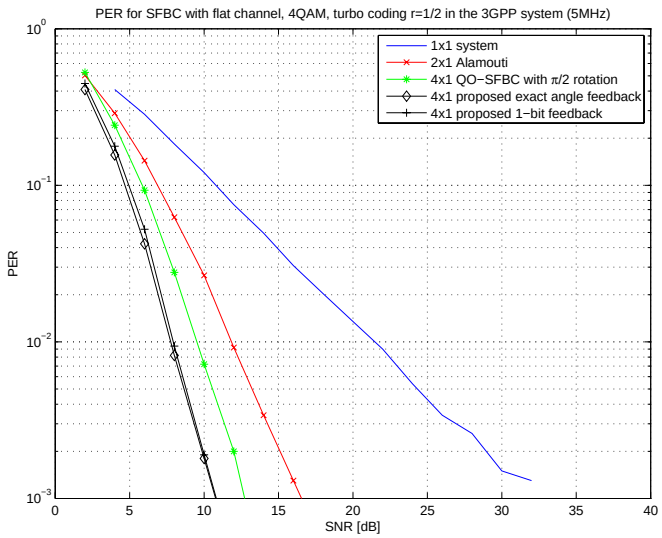

Fig. 2. PER curves of proposed 1-bit feedback and exact angle feedback for a 3GPP-LTE flat channel with 4 QAM and turbo coding r=1/2

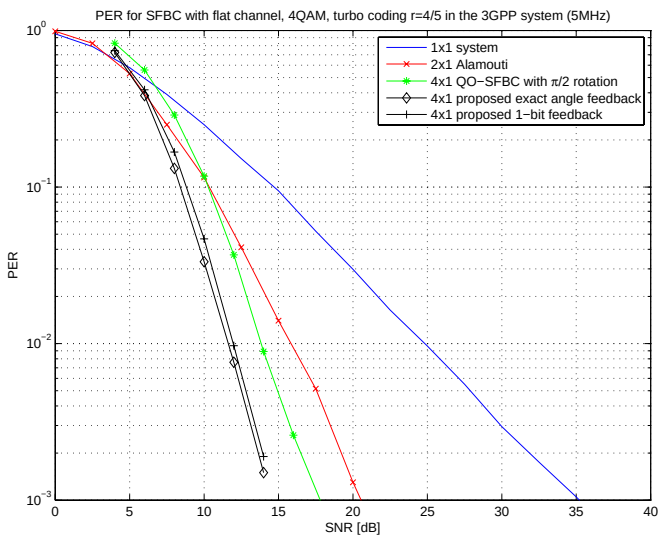

Fig. 3. PER curves of proposed 1-bit feedback and exact angle feedback for a 3GPP-LTE flat channel with 4 QAM and turbo coding r=4/5

modulo, *i.e.*, all four antennas need to always transmit at the same power, antenna selection scheme is not considered for comparison in our simulation.

Figure 2 and Figure 3 show the packet error rate (PER) results as a function of the average received signal-to-noise ratio (SNR) for a flat fading channel. For reference, we plot the performances of a single antenna (1×1) system; 2-transmit, 1-receive antenna (2×1) system using the Alamouti code and 4×1 QO-SFBC with fixed $\pi/2$ rotation (scheme in [6]). The simulation results show that the proposed scheme has the full transmit diversity gain with exact angle feedback. Compared to QO-SFBC with $\pi/2$ rotation, it shows $2 \sim 3$ dB of performance improvement. Notably, with 1-bit feedback, it performs to within about 0.5 dB of the optimum exact angle feedback.

With a frequency selective channel, the proposed scheme suffers from non-orthogonality of the effective channel, which may result in slight performance loss. We consider a Pedestrian A (PEDA) channel, which has been considered as a moderate frequency selective channel to test Wideband Code Division Multipla Access (WCDMA) and High-Speed Downlink Packet Access (HSDPA) modem performance. Typically, a PEDA channel has 46 nsec rms delay spread. The simulation results for a PEDA channel are shown in Figure 4 and Figure 5. While the results illustrate some degree of performance loss due to non-orthogonality of the effective channel, the penalty is not significant. The proposed scheme maintains the gain of $1 \sim 2$ dB or $3 \sim 4$ dB compared to QO-SFBC with $\pi/2$ rotation or the 2×1 system using the Alamouti code, respectively. We also observe that the gain of the proposed scheme over QO-SFBC is bigger when the coding rate is higher. This is because QO-SFBC with $\pi/2$ cannot utilize the coding gain enough to overcome the interference from its non-orthogonality when a weaker code is employed. In addition, 1-bit feedback per PRB with 12 adjacent tone grouping is also considered in order to reduce the feedback overhead. Simulation shows that the performance loss of this grouping is negligible; this indicates only 6 bits are required per user to achieve the suboptimal performance for the 5 MHz operation in 3GPP-LTE.

With multiple receive antennas, the proposed scheme can be applied as described in section III. Figure 6 shows PER curves for 4-transmit and 2-receive antenna (4×2) system. The results illustrate that the proposed scheme has $1 \sim 2$ dB improvement over QO-SFBC with $\pi/2$ rotation. This gain is smaller compared to the gain for the 4×1 system, since the 4×2 system has additional receive diversity gain which mitigates the loss from non-orthogonality in QO-SFBC.

VI. Conclusions

In this paper, new STBC/SFBC with angle feedback is introduced. This can be applied to an existing QO-STBC/SFBC, either ABBA scheme [4] or Jafarkhani's scheme [5], with single angle rotation on QAM symbols in the cordwords. With full resolution of angle feedback, the optimum 4-transmit-antenna STBC/SFBC with full diversity gain at full rate can be achieved. The proposed scheme can be generalized with arbitrary number of receive antennas. We have also proposed a 1-bit feedback scheme based on the observed condition to eliminate the larger between the real and imaginary parts of the interference term. Simulation for a flat fading channel and a frequency selective channel shows that the 1-bit scheme performs to within 0.5 dB of the optimum performance (achieved with exact angle feedback), and gives a 2 to 3 dB improvement over a best-known open-loop QO-STBC/SFBC code.

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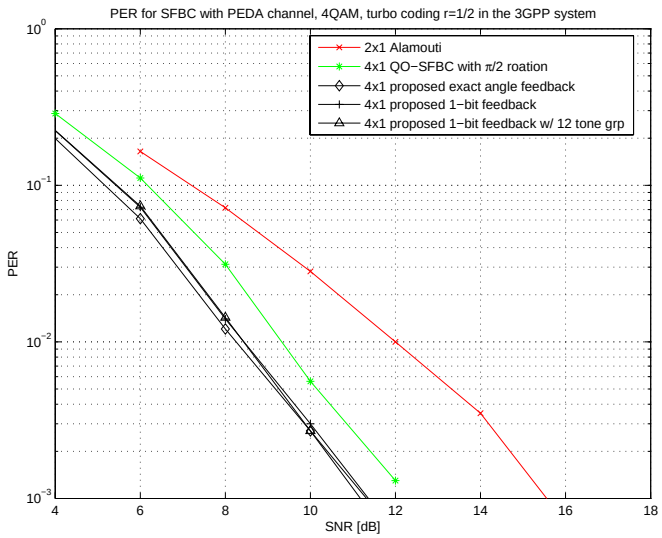

Fig. 4. PER curves of proposed 1-bit feedback and exact angle feedback for a 3GPP-LTE PEDA channel with 4 QAM and turbo coding r=1/2

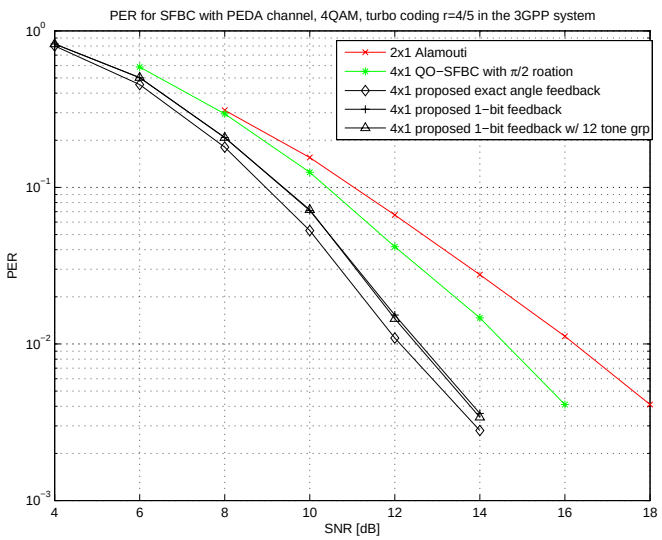

Fig. 5. PER curves of proposed 1-bit feedback and exact angle feedback for a 3GPP-LTE PEDA channel with 4 QAM and turbo coding r=4/5

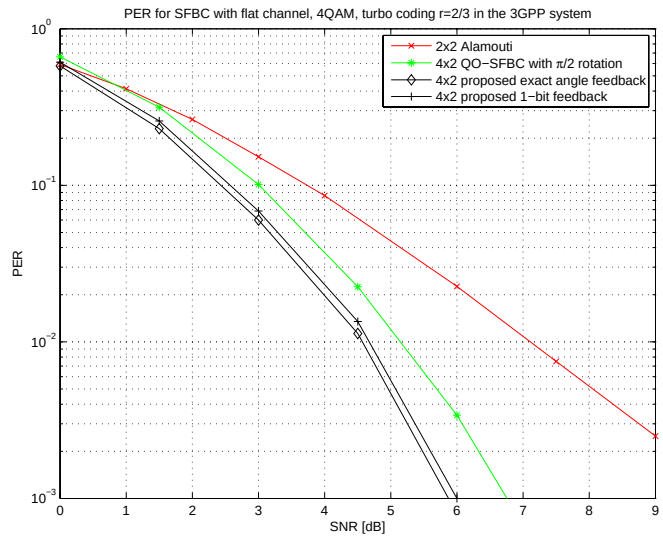

Fig. 6. PER curves of proposed 4×2 scheme with 1-bit feedback and exact angle feedback for a 3GPP-LTE flat channel with 4 QAM and turbo coding r=2/3

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