A Broadcast Approach to Robust Communications over Unreliable Multi-Relay Networks

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***Abstract*—A multi-relay network is studied in which communication from source to relays takes place over a Gaussian broadcast channel, while the relays are connected to the receiver via orthogonal finite-capacity links. Unbeknownst to the source and relays, link failures may take place between any subset of relays and the destination in a non-ergodic fashion. Upper and lower bounds are derived on average achievable rates with respect to the prior distribution of the link failures. It is first assumed that relays are oblivious to the codebook shared by source and destination, and then the results are extended to the non-oblivious case. The lower bounds are obtained via strategies that combine the broadcast coding approach, previously investigated for quasi-static fading channels, and different robust distributed compression techniques.**

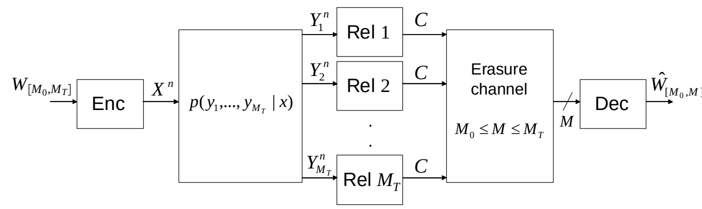

Fig. 1. A single transmitter communicates to a remote receiver via M_T relays connected to the destination through unreliable finite-capacity links (non-ergodic erasures). The number of functioning links M is unknown to source and relays (*uninformed source and relays*).

I. Introduction

This work is motivated by two major characteristics of modern packet data networks. On the one hand, in the presence of delay-sensitive applications, link failures are often appropriately modelled as being unpredictable and non-ergodic. While the conventional transmission design is based on constant-rate data delivery (possibly with an associated outage probability), it is often feasible, and desirable, to deploy transmission strategies that are able to provide *variable-rate* data delivery depending on the current state of the involved links [1]-[3]. On the other hand, data communication networks are typically envisaged to include distributed nodes, whose operation is *decentralized*. In this paper, we consider a baseline model for communication networks that include these two basic elements of non-ergodic link failures and decentralized operation.

Consider a scenario in which a single source communicates with a remote destination via a number of relays (also referred to as "agents" in related literature), with no multi-access interference at the destination (i.e., orthogonal finite-capacity links, see Fig. 1). This model provides a basic framework to address the problem of *decentralized processing*. In [6] the multi-relay network described above was studied under the assumption that the relays are either *oblivious* to or informed about the codebook shared by the source and destination. In the former case, unlike the latter, processing at the relays cannot depend on the specific codebook selected by the source (as in, e.g., compress-and-forward or amplify-and-forward achievable strategies). This assumption is of particular relevance for nomadic applications (in which no signalling is in place to exchange information regarding modulation and coding used at the source) or in networks with inexpensive relays whose processing cannot adapt to the specific source operation. A related model that includes also *non-ergodic* failures was studied in [4] and [5] in the context of *distributed indirect source coding*, also referred to as the CEO problem, in which the source is a given random process to be reproduced at the destination. Therein, it is assumed that, unbeknownst to the agents, the links to the destination may not be active. In this work, we are interested in the communication (channel coding) scenario of Fig. 1, and we extend the analysis in [6] by accounting for *unreliable links* between relays and destinations (*non-ergodic failures*) in the sense of [4] and [5].

The basic idea behind our approach to the analysis of the system in Fig. 1 is to exploit the synergy between the broadcast (BC) coding approach of [3] at the source, which allows for variable-data delivery to the destination depending on the current connectivity conditions, and the robust distributed compression strategies of [4] and [5]. It is noted that a related idea was put forth in [1] and [2] (see also references therein), in which the BC coding approach was combined with successive-description compression techniques for transmission of a Gaussian source over a slowly fading channel without channel state information. For lack of space, in this paper results are provided only for the Gaussian model and without formal proofs. Extension to the discrete model and full proofs can be found in [7].

Notation: The notation $[a, b]$ with a and b integers represents the interval $[a, a+1, ..., b]$, with the convention that if $a > b$ then $[a, b] = \emptyset$. Similarly, the subscript notation $X_{[a,b]}$ denotes the vector $[X_a, ..., X_b]$ with the same convention that, if $a > b$, $X_{[a,b]} = \emptyset$. In general, lower-case letters represent instances of the random variables denoted by the corresponding upper-case letters. Moreover, using standard notation, we will sometimes use superscripts to denote index bounds in sequences as in $x^i = [x_1, \cdots, x_i]$. The use of the superscript will be made clear by the context. Probability distributions are identified by their arguments, e.g., $p_X(x) = \Pr[X = x] \triangleq p(x)$.

II. System Model

We consider the decentralized communication scenario of Fig. 1, in which a source communicates to a destination via M_T "agents" or relays, connected to the receiver via orthogonal finite-capacity (backhaul) links of capacity C. No direct connection from the source to the destination is available. The channel from source to relays is memoryless and Gaussian. The signal $Y_{i,j} \in \mathbb{R}$ received by the agent $i \in [1, M_T]$ at time instant $j \in [1, n]$ is given by

$$Y_{i,j} = X_j + Z_{i,j}, \tag{1}$$

with X_j being the jth transmitted symbol and the noise $Z_{i,j} \sim \mathcal{N}(0, 1)$ being independent and identically distributed (i.i.d.) over both i and j. Notice that the observations $Y_{i,j}$ for different i are statistically exchangeable (see, e.g., [4]). We assume an average input power constraint of P: $1/n \sum_{j=1}^{n} x_j^2 \leq P$. To account for a nomadic scenario and/or to simplify the operations at the relays, we assume at first, as in [6], that the relays are not informed about the codebooks used by the transmitter (**oblivious agents**); see below for details. The non-oblivious case is then considered in Sec. V.

The model described above coincides with the one studied in [6]. Here, however, we are interested in investigating the scenario in which the backhaul links from relays to destination present **non-ergodic failures**. Specifically, following [4], we assume that only a number $M \leq M_T$ of links are functioning at a given coding block, while the remaining $M_T - M$ are *erased* (e.g., in outage) for the entire duration of the current transmission (non-ergodic scenario). We define the probability that $M = m$ as p_m and collect the probabilities p_m in vector $\mathbf{p} = (p_{M_0}, ..., p_{M_T})$, where M_0 represents the minimum guaranteed number of active links [4] (this implies $p_m = 0$ for $m \leq M_0$). We remark that, by the symmetry of (1), the system configuration for a given M depends only on the number M of active links and not on which links are active. Finally, in keeping with the models of [4] and [5] (for distributed source coding), we are interested in scenarios in which *no instantaneous information regarding the current state of the unreliable links (i.e., the value of M) is available a priori to the source and the agents* (**uninformed source and agents**). More precisely, the only information that is available at source and relays is the probability mass function $\mathbf{p}$.

A. Average Rate and Formal Setting

We are interested in *average* achievable rates, where the average is taken with respect to the a priori connectivity probability vector $\mathbf{p}$. Specifically, we consider a *degraded message* structure in which the overall source message of rate T_{M_T} [bits/ channel use] is split into submessages $(W_{M_0}, ..., W_{M_T}) \triangleq W_{[M_0,M_T]}$ of rates $R_{M_0}, ..., R_{M_T}$, respectively, i.e., $W_m \in [1, 2^{nR_m}]$. When $M = m$ links are active, with $m \in [M_0, M_T]$, the receiver decodes messages $W_{[M_0,m]} = (W_{M_0}, ..., W_m)$ of total rate $T_m = \sum_{i=M_0}^{m} R_i$. Notice that the more links are active the more bits (and messages) are decoded. The *average rate* R is defined as

$$R = \sum_{m=M_0}^{M_T} p_m T_m. \tag{2}$$

We remark that, as in [3], the average rate (2) does not have the operational significance of an ergodic rate, the channel being non-ergodic. It is instead a measure of the rate that could be accrued with repeated, and independent, transmission blocks, or of the "expected" rate or throughput. The setting is briefly formalized in the following (see [7] for details) for the oblivious case, which is studied throughout the paper, except in Sec. V.

(*i*) The *encoder* performs a (stochastic) mapping $\phi_F^{(E)}$ (the superscript (E) denotes the encoder) from the messages $W_{[M_0,M_T]}$ to a codeword x^n, namely $x^n = \phi_F^{(E)}(W_{[M_0,M_T]})$ with $F \in \mathcal{F} = [1, |\mathcal{X}|^{n2^{nT_{M_T}}}]$ being a random key that runs over all possible codebooks of size $2^{nT_{M_T}}$. The key $F \in \mathcal{F}$ is revealed to the destination, but *not* to the relays (oblivious relays), and formalizes the fact that the relays have no prior knowledge of the codebook. As detailed in [6], by appropriately choosing the probability $\Pr[F = f]$ of selecting a given codebook $\phi_f^{(E)}$, one can model a scenario in which the signal transmitted by the source X^n, in the absence of knowledge of F (i.e., at the relays), is distributed i.i.d. according to the distribution $p_{X^n}(x^n) = \prod_{i=1}^{n} p_X(x_i)$, where $p_X(x_i)$ is constrained to be a Gaussian distribution with zero mean and power P, and similarly the received signals Y_j^n at the relays appear i.i.d.; (*ii*) Each ith *relay* ($i \in [1, M_T]$), unaware of the codebook F (oblivious relays) and of M, maps the received sequence y_i into an index $s_i \in [1, 2^{nC}]$ via a given mapping $s_i = \phi^{(i)}(y_i^n)$; (*iii*) The *decoder, if $M = m$ links are active,* decodes messages $W_{[M_0,m]} = (W_{M_0}, ..., W_m)$ based on its knowledge of the codebook key F and the received indices s_i over the m active links (these can be assumed by symmetry to be $s_1, ..., s_m$) via a decoding function $\phi_F^{(D)}$; (*iv*) The probability of error when $M = m$ links are active (averaged over F) is defined as $P_{e,m}^n = \Pr[\phi_F^{(D)}(S_{[1,m]}) \neq W_{[M_0,m]}]$. An average rate R (2) is *achievable* if there exists a sequence of codes such that all rates $T_m = \sum_{j=M_0}^{m} R_j$ for $m \in [M_0, M_T]$ are achievable, i.e., $\max_m P_{e,m}^n \to 0$ as $n \to \infty$. The *average capacity* $\mathrm{C}_{\mathrm{avg}}$ is the supremum of all average achievable rates (2).

III. Upper Bounds

In this section, we start the study of the system presented above by deriving upper bounds on the capacity $\mathrm{C}_{\mathrm{avg}}$. It is recalled from the previous section that, as in [6], *we a priori restrict the input distribution to be Gaussian*.

Proposition 1: (Cooperative relays) The following is an upper bound on the capacity C_{avg}

$$\mathrm{C}_{\text{avg}} \leq \max \sum\nolimits_{m=M_0}^{M_T} p_m \left(\sum\nolimits_{j=M_0}^{m} R_j \right), \quad (3)$$

with

$$R_m = \frac{1}{2}\log_2 \left(1 + \frac{m\beta_m P}{1 + m\sigma_m^2 + mP\sum\nolimits_{k=m+1}^{M_T} \beta_k} \right), \quad (4)$$

for $m \in [M_0, M_T]$, where the maximization is taken with respect to parameters $\beta_{M_0}, ..., \beta_{M_T} \geq 0$ with $\beta_{M_0} + ... + \beta_{M_T} = 1$ and $\sigma_m^2 = (1/m + P)/(2^{2mC} - 1)$.

Remark 1: The upper bound of Proposition 1 is obtained by assuming that all of the M relays that are connected to the corresponding active links can fully cooperate in processing their received signals (notice that this implies that they are also informed of which links are active). The upper bound can be interpreted as stating that, under this assumption: (*i*) The best way to operate at the source is to use a standard BC code characterized by powers $\beta_m P$ ($m \in [M_0, M_T]$): such powers correspond to the transmission of message W_m to be decoded at the receiver when $M = m$; (*ii*) The $M = m$ fully cooperative relays can employ without loss of optimality compress-and-forward (CF) techniques with a Gaussian test channel (see also sketch of proof below) to communicate to the receiver, where the parameter σ_m^2 is the compression noise power. Notice that the optimality of CF in this context is a consequence of the obliviousness assumption (see also [6]).

Proof: (sketch) Assume that the relays are perfectly cooperating so that, when $M = m$ links are active, they can be seen as a unique compound agent with m measurements $Y_{[1,m]}^n$ (since the signals are statistically equivalent, there is no loss in generality in this choice of Y_j^n). It is easy to see that the compound agent can be equivalently considered as having scalar measurements (recall (1)): $Y_i^{(m)} = 1/m \sum\nolimits_{j=1}^{m} Y_{j,i}$, since there is no performance loss in projecting the received signal over the signal space, the noise in (1) being uncorrelated over the agents. From [6], it is known that, under the given assumptions, the optimal operation at the compound agent (which is clearly aware of the capacity mC toward the destination) is to quantize to a rate mC bits/ source symbol the received signal via a Gaussian test channel $V_m = Y^{(m)} + Q_m$ with $Q_m \sim \mathcal{N}(0, \sigma_m^2)$ independent of $Y^{(m)}$. From standard arguments in rate-distortion theory, in order to have vanishing probability of error in the quantization process (as the block size n increases), we can set $mC = I(V_m; Y^{(m)})$, thus obtaining $\sigma_m^2 = (1/m + P)/(2^{2mC} - 1)$. As a result, since the source is not informed about the current value of $M = m$, the equivalent channel can be seen as a *degraded Gaussian broadcast channel*, in which the $M_T - M_0 + 1$ destinations observe received signals V_m with equivalent noise variances $1/m + \sigma_m^2$ for $m \in [M_0, M_T]$. Notice that such variances are clearly decreasing with m. Recalling the capacity region for the Gaussian broadcast channel, bound (4) then easily follows. ■

It is interesting two consider two further enhanced systems that provide alternative upper bounds. The first is obtained by assuming an "ergodic" system, in which any ith link is always active (fully reliable) with capacity equal to the average $E[C_i] = C \cdot (1 - \Pr[A_i = 0])$, where A_i is a random variable accounting for whether the ith link is active ($A_i = 1$) or not ($A_i = 0$). One can choose different joint distributions $\Pr[A_1, ..., A_{M_T}]$ that are compatible with the probabilities $\mathbf{p}$ (i.e., $\Pr[M = m] = p_m$), and any such distribution generally leads to different $E[C_i]$ and thus different upper bounds. Here, for simplicity, we focus on joint distributions such that the marginals $\Pr[A_i]$ are the same for each links (it can be seen that these can be found for any M_T).

Proposition 2: (Ergodic) The following is an upper bound on the average capacity:

$$\begin{aligned} \mathrm{C}_{\text{avg}} \quad \leq \quad & \max_{r \geq 0} \min_{k \in [0, M_T]} \{ k(E[C] - r) \quad (5) \\ & + \frac{1}{2}\log_2 \left(1 + P\left(M_T - k\right)\left(1 - 2^{-2r}\right) \right) \Big\}, \end{aligned}$$

where $E[C] = C \cdot (1 - \Pr[A_i = 0])$ with $\Pr[A_i]$ being the marginal distribution of the ith link corresponding to any joint distribution $\Pr[A_1, ..., A_{M_T}]$ such that $\Pr[A_i = 0] = \Pr[A_j = 0]$, for $i \neq j$, and $\Pr[M = m] = p_m$ for $m = 0, ..., M_T$.

Proof: Rate (5) is the (Shannon) capacity of the system for the case in which $p_{M_T} = 1$ (fully reliable links) and the links have the same capacity $E[C]$ ("ergodic" system) [6]. Now, it can be seen that the (Shannon) capacity for the "ergodic" system provides an upper bound to the average rate of the non-ergodic model at hand. In fact, any average rate achievable in the non-ergodic model can be achieved in the "ergodic" system by time-sharing. ■

A second alternative upper bound can be obtained by assuming informed source and relays.

Proposition 3: (Informed source and relays) The following is an upper bound on the average capacity: $\mathrm{C}_{\text{avg}} \leq \sum\nolimits_{m=M_0}^{M_T} p_m T'_m$ with

$$\begin{aligned} T'_m \quad = \quad & \max_{r_m \geq 0} \min_{k \in [0,m]} \{ k(C - r_m) \quad (6) \\ & + \frac{1}{2}\log_2 \left(1 + P(M_T - k)(1 - 2^{-2r_m}) \right) \Big\}. \end{aligned}$$

Moreover, rate $\sum\nolimits_{m=M_0}^{M_T} p_m T'_m$ with (6) is the capacity for the system at hand if we assume that *both source and relays are informed about the value of* M (while the relays are still oblivious to the source codebook).

Proof: Assume that the relays are aware of the current number $M = m$ of active links (while still being oblivious to the source codebook). The upper bound then follows from Theorem 5 in [6]. Moreover, the capacity result is a consequence of the fact that assuming informed source and relays amounts to considering a model with fully reliable links, which was solved for the Gaussian case in [6]. ■

IV. Achievable Rates

In the following, motivated by the upper bound of Proposition 1, we propose achievable schemes based on the BC coding strategy of [3] and CF at the relays. The source transmits a superposition of $M_T - M_0 + 1$ codewords of rates R_m for $m \in [M_0, M_T]$. When $M = m$, the receiver decodes $W_{[M_0,m]}$. The two techniques proposed in the following differ in the way the CF strategy is implemented in terms of compression at the agents and decompression/ decoding at the receiver, and entail increasing levels of complexity.

A. Broadcast Coding and Single-Description Compression (BC-SD)

In this section, we consider a transmission strategy based on BC coding and single-description (SD) compression at the relays. In other words, each relay sends over the backhaul link a single index (description), which is a function of the received signal. The compression/decompression scheme is inspired by the technique used in [4] for robust distributed source coding in a CEO problem. The technique works by performing random binning at the agents, as is standard in distributed compression. Moreover, the test channel (i.e., equivalent compression noise) and binning rate are selected so that the receiver can recover with high probability the compressed signals on the M active links irrespective of the realized value of M as long as it is $M \geq M_0$ (as guaranteed by assumption). In other words, design of the compression scheme targets the *worst-case scenario* of $M = M_0$. Notice that, should more than M_0 links be active ($M > M_0$), the corresponding compressed signals would also be recoverable at the receiver, since, by design of the binning rate, any subset of M_0 descriptions can be decompressed [4]. After decompression is performed, the receiver uses all the M signals obtained from the relays to decode the codewords up to the Mth layer (that is, the layers with rates R_m with $M_0 \leq m \leq M$).

Proposition 4: (BC-SD) The average rate (2) is achievable for

$$R_m \leq \frac{1}{2}\log_2\left(1+\frac{m\beta_m P}{1+\sigma^2+mP\sum_{k=m+1}^{M_T}\beta_k}\right) \quad (7)$$

and σ^2 satisfying

$$C \geq \frac{1}{2}\log_2\left[\left(1+\frac{M_0 P}{1+\sigma^2}\right)^{\frac{1}{M_0}}\left(1+\frac{1}{\sigma^2}\right)\right], \quad (8)$$

for any power allocation $\beta_{M_0}, ..., \beta_{M_T} \geq 0$ with $\beta_{M_0} + ... + \beta_{M_T} = 1$.

Remark 2: Similar to the discussion around Proposition 1, the power $\beta_m P$, $m \in [M_0, M_T - 1]$, accounts for the codebook used for the transmission of the mth layer to be decoded at the receiver when $M = m$. Moreover, Gaussian test channels are used at each relay for compression, and condition (8) is shown in [4] to guarantee that the decoder is able to decompress the descriptions corresponding to any subset of M_0 agents. We finally notice that the only difference between the achievable rate of Proposition 4 obtained with BC-SD and the upper bound of Proposition 1 is related to the powers of the equivalent compression noise (compare (7) with (4)).

Remark 3: For $M_T = M_0$ (fully reliable links), the achievable rate of Proposition 4 coincides with the one presented in Theorem 1 of [6].

Remark 4: (Joint Decompression/Decoding) A potentially more efficient (but also more complex) implementation of a system working with BC coding and SD compression can be designed based on joint decompression/decoding, similarly to the scheme proposed in [6]. We refer to [7] for further analysis and discussion.

B. Broadcast Coding and Multi-Description Robust Compression (BC-MD)

In this section, we propose to couple the BC coding approach considered throughout the paper with multi-description (MD), rather than SD, compression at the agents. The idea follows the work in [5] , which focused on the CEO problem. Accordingly, each relay shares the nC bits it can convey to the destination between multiple descriptions of the received signal to the decoder. The basic idea is that different descriptions are designed to be recoverable only if certain connectivity conditions are met (that is, if the number of functioning links M is sufficiently large). This adds flexibility and robustness to the compression strategy.

To simplify the presentation, here we focus on the two-agent case ($M_T = 2$). Dealing with the more general setup requires a somewhat more cumbersome notation, but is conceptually a straightforward extension. Moreover, without loss of generality, we assume $M_0 = 0$ or $M_0 = 1$, since with $M_0 = M_T = 2$ the system coincides with the one with fully reliable links studied in [6]. The two agents send two descriptions: a basic one to be used at the receiver in case the number of active links turns out to be $M = M_0 = 1$ and a "refined" one that will be used only if $M = M_T = 2$. It is also noted that for the scheme at hand the only difference between the cases $M_0 = 0$ and $M_0 = 1$ is in the prior $\mathbf{p} = (p_0, p_1, p_2)$, where in the former case, unlike the latter, we have $p_0 > 0$.

Proposition 5: (*BC-MD*) For $M_T = 2$, $M_0 = 0$ or 1, the average rate (2) is achievable for

$$R_1 \leq \frac{1}{2}\log_2\left(1+\frac{\beta P}{1+(1-\beta)P+\sigma_1^2+\sigma_2^2}\right) \quad (9a)$$

$$\text{and } R_2 \leq \frac{1}{2}\log_2\left(1+\frac{2(1-\beta)P}{1+\sigma_2^2}\right) \quad (9b)$$

with any power allocation $0 \leq \beta \leq 1$, and any σ_1^2 and σ_2^2 such that

$$C \geq \frac{1}{2}\log\left(1+\frac{P+1}{\sigma_1^2+\sigma_2^2}\right) + \frac{1}{4}\log\left(\frac{\left(\sigma_1^2+\sigma_2^2\right)^2\left(2P+\sigma_2^2+1\right)\left(\sigma_2^2+1\right)}{\left(2P+\sigma_1^2+\sigma_2^2+1\right)\left(\sigma_1^2+\sigma_2^2+1\right)\sigma_2^4}\right). \quad (10)$$

Remark 5: In the MD scheme achieving the rate above, each transmitter divides its capacity C in two parts, say with a fraction $0 \leq \lambda \leq 1$ devoted to the first ($m = 1$) and $(1-\lambda)$ to the second ($m = 2$) description. Denote as V_{mi} the auxiliary random variable defining the mth description of the ith agent. For both descriptions, Gaussian test channels are used. For the first description, no random binning is used and the rate of the compression codebook is $\lambda C = I(V_{1i}; Y_1)$ to guarantee correct compression from standard rate-distortion theoretic arguments. For the second description, test channel and binning rate are selected so that the two descriptions of both agents are recoverable at the destination whenever $M = 2$. To ensure this, it is sufficient to impose the condition $2(1 - \lambda)C \geq I(V_{21}, V_{22}; Y_1, Y_2|V_{11}, V_{12})$ from distributed lossy rate-distortion theory, see, e.g., [5]. Notice that the latter inequality exploits the fact that the first descriptions V_{11} and V_{12} have been correctly decompressed at the decoder when $M = 2$, and thus provide side information. Variances σ_1^2 and σ_2^2 in (9)-(10) account for the compression noises for the first and second description, respectively, and condition (10) follows from the discussion above. The powers $(\beta P, (1-\beta)P)$ represent, as in the rest of the paper, the BC code.

Remark 6: On letting $\sigma_2^2 \to \infty$ for the Gaussian model, Proposition 5 reduces to Proposition 4 for $M_T = 2$, $M_0 = 0$ or 1.

V. Non-Oblivious Agents

In this section, we briefly consider the model in which the agents are informed about the codebook used at the source, that is, equivalently, about the key F. A similar model was considered in [6] for the case of fully reliable links, $M_0 = M_T$. We first consider a simple upper bound on the capacity (i.e., maximum average achievable rate) for this scenario that is a direct consequence of cut-set arguments. Specifically, it can be seen that the average capacity for the setup at hand is upper bounded by

$$\mathrm{C}_{\mathrm{avg}} \leq \sum_{m=M_0}^{M_T} p_m \min\left\{\frac{1}{2}\log_2\left(1+mP\right),\ mC\right\}, \tag{11}$$

where the first term in the $\min\{\cdot,\cdot\}$ follows by considering the cut between source and agents (agents not connected to the destination cannot contribute to the rate) and the second depends on the cut from agents to destination.

As for an achievable strategy, we propose the following scheme that generalizes the BC-SD strategy considered in the previous section. In the proposed scheme, the source uses BC coding with Gaussian codebooks as considered throughout the rest of the paper. However, on top of the $M_T - M_0 + 1$ layers assumed in the schemes described in Sec. IV, here the source superimposes a further layer carrying a common message, say W_0, with rate R_0, to be *decoded* by all agents (recall that in our model all agents are statistically equivalent) and then forwarded to the destination. We would like the destination to be able to recover such a message at all times, that is, as long as the number of active link M satisfies $M \geq M_0$. This is akin to the SD approach to compression studied in Sec. IV-A. Towards this goal, each agent reserves a rate of R/M_0 on its outgoing links to send an index computed as a random function of the decoded W_0. It can be easily seen that, even though the agents are unaware of which links are currently active (but only that $M \geq M_0$), the receiver will be able to recover W_0 with vanishing probability of error as $n \to \infty$ (this is a special case of the Slepian-Wolf problem). The extra layer carrying W_0 is decoded first by the agents and cancelled, and the rest of coding/ decoding takes place as for the BC-SD scheme of Sec. IV-A with the caveat that now the remaining link capacity to forward compression indices is $C - R/M_0$.

Proposition 6: The average rate (2) is achievable in the presence of non-oblivious relays for the Gaussian model with

$$R_{M_0} \leq \tilde{R}_{M_0} + R_0, \tag{12a}$$

$$R_m \leq \tilde{R}_m \text{ for } m = M_0+1, ..., M_T \tag{12b}$$

and

$$R_0 = \frac{1}{2}\log_2\left(1 + \frac{\beta_0 P}{1+(1-\beta_0)P}\right), \tag{13}$$

with rates $\tilde{R}_m$ satisfying the inequalities in (7), and σ^2 satisfying

$$C - \frac{R_0}{M_0} \geq \frac{1}{2}\log_2\left[\left(1+\frac{M_0 P(1-\beta_0)}{1+\sigma^2}\right)^{\frac{1}{M_0}}\left(1+\frac{1}{\sigma^2}\right)\right], \tag{14}$$

for any power allocation $\beta_0, \beta_{M_0}, ..., \beta_M \geq 0$ with $\beta_0 + \beta_{M_0} + ... + \beta_M = 1$.

Proof: This result follows easily from the proof of Proposition 4 and the description of the scheme provided above. ■

Remark 7: The parameter β_0 in (13)-(14) represents the amount of power spent for transmission of message W_0. Moreover, if $\beta_0 = 0$, the rate of the proposition above reduces to the BC-SD scheme of Proposition 4.

VI. Numerical results

Consider a two-agent system $(M_T = 2)$ with $M_0 = 1$ guaranteed functioning links. We start by considering the performance with oblivious agents. We compare the performance of the schemes described above, with single description (SD) or multi-description (MD) compression. For reference, we consider the upper bound (4) corresponding to cooperative relays (labelled as "cooperative"). To assess the impact of non-ergodic link outage, we also show the ergodic upper bound (5) with average link capacity $E[C] = (1 - p_1/2)C$ (which corresponds to assuming identically distributed links), which leads to [6] $\mathrm{C}_{\mathrm{avg}} \leq 1/2\log_2(1 + 2P(1 - 2^{-4E[C]}(\sqrt{P^2 + 2^{4E[C]}(1+2P)} - P)))$ (labelled as "ergodic"). The informed upper bound is not shown since for this example it is very close to the ergodic one. Finally, the rate of a baseline single-layer (SL), or non-broadcast, transmission in which the source only sends one information layer to be decoded in the worst case scenario $M_0 = 1$ and the relays perform SD compression is shown for reference. The rate of this SL-SD scheme is easily seen to be $R_{SL-SD} = \frac{1}{2}\log_2\left(1 + P/(1+\sigma^2)\right)$, with $\sigma^2 = (1+P)/(2^{2C} - 1)$.

Fig. 2 shows the average rates of the proposed schemes for $P = 15\text{dB}$ and $C = 0.5$ versus the probability $p_2 = 1 - p_1$ of having $M = 2$ active links (rather than the minimum

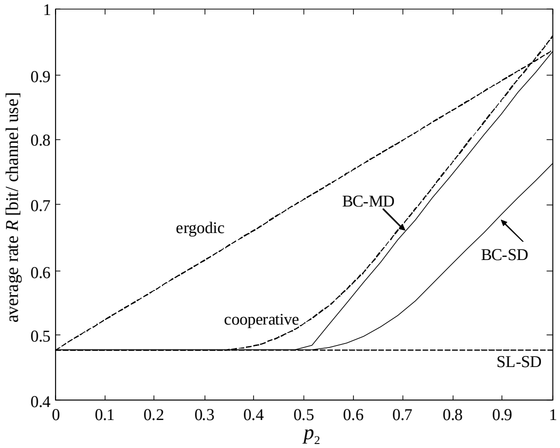

Fig. 2. Average achievable rates (2) for the proposed BC-based schemes with single description (SD) or multi-description (MD) compression, versus the probability $p_2 = 1 - p_1$ of having $M = 2$ active links. For reference, the upper bound (4) achievable with cooperative relay, the upper bound corresponding to ergodic link failures and the rate of single-layer (SL), or non-broadcast, transmission with SD compression are also shown ($P = 15\text{dB}$ and $C = 0.5$).

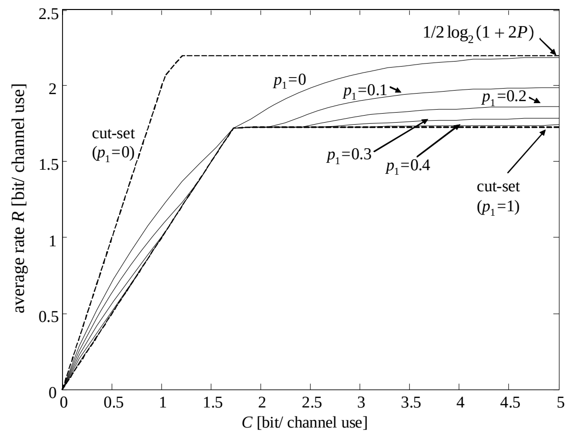

Fig. 3. Average achievable rate (12) with non-oblivious agents versus capacity C, for various values of p_1 and $P = 10dB$. ($M_T = 2$ with $M_0 = 1$). Also shown is the cut-set bound (11) for $p_1 = 0$ and $p_1 = 1$.

guaranteed $M_0 = 1$). The rates are optimized numerically over the parameters at hand (i.e., the compression noise variances σ_i^2 and power allocation β). It can be seen that the BC coding strategy provides relevant advantages over SL as long as the probability p_2 is sufficiently large, since it offers the possibility to exploit better connectivity conditions when they arise. Moreover, MD compression clearly outperforms the SD-based approach for all values of p_2 for which BC coding is advantageous, due to the added flexibility in allocating part of the backhaul link rate for the case of full connectivity ($M = M_T$). In particular, BC-MD performs very close to the upper bound of cooperative relays and for $p_2 = 1$ achieves the capacity for $M_0 = M_T = 2$ of [6] (that is, the ergodic bound above with $p_1 = 0$).

We now consider non-oblivious agents. Fig. 3 shows the average achievable rate for a two-agent system $(M_T = 2)$ versus capacity C with $M_0 = 1$, for various values of p_1 (probability of $M = 1$) and $P = 10dB$. The rates are compared with the upper bound (11) drawn for two representative values of p_1, namely 0 and 1. From (11) and Proposition 6, it is noted that the cut-set bound for $p_1 = 1$ coincides with the rate achievable by sending only the message W_0, that is, by setting $\beta_j = 0$ for $j \in [M_0, M_T]$ in (12). Therefore, for $p_1 = 1$, the proposed scheme is optimal for any value of C, and there is no need for compression of the received signal. Considering then the other limiting case, $p_1 = 0$, it is seen that the proposed scheme achieves the cut-set bound, and specifically the fully cooperative rate $1/2 \log_2 (1 + 2P)$, for C sufficiently large. Moreover, this result is achieved by setting $\beta_0 = 0$ or equivalently $R_0 = 0$, that is, by not exploiting the decoding capability of the agents. This fact is immediate if one notices that for C large and $p_1 = 0$, the two received signals can be sent by two agents with full reliability to the destination via quantization (see also [6]). Increasing p_1, the proposed scheme does not achieve the cut-set bound (not shown), even though the loss is rather limited. Furthermore, in general, for $p_1 < 1$ one can gain by using the backhaul links to send "soft" (quantization) information, along with the "hard" information on W_0, as is clear by comparing the performance with the cut-set bound with $p_1 = 1$ (see discussion above). In the example at hand, for $p_1 \gtrsim 0.4$ such gain vanishes.

VII. Concluding Remarks

Focusing on a multi-relay network with one transmitter-receiver pair and unreliable orthogonal link between each relay and the destination, we have exploited the synergy between the BC approach of [3] and the distributed source coding techniques of [4] and [5] to propose a number of robust communication strategies. This work opens a number of possible avenues for future research, such as the extension to multi-user scenarios with more than one source.

VIII. Acknowledgment

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