Closing the Capacity Gap in Wireless Ad Hoc Networks Using Multi-packet Reception

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***Abstract*—In this paper, we compute the throughput capacity of random wireless ad hoc networks in which nodes are endowed with multipacket reception (MPR) capabilities. We show that $\lambda(n) = \Theta\left(\frac{(R(n))^{(1-2/\alpha)}}{n^{1/\alpha}}\right)$ bits per second constitutes a tight bound for the throughput capacity of random wireless ad hoc networks using physical model, where $\alpha > 2$ is the path loss parameter in the physical model, n is the total number of nodes in the network, and $R(n)$ is the MPR receiver range. Compared to the original result derived for plain routing by Gupta and Kumar, MPR closes the capacity gap and achieves a capacity gain of at least $\Theta\left((\log n)^{\frac{\alpha-2}{2\alpha}}\right)$ when $R(n) = \Theta\left(\sqrt{\log n/n}\right)$.**

I. Introduction

The seminal work by Gupta and Kumar [1] on the scaling laws of wireless ad hoc networks show that forwarding information from sources to destinations over multihop paths in which each relay is able to transmit or receive at most one packet at a time is not scalable. As a result, there has been a growing interest in the study of the capacity of wireless ad hoc networks and methods that can be used to improve the order capacity of such networks.

Gupta and Kumar showed that, under the physical model, the throughput capacity of a wireless network has lower and upper bounds of $\Theta(\sqrt{1/n \log n})$ and $\Theta(\sqrt{1/n})$, respectively [1]. Subsequently, Franceschetti et al. [2] closed the gap between these two bounds and obtained a tight bound of $\Theta(\sqrt{1/n})$ under the physical model using percolation theory. In this approach, the communication between relays is kept at short distance with multi-hop transmission inside backbone paths while nodes require to transmit longer distance to reach these backbone paths. In using percolation approach, communication is simple point-to-point without any cooperation between senders and receivers.

A number of techniques have been proposed to improve the capacity of wireless networks. Grossglauser and Tse [3] demonstrated that a non-vanishing capacity can be attained at the price of long delivery latencies by taking advantage of long-term storage in mobile nodes. We can also increase the throughput capacity by using multiple channels [4] or sender-receiver cooperation [5]. Recently, Ozgur et al. [6] demonstrated that the capacity of random wireless ad hoc network scales linearly with n by allowing nodes to cooperate intelligently using distributed MIMO communications.

Multi-packet reception (MPR) is a cooperative approach that enables each receiver to decode multiple concurrent transmissions within its reception radius. Ghez et al. [7], [8] and Tong et al. [9] present the first model of MPR in a framework for many-to-one communication. In this context, multiple nodes cooperate to transmit their packets simultaneously to the same node using directional antennas, multiuser detection (MUD), or multiple input multiple output (MIMO) techniques [10], [11]. The receiver node utilizes MUD and successive interference cancelation (SIC) to decode multiple packets [12]. Recently, Garcia-Luna-Aceves et al. [13] have shown that the throughput capacity with MPR is tightly bounded by $\Theta(R(n))$ under the protocol model. This represents a minimum gain of $\Theta(\log n)$ compared to the capacity bounds obtained by Gupta and Kumar for point-to-point communication under the protocol model. However, this work does not address the physical model.

The contribution of this paper is to compute the throughput capacity of random wireless ad hoc networks under the physical model assumption when all nodes are endowed with MPR. Section II presents the network model we use to obtain the upper and lower bounds on the throughput capacity of wireless networks with MPR, which are derived in Section III. We show that $\Theta\left(\frac{(R(n))^{(1-2/\alpha)}}{n^{1/\alpha}}\right)$ bits per second constitutes a tight bound for the throughput capacity per node in random wireless ad hoc networks. When $R(n) = \Theta\left(\sqrt{\frac{\log n}{n}}\right)$, the throughput capacity is tight bounded by $\Theta\left(\frac{(\log n)^{\frac{1}{2}-\frac{1}{\alpha}}}{\sqrt{n}}\right)$. This is a gain of $\Theta\left((\log n)^{\frac{\alpha-2}{2\alpha}}\right)$ compared to the bound $\Theta(1/\sqrt{n})$ in [1] and [2]. The assumptions we use to obtain these results are similar to those made by Gupta and Kumar [1], except that each node is equipped with MPR capabilities. Furthermore, the results of this paper closed the gap between the upper and lower bounds on the throughput capacity of wireless networks with MPR under the physical model.

II. Network Model

We consider a dense wireless ad hoc network with n nodes distributed uniformly in a square of unit area. Hence, in our model, as n goes to infinity, the density of the network also goes to infinity. Our capacity analysis is based on the extension of physical model for dense networks introduced by Gupta and Kumar [1].

Definition 2.1: Physical Model with Plain Routing
In the physical model of dense random wireless ad hoc networks [1], a successful communication occurs if signal to interference and noise ratio (*SINR*) of the pair of transmitter i and receiver j satisfies

$$\textit{SINR}_{i\to j}=\frac{Pg_{ij}}{BN_0+\sum_{k\neq i,k=1}^{n}Pg_{kj}}\geq\beta, \qquad (1)$$

where P is the transmit power of a node, g_{ij} is the channel attenuation factor between nodes i and j, and BN_0 is the total noise power. The channel attenuation factor g_{ij} is only a function of the distance (the simple path loss propagation model) which is the same as [1]. Therefore, $g_{ij}=|X_i-X_j|^{-\alpha}$ in which $\alpha>2$ is the path loss parameter.

However, in the physical model of MPR, each receiving node has a receiver range such that all the nodes transmitting within this range will be decoded by the receiver. Consequently, the definition of physical model should incorporate this fact in order to better represent this new many-to-one communication scheme. The following proposition states the decoding procedure for MPR. Note that with MPR, we can either decode the received signal for multiple transmitters jointly using maximum likelihood decoding or decode transmitters sequentially using SIC as long as the *SINR* condition is satisfied. We will describe the condition that will satisfy the minimum required *SINR* in definition 2.3.

Proposition 2.2: The transmitter-receiver pair with maximum *SINR* is the nearest set of transmitters, after decoding and subtracting this group from the received signal, the set with the next highest *SINR* is the second nearest group of transmitters, and this continues; i.e., receivers decode the information from the nearest transmitters to farthest ones whose positions are the maximum distance inside of communication range.

Because the channel propagation model is based on the path-loss parameter, it is clear from (1) that the node (or group of nodes) with the closest distance to the receiver has the highest *SINR*. After decoding this (their) packet(s) and subtracting it (them) from the received data, it is obvious that the next packet(s) with highest *SINR* is (are) from the second closest node(s) to the receiver node and this procedure can continue. At a given time t, the decoding procedure for any receiver j in MPR scheme is sequential, i.e., a receiver decodes the information from the highest *SINR* to the lowest *SINR* for MPR with SIC.

Essentially, this proposition states that each group of transmissions from some transmitters can be decoded if and only if the previous group of transmissions from transmitters that are closer to the receiver node was decoded first by the receiver node. The last decoded node occurs at the edge of the circle whose radius is $R(n)$.

Definition 2.3: Physical Model with Multipacket Reception
In the physical model of dense random wireless ad hoc networks [1], the transmissions from all of the transmitters centered around a receiver j with a distance smaller or equal to $R(n)$ occur successfully if the *SINR* of the transmitter $Z(R(n))$ at the edge of this receiver circle satisfies

$$\textit{SINR}_{Z(R(n))\to j}=\frac{Pg_{Z(R(n))j}}{BN_0+\sum_{k\notin A_{Z(R(n))}}Pg_{kj}}\geq\beta, \qquad (2)$$

where $g_{Z(R(n))j}$ is the channel attenuation factor between nodes $Z(R(n))$ and j, $A=\pi R^2(n)$ is the receiver communication range (circle) centered around the receiver j, and $g_{Z(R(n))j}$ and g_{kj} are the channel attenuation factors which are defined earlier.

Any transmission outside the receiver range is considered interference while all the transmissions inside receiver range will be decoded jointly or separately. Note that for the MPR model, the receiver range $R(n)$ defines the area where the receiver is capable of decoding, which contrasts with point-to-point communication [1], for which the transmission range $r(n)$ defines the possible area where the receiver can decode, given that only one transmission is successful at a receiver. Since any transmitter that is closer to the receiver has smaller channel attenuation compared to the edges of the circle, it is easy to show that the *SINR* of these transmitter nodes satisfy equation (2) if these nodes are decoded jointly or separately depending on the distribution of these nodes around the receiver node j.

We assume that nodes cannot transmit and receive at the same time, which means half-duplex communication. The capacity between transmit node i and receive node j is defined as $C_{ij}=B\log(1+\textit{SINR})$ bits/sec. In [1], C_{ij} can be a constant value W if and only if *SINR* is guaranteed to be larger than a constant β. We follow a similar assumption in this paper.

We use the same definition for throughput capacity of unicast as defined in Gupta and Kumar [1] paper.

Definition 2.4: Order of throughput capacity: $\lambda(n)$ is said to be of order $\Theta(f(n))$ bits per second if there exist deterministic positive constants c and c' such that

$$\begin{cases}\lim_{n\to\infty}\text{ Prob }(\lambda(n)=cf(n)\text{ is feasible})=1\\ \lim_{n\to\infty}\text{ Prob }(\lambda(n)=c'f(n)\text{ is feasible})<1.\end{cases} \qquad (3)$$

The distribution of nodes in random networks is uniform, so if there are n nodes in a unit square, then the density of nodes equals n. Hence, if $|S|$ denotes the area of space region S, the expected number of the nodes, $E(N_S)$, in this area is given by $E(N_S)=n|S|$.

An event occurs with high probability (w.h.p.) if its probability tends to one as $n\to\infty$. It follows that, w.h.p., we can get a very sharp concentration on the number of nodes in an area, so we can find the achievable lower bound w.h.p., provided that the upper bound (mean) is given. In the next section, we first derive the upper bound, and then use the Chernoff bound [14] to prove the achievable lower bound w.h.p..

III. Throughput Capacity with MPR

A. Upper Bound With MPR Scheme

In order to compute the upper and lower bounds, we first need to describe some definitions and preliminary results from the earlier work of Garcia-Luna-Aceves et al. in [13].

The per-node throughput capacity of the network is defined as the number of bits per second that every node can transmit w.h.p. to its destination. Note that throughput capacity is equivalent to transport capacity in this paper. Transport capacity is defined in units of bits per second in random networks [1].

A cut Γ is a partition of the vertices (i.e. nodes in the wireless networks) of a graph into two sets. The cut capacity is defined to be the sum of bandwidth of all the edges crossing the cut. Min-cut is a cut whose capacity is the minimum value among the capacity of all cuts. For the wireless networks, we use the concept of *sparsity cut*, as defined by Liu et al. [15], instead of min-cut, to take into account the differences between wired and wireless links.

In the 2-D case, the cut length l_Γ is defined as the length of the cut line segment. For the square region illustrated in Fig. 1, the middle line induces a sparsity cut Γ. Because nodes are uniformly deployed in a random network, such a sparsity cut captures the traffic bottleneck of these random networks on average.

The sparsity-cut capacity is upper bounded by deriving the maximum number of simultaneous transmissions across the cut.

Let $R(n)$ be the radius of the receiver range A, i.e., $A = \pi R^2(n)$. Given that we assume omni-antenna broadcasting, this is the radius that distinguishes the decodable transmitter nodes from the interference.

Lemma 3.1: The disk with radius $R(n)$ centered at any receiver should be disjoint from the other disks centered at the other receivers.

Proof: The proof is by contradiction. If the disks of different receivers overlap, then there exists some transmitters that are within the receiver range of two receiver nodes. Because this node can send different information $2W$ at one time to two different receivers, this contradicts the assumption that each node only transmits W information at a given time. ∎

Lemma 3.2: The asymptotic throughput capacity of a sparsity cut Γ for a unit square region has an upper bound of $\frac{\pi l_\Gamma n W}{2}\frac{R^2(n)}{D(n)}$, where, $R(n)$ and $D(n)$ are the receiver range and the distance between two receiver nodes of MPR respectively. Fig. 2 illustrates these two variables.

Proof: The cut capacity is upper bounded by the maximum number of simultaneous transmissions across the cut. We observe from Fig. 1 that all the nodes located in the shaded area S_{xy} can send their packets to the receiver node located at (x, y). These nodes lie in the left side of the cut Γ within an area called S_{xy} and the assumption is that all these nodes are sending packets to the right side of the cut Γ.

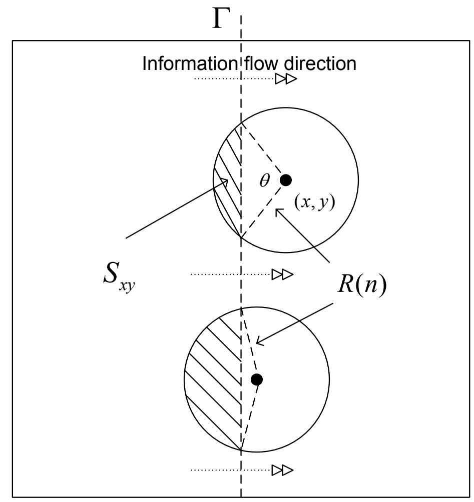

Fig. 1. For a receiver at location (x, y), all the nodes in the shaded region S_{xy} can send messages successfully and simultaneously.

For a node at location (x, y), any node in the disk of radius $R(n)$ can transmit information to this receiver simultaneously and the node can successfully decode those packets. In order to obtain an upper bound, we only need to consider edges that cross the cut. Let us first consider all possible nodes in the S_{xy} region that can transmit to the receiver node. By drawing a circle of radius $R(n)$ centered at (x, y), this region is illustrated in Fig. 1 as S_{xy}. Because nodes are uniformly distributed, the average number of transmitters located in S_{xy} is $n \times S_{xy}$. The number of nodes that are able to transmit at the same time from left to right is upper bounded as a function of S_{xy}.

The area of S_{xy} is $S_{xy} = \frac{1}{2}R^2(n)(\theta - \sin\theta)$. This area is maximized when $\theta = \pi$, $\max_{0\le\theta\le\pi}[S_{xy}] = \frac{1}{2}\pi R^2(n)$.

We can compute the total information capacity C_j for one receiver j at the right side of the cut as $C_j = \frac{1}{2}\pi n W R^2(n)$. In order to guarantee that this statement is true for all of the nodes inside the circle of radius $R(n)$, is to satisfy $SINR_{i\in S_{xy}} \ge \beta$. This is equivalent of physical model for MPR approach. For this reason, the circles whose nodes are transmitting concurrently must be away from each other far enough $(D(n) \ge 2R(n))$ as shown in Fig. 2. Therefore, the total throughput capacity $C(n)$ across the sparsity cut is

$$C(n) \le \left(\left\lfloor \frac{l_\Gamma}{D(n)} \right\rfloor + 1\right) C_j < \frac{\pi n W R^2(n)(l_\Gamma + D(n))}{2D(n)}. \quad (4)$$

Note that $D(n)$ and $R(n)$ are decreasing functions of n, and $\lim\,(l_\Gamma + D(n)) = l_\Gamma$ asymptotically because $\lim D(n) = 0$ as $n \to \infty$. This proves the lemma. ∎

Lemma 3.3: The per source-destination throughput of MPR scheme in a 2-D random network is upper bounded by $O\left(\frac{R^2(n)}{D(n)}\right)$.

Proof: From lemma 3.2, there are $l_\Gamma/D(n)$ different circles of radius $R(n)$ each of them having $\Theta(nR^2(n))$ nodes w.h.p.. Therefore, the average per node throughput capacity

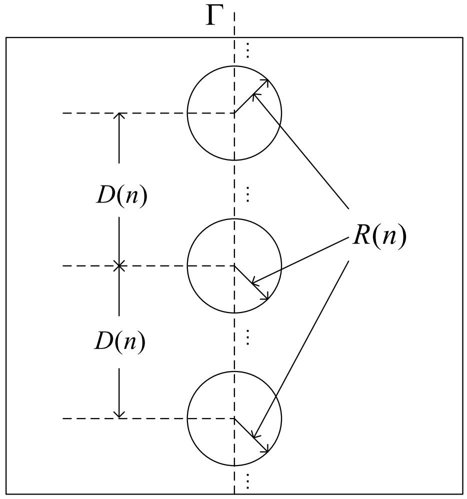

Fig. 2. Upper bound design of the network

can be derived as

$$\lambda(n) = \frac{C(n)}{n} = O\left(\frac{R^2(n)}{D(n)}\right). \quad (5)$$

■

To derive an upper bound for the throughput capacity, we need to obtain a minimum $D(n)$, such that it guarantees $SINR_{Z(R(n))} \geq \beta$. From Proposition 2.2, the decoding sequence of transmissions is from nearest nodes to farthest nodes, i.e., the information of the next transmitter in the communication range can be decoded if and only if the previous one is decoded successfully and then it is subtracted from the received data. Hence, if the *SINR* of the outmost node can be decoded, then all of the nodes inside that circle can be decoded separately or at least jointly. Based on this assumption, we only need to compute the *SINR* of the farthest nodes $Z(R(n))$ (i.e., at the conjunction edge of the communication circle) to make sure $SINR_{Z(R(n))} \geq \beta$. Therefore, the upper bound capacity exists and maximizing this capacity is equivalent of maximizing the following function.

$$\max_{SINR_{Z(R(n))} \geq \beta} \lambda(n) = \max_{SINR_{Z(R(n))} \geq \beta} \frac{R^2(n)}{D(n)} \quad (6)$$

Note that the throughput capacity is maximized by minimizing $D(n)$, while if this value is too small, then Eq. (2) will not be satisfied. Our aim is to find the optimum value for $D(n)$ such that both conditions are satisfied. The following theorem establishes the optimum value that will satisfy Eq. (2).

Theorem 3.4: The per source-destination throughput of MPR scheme in a 2-D random network is upper bounded by $O\left(\frac{(R(n))^{(1-2/\alpha)}}{n^{1/\alpha}}\right)$.

Proof: In order to compute the upper bound, we derive the *SINR* for the node that is in a circle close to the edge of the network. For this receiver node, the Euclidean distances of interfering nodes are at $(iD(n) + R(n))$ if we assume all interfering nodes are at the farthest distance from the receiver node. Then the *SINR* of the transmitter node that is located at the circumference of the communication circle is given by

$$\begin{aligned} SINR_{Z(R(n))} &\leq \frac{P/R^{\alpha}(n)}{\frac{\pi}{2} n R^2(n) \sum_{i=1}^{l_\Gamma/D(n)} \frac{P}{(iD(n)+R(n))^{\alpha}}} \quad (7) \\ &\leq \left(\frac{D(n)}{R(n)}\right)^{\alpha} \frac{1}{\frac{\pi}{2} n R^2(n) \sum_{i=1}^{l_\Gamma/D(n)} \frac{1}{(i+\frac{1}{2})^{\alpha}}}. \end{aligned}$$

The second inequality above stems from the fact that $\frac{R(n)}{D(n)} \leq \frac{1}{2}$. Note that $l_\Gamma/D(n)$ approaches infinity when $n \to \infty$; therefore, the summation $\sum_{i=1}^{l_\Gamma/D(n)} \frac{1}{(i+\frac{1}{2})^{\alpha}}$ converges to a bounded value. This means that there are constant values c_3 and c_4 such that

$$c_3 \leq \sum_{i=1}^{l_\Gamma/D(n)} \frac{1}{(i+\frac{1}{2})^{\alpha}} \leq \sum_{i=1}^{l_\Gamma/D(n)} \frac{1}{(i)^{\alpha}} \leq c_4. \quad (8)$$

Combining (7) and (2), the *SINR* constraint can be revised as

$$\beta \leq SINR_{Z(R(n))} \leq \left(\frac{D(n)}{R(n)}\right)^{\alpha} \frac{2}{\pi c_3 n R^2(n)}. \quad (9)$$

Then the relationship between $R(n)$ and $D(n)$ can be expressed as

$$D(n) \geq \left(\frac{c_3 \beta \pi}{2}\right)^{\frac{1}{\alpha}} n^{\frac{1}{\alpha}} (R(n))^{(1+2/\alpha)}. \quad (10)$$

From Eqs. (5) and (10), the upper bound of the throughput capacity is computed as

$$\lambda(n) = O\left(\frac{R^2(n)}{D(n)}\right) = O\left(\frac{(R(n))^{(1-2/\alpha)}}{n^{1/\alpha}}\right). \quad (11)$$

■

The above bound may seem a loose bound because we used the largest $SINR_{Z(R(n))}$ to ensure that the *SINR* of other nodes inside the communication circle, A, satisfies physical model condition, i.e., $SINR_{U_i} \geq \beta$. $U_i \in A$ is defined as a subset of the set A that contains a group of nodes in the communication circle with the closest distance to the receiver j that will be decoded in the next step. At each time, this subset may contain a single node or multiple nodes to be decoded next. Note that in order to satisfy the physical model, it may be necessary that for some nodes inside the communication circle, we need to decode them jointly using maximum likelihood decoding. We will prove in the next section that indeed $SINR_{U_i} \geq SINR_{Z(R(n))} \geq \beta$ is an achievable lower bound.

B. Lower Bound With MPR Scheme

Before proving the lower bound, we first compute the number of nodes that transmit simultaneously from each communication circle.

We have derived the upper bound in the previous section and then the Chernoff Bound is used to prove the achievable lower bound w.h.p..

Next we prove that, when n nodes are distributed uniformly over a square area, we have simultaneously at least $\frac{l_\Gamma}{D(n)}$ circular regions (see fig. 1), each one containing $\Theta(nR^2(n))$

nodes w.h.p.. The objective is to find the achievable lower bound using the Chernoff bound, such that the distribution of the number of edges across the cut is sharply concentrated around its mean, and hence in a randomly chosen network, the actual number of edges crossing the sparsity cut is indeed $\Theta\left(\frac{(R(n))^{(1-2/\alpha)}}{n^{(1/\alpha)}}\right)$ w.h.p..

Theorem 3.5: Each area A_j with circular shape of radius $R(n)$ contains $\Theta(nR^2(n))$ nodes w.h.p. and uniformly for all values of $j, 1 \leq j \leq \frac{l_\Gamma}{D(n)}$ under the condition that $R(n) \geq \Theta\left(\sqrt{\frac{\log n}{n}}\right)$. Equivalently, this can be expressed as

$$\lim_{n\to\infty} P\left[\bigcap_{j=1}^{l_\Gamma/D(n)} |N_j - E(N_j)| < \delta E(N_j)\right] = 1, \qquad (12)$$

where δ is a positive arbitrarily small value close to zero.

Proof: From the definition of Chernoff bound [14], for any given $0 < \delta < 1$, there exists a $\theta > 0$ such that

$$P\left[N_j - |E(N_j)| > \delta E(N_j)\right] < e^{-\theta E(N_j)} = e^{-\theta n |A_j|}. \quad (13)$$

Thus, we can conclude that the probability that the value of the random variable N_j deviates by an arbitrarily small constant value from the mean tends to zero as $n \to \infty$. This is a key step in showing that when all the events $\bigcap_{j=1}^{l_\Gamma/D(n)} |N_j - E(N_j)| < \delta E(N_j)$ occur simultaneously, then all N_js converge uniformly to their expected values. Utilizing the union bound, we arrive at

$$\begin{aligned} & P\left[\bigcap_{j=1}^{l_\Gamma/D(n)} |N_j - E(N_j)| < \delta E(N_j)\right] \\ = \;& 1 - P\left[\bigcup_{j=1}^{l_\Gamma/D(n)} |N_j - E(N_j)| > \delta E(N_j)\right] \\ \geq \;& 1 - \sum_{j=1}^{l_\Gamma/D(n)} P\left[|N_j - E(N_j)| > \delta E(N_j)\right] \\ > \;& 1 - \frac{l_\Gamma}{D(n)} e^{-\theta E(N_j)}. \qquad (14) \end{aligned}$$

Because $E(N_j) = \frac{\pi}{2} n R^2(n)$, the final result is

$$\begin{aligned} & \lim_{n\to\infty} P\left[\bigcap_{j=1}^{l_\Gamma/D(n)} |N_j - E(N_j)| < \delta E(N_j)\right] \\ & \geq 1 - \frac{l_\Gamma}{D(n)} e^{-\frac{\theta\pi n R^2(n)}{2}} \geq 1 - \frac{l_\Gamma}{2R(n)} e^{-\frac{\theta\pi n R^2(n)}{2}} \quad (15) \end{aligned}$$

If $R(n) \geq \sqrt{\frac{c_5 \log n}{n}} = \Theta\left(\sqrt{\frac{\log n}{n}}\right)$ and as $n \to \infty$, then $\frac{e^{-\frac{\theta\pi n R^2(n)}{2}}}{R(n)} \to 0$, when $\theta > 1/\pi c_5$. Here, the key constraint of $R(n)$ is given as

$$R(n) \geq \Theta\left(\sqrt{\frac{\log n}{n}}\right). \qquad (16)$$

■

Eq. (16) is equivalent to the connectivity condition in the protocol model [1], [13]. It is interesting to note that we did not really use connectivity criterion in the physical model, however, it turns out that the minimum distance for the receiver range in MPR model is equivalent to the connectivity constraint in random networks.

The above theorem demonstrates that w.h.p., there are indeed $\Theta(nR^2(n))$ nodes in each communication region with the constraint in (16). The achievable capacity is only feasible when the receiver range of each node in MPR scheme is at least equal to the connectivity criterion of transmission range in point-to-point communication [1]. Combining the result of Eq. (11) in Theorem 3.4 and (16) in Theorem 3.5, we can state the following theorem for the lower bound of throughput capacity, which implies the lower bound order capacity achieves the upper bound.

Theorem 3.6: The per source-destination throughput capacity of MPR scheme in a 2-D static wireless ad hoc network is lower bounded by $\Omega\left(\frac{(R(n))^{(1-2/\alpha)}}{n^{1/\alpha}}\right)$ provided that $R(n) \geq \Theta\left(\sqrt{\frac{\log n}{n}}\right)$, which means the tight bound is at least $\Theta\left(\frac{(\log n)^{\frac{1}{2}-\frac{1}{\alpha}}}{\sqrt{n}}\right)$ for $\alpha > 2$.

Proof: We first prove that Eq. (11) is an achievable bound and then by applying the minimum receiver range constraint in Eq. (16), we derive the lower bound for this theorem.

In order to derive the achievable lower bound, we design a scheme for separating decodable transmitter nodes inside the communication circle and interference, such that $SINR_{Z(R(n))} \geq \beta_1$. Similar to the derivations in Eq. (7) and using Fig. 2, it is clear that the *SINR* is minimized when the largest value for interference is considered. This value is achieved when we compute the interference for a receiver node in the middle of the network and use the closest possible distance to the receiver node[1]. This lower bound can be written as

$$SINR_{Z(R(n))} \geq \frac{\frac{P}{R^\alpha(n)}}{BN_0 + \frac{\pi}{2} n R^2(n) \sum_{i=1}^{l_\Gamma/2D(n)} \frac{2P}{(iD(n)-R(n))^\alpha}}. \qquad (17)$$

Assume that $D(n)$ satisfies the condition in Eq. (10). If we use the constraint for $R(n)$ in (16), we arrive at

$$\frac{D(n)}{R(n)} \geq \left(\frac{c_3\beta\pi}{2}\right)^{\frac{1}{\alpha}} n^{\frac{1}{\alpha}} (R(n))^{2/\alpha} \geq \Theta\left((\log n)^{\frac{1}{\alpha}}\right), \quad (18)$$

which illustrates that $R(n)$ can be ignored compared with $D(n)$ for large values of n, i.e., $n \to \infty$. We now evaluate the asymptotic behavior of (17) when $n \to \infty$. Combining Eqs. (18) and (17), $SINR_{Z(R(n))}$ can be lower bounded by

$$\begin{aligned} \lim_{n\to\infty} SINR_{Z(R(n))} & \geq \left(\frac{D(n)}{R(n)}\right)^\alpha \frac{1}{\pi n R^2(n) \sum_{i=1}^{l_\Gamma/D(n)} \frac{1}{i^\alpha}} \\ & \geq \left(\frac{D(n)}{R(n)}\right)^\alpha \frac{1}{\pi c_4 n R^2(n)} \geq \frac{c_3}{2c_4}\beta = \beta_1. \end{aligned}$$

[1]Note that the difference between maximum and minimum value of interference is a constant value

This inequality is derived using Eqs. (10) and (8), together with the fact that the second term in the denominator of *SINR* goes to infinity when $n \to \infty$ and, therefore, we can drop the first term related to the noise. Using the same arguments introduced for the computation of the upper bound, we can show that a non-zero value for $SINR_{Z(R(n))}$ can be achieved which implies that the throughput capacity can be achieved asymptotically. ∎

The above theorem demonstrates that gain of at least $\Theta\left((\log n)^{\frac{\alpha-2}{2\alpha}}\right)$ can be achieved compared with the results of Gupta and Kumar [1] and Franceschetti et al. [2]. Combining Theorems 3.4 and 3.6, we arrive at our first major contribution of this paper.

Theorem 3.7: The per source-destination throughput capacity of MPR scheme in a 2-D static wireless ad hoc network is tight bounded as $\Theta\left(\frac{(R(n))^{(1-2/\alpha)}}{n^{1/\alpha}}\right)$. The minimum receiver range is lower bounded as $R(n) \geq \Theta\left(\sqrt{\frac{\log n}{n}}\right)$, which implies a lower tight bound of $\Theta\left(\frac{(\log n)^{\frac{1}{2}-\frac{1}{\alpha}}}{\sqrt{n}}\right)$.

Note that this result shows that we can close the gap in the physical model similar to the results derived by Franceschetti et al. [2] but achieving higher throughput capacity with MPR.

IV. Discussion and Conclusion

The reason for significant increase in capacity with MPR is because unlike point-to-point communications that nodes compete to access the channel, MPR embraces the (strong) interference by allowing higher decoding complexity for all nodes. Interference is the major impeding factor for diminishing behavior of capacity in point-to-point communications. However, MPR approach reduces the negative effect of interference significantly.

It is interesting to note that other researchers have also adopted a version of MPR in conjunction with network coding [16], [17] to increases the capacity of wireless ad hoc networks. These results clearly demonstrate that embracing interference is crucial to improve the performance of wireless ad hoc networks. This paper particularly concentrated on studying the importance of MPR for random wireless ad hoc networks with physical model assumption. Another interesting observation is the fact that increasing the receiver range $R(n)$ increases the throughput capacity. This is in sharp contrast with point-to-point communication in which increasing the communication range actually decreases the throughput capacity and it is again due to the fact that MPR embraces the interference.

There are certain issues that we did not discuss in this paper. Our analysis does not discuss the increased decoding complexity which is necessary for MPR. Our analysis also does not include additional required overhead related to cooperation among nodes. Such comprehensive analysis is the subject of future studies.

This paper shows that the use of MPR can close the gap for the transport (throughput) capacity in random wireless ad hoc networks under the physical model, while achieving much higher capacity gain than that of [2]. The tight bound is $\Theta\left(\frac{(R(n))^{1-2/\alpha}}{(n^{(1/\alpha)})}\right)$ where $R(n)$ is the receiver range in MPR model. For the minimum value of $R(n)$, a gain of $\Theta\left((\log n)^{\frac{\alpha-2}{2\alpha}}\right)$ is achievable in MPR scheme.

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