Insights Into Feedback and Feedback Signaling for Beamformer Design

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Abstract— **Multi-site cooperative transmission is gaining industrial support in latest standard development. One key criteria to sucessfully implement a multi-site cooperative transmission system is the need to quantize an expanded set of channels. To maintain quantization precision, codebook size needs to scale roughly exponentially with the number of total antennas and renders quantizing the expanded channel set a computationally demanding task. In this paper, we identify a connection between noncoherent communication and the phase invariant property of beamforming vectors. We leverage techniques from the noncoherent communication literature to design an efficient beamforming vector quantization scheme. Our proposed scheme has a quantization complexity that grows linearly with number of antennas. Simulation results indicate this method has slightly more quantization distortion than RVQ while being significantly more efficient.**

Index Terms— **beamforming vector quantization, efficient vector quantization, trellis coded vector quantization**

I. INTRODUCTION

Recent standardization activities in the 3GPP Long Term Evolution (LTE) identified cooperative transmission from multiple base stations as a candidate technology for future releases. It is well known that cooperative transmission gives significant performance boost to mobile stations (MS) that are close to the edges of the cooperating cells [1], [2]. Typically, these cooperating cells are connected via a high-rate and reliable backbone so that information known by one station can be rapidly shared with the other cooperating stations. Many early works that studied cooperative transmission treated the multiple transmitting sites as an augmented multiple-antenna transmission system and directly applied well-known MIMO techniques in a single-cell setup. One prerequisite to effective cooperative transmission is the availability of channel state information at the cooperating base stations (CSIT). Many systems (including LTE) employ frequency division duplexing to manage the uplink and downlink channels where channel reciprocity does not usually hold. The receiver may provide the transmitter with CSIT by feeding back channel information to the transmitter. Typically this feedback channel has a much lower transmission rate than the forward channel.

An important type of CSIT is the beamforming vector preferred by the receiver. It is usually determined by finding the vector in a predefined vector codebook known to both the transmitter and the receiver that has the smallest chordal distance from the ideal vector. During operation, the receiver feeds back the index of the preferred vector in the codebook to the transmitter. The vector ordering within a codebook is typically random as the ordering does not affect the minimum distance of the codebook. Indeed, in spatially uncorrelated Rayleigh fading, performance of a quantization codebook is primarily determined by its minimum distance when feedback is received perfectly. In reality, feedback channels are never exempt from channel effects. Furthermore, CSIT feedback cannot be coded over arbitrarily many feedback channel uses as the feedback information becomes outdated and useless. Thus it is desirable that if feedback is decoded erroraneously to a codeword that is a neighbor to the intended codeword, the performance degradation is as small as possible. This is ensured when feedback channel codewords separated by small Euclidean distance represent beamforming vectors separated by small chordal distance.

Comparing to single-cell CSIT feedback, each MS needs to feed back an expanded set of CSIT with a number of bits roughly proportional to the number of cooperating base stations. To maintain quantization precision, the codebook size grows exponentially with the total number of transmit antennas. This leads to exponential growth in quantization complexity and can put a strain on mobile processing.

In this paper, we identify a crucial connection between the design of noncoherent additive white Gaussian noise (AWGN) communication systems and the rotation-invariant property of beamforming vectors. Using this property, we propose an efficient quantization scheme that maps points in the Grassmann manifold to points on a sphere that are appropriate for protection against channel effects. Specifically, we leverage existing trellis coded quantization (TCQ) technique for the noncoherent communication in our vector encoder [3]–[6]. This scheme enjoys efficient encoding and decoding with complexity linear to the total number of antennas, while providing decent performance.

The rest of the paper proceeds as follow. Section II provides details to our system assumptions. Section III discusses conventional quantization methods, while Section IV discusses several simple quantization algorithms. Section V provides simulations to illustrate various system properties, and Section

VI has concluding remarks and potential future directions.

II. SYSTEM MODEL

We consider a beamforming transmission system where the transmitter has M antennas and the receiver has a single antenna. The M transmit antennas can be co-located or spread over multiple sites. We assume that downlink and uplink transmissions use different frequency spectrums such that channel reciprocity does not hold. The input-output relationship of the forward channel can be described by

$$y = \mathbf{h}^H \mathbf{w} s + n, \tag{1}$$

where y is the received symbol, $\mathbf{h} \in \mathbb{C}^{M \times 1}$ is the channel vector, $\mathbf{w}$ is a unit-norm beamforming vector, s is the transmit symbol, and n is the zero-mean additive white Gaussian noise (AWGN). The transmit symbol s satisfies the power constraint $E(|s|^2) \leq P$, where P is the available power to the transmitter. We assume the communication system experiences i.i.d. Rayleigh fading where each channel coefficient undergoes complex normal fading $\mathcal{CN}(0,1)$. We assume the noise power satisfies $E(|n|^2) = 1$.

We do not focus on channel estimation effects in this paper and thus we assume the receiver has instantaneous and perfect knowledge of $\mathbf{h}$. Likewise, the transmitter has perfect knowledge of the reverse channel at the time feedback is transmitted. With an appropriate combining vector, the feedback channel can be treated as an additive white Gaussian noise (AWGN) system. Finally, we assume feedback takes N_f feedback channel uses and the average feedback rate is B/N_f bits per feedback channel uses. Hence the total feedback consists of B bits.

III. CONVENTIONAL QUANTIZATION ALGORITHMS

The common approach to beamforming vector feedback taken by many papers in the limited feedback literature (see [7] and references therein) assumes a predefined codebook of beamforming vectors $\mathcal{W} = \{\mathbf{w}_1, \ldots, \mathbf{w}_{2^B}\}$ is known to both the transmitter and the receiver. Based on the chordal distance, the receiver selects the preferred beamforming vector by

$$\mathbf{w} = \underset{\mathbf{w}' \in \mathcal{W}}{\operatorname{argmin}}\, 1 - \frac{|\mathbf{h}\mathbf{w}'|^2}{\|\mathbf{h}\|^2} = \underset{\mathbf{w}' \in \mathcal{W}}{\operatorname{argmax}}\, |\mathbf{h}\mathbf{w}'|^2. \tag{2}$$

It then feeds back the index of $\mathcal{W}$ to the transmitter using B bits of feedback. This approach incurs a quantization complexity of $O(2^B)$.

Channel quantization complexity does not usually raise concern because typical codebook size is usually small (LTE codebook can have 4 bits for 4 antennas). With the recent research interest into multi-site cooperative communication, the effective number of transmit antennas increases and so do the codebook size and the quantization complexity. For a practical example, a three-site cooperative beamforming system in LTE has a maximum of 12 antennas and requires roughly 10 bits of feedback to maintain the same level of quantization performance if the channel is quantized jointly. This requires searching through more than a thousand vectors and can put a huge computational strain on the receiver. As such, we want to find a more efficient algorithm to perform vector quantization and channel coding.

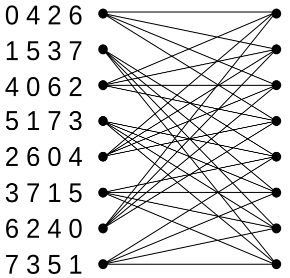

Fig. 1. Trellis of the 8-state TCM in [9].

IV. PROPOSED QUANTIZATION ALGORITHM

Before describing the proposed quantization process in detail, we first establish the connection between quantizing beamforming vectors and encoding/decoding in a noncoherent transmission scheme that can be exploited.

A. Connection to Noncoherent Communication

Noncoherent communicaiton is an important problem and attracted a large body of research literature, where the noncoherent nature of the channel has a number of different assumptions. Of particular interest to our problem is the assumption where the transmitted signal is corrupted by a fixed but unknown phase rotation. Specifically, the single-input-single-output noncoherent system over T transmission uses is described by

$$\mathbf{y}_{\text{non}} = e^{j\phi} \mathbf{s}_{\text{non}} + \mathbf{n}_{\text{non}}, \tag{3}$$

where ϕ is a constant phase rotation with uniform probability in $[0, 2\pi)$ and is unknown to both the transmitter and the receiver, $\mathbf{y}_{\text{non}}, \mathbf{s}_{\text{non}}, \mathbf{n}_{\text{non}}$ are the received vector, transmit vector, and noise vector in the noncoherent communication system. From [8, P.290], the best noncoherent decoder is the codeword that maximizes the magnitude of correlation with $\mathbf{y}$. One effective way to accomplish this is to use a specially designed trellis to modulate the signal without using differential signaling [3]. A simple 8PSK version is proposed in [6], where the authors converted Ungerboeck's 8-state trellis [9] shown in Fig. 1 into a trellis suitable for noncoherent communicaiton without altering the free distance of the trellis. This conversion is simply modifying the output of the t-th channel use by

$$p^*(t) = \begin{cases} p & , t \text{ is odd} \\ \text{(8-p) mod 8} & , t \text{ is even} \end{cases}, \tag{4}$$

where p is the output symbol in the trellis from Fig. 1. The purpose of (4) is to remove the noncoherently equivalent codewords (sequences that differ by a common phase shift).

The noncoherent communication system has a connection with the phase invariant property of beamforming vectors. Specifically, the vector $\mathbf{w}$ and the vector $\mathbf{w}e^{j\phi}$ have the same performance for any $\phi \in [0, 2\pi)$. To make this connection explicit, we can specify the quantization system as

$$\tilde{\mathbf{w}} = \sqrt{P} e^{j\phi} \mathbf{w} + \tilde{\mathbf{n}}, \tag{5}$$

where $\mathbf{w}$ is the quantized beamforming vector, $\tilde{\mathbf{n}}$ is the quantization noise, and $\tilde{\mathbf{w}} = \mathbf{h}/\|\mathbf{h}\|$ is ideal (unquantized) beamforming vector. Writing the quantization equation in this way, it is easy to see the similarity between (3) and (5).

B. Sample Algorithms

In this section, we describe a few efficient joint quantization-channel coding algorithms. Specifically, the quantization component is a scalar quantizer similar to [10], [11]. For expositional simplicity, we restrict ourselves to information codes with average $R = 2$ bits per feedback channel use. The first algorithm is simply an entry-by-entry QPSK encoder. It quantizes $\mathbf{h} = [h_1, \ldots, h_M]$ and encodes into $\mathbf{w} = [w_1, \ldots, w_M]$ by

$$w_k = \operatorname*{argmax}_{v \in \{\pm 1/\sqrt{M}, \pm j/\sqrt{M}\}} Re\{h_k^H v\}. \tag{6}$$

This algorithm is very simple and efficient but has no channel coding capability.

The second scheme is an improved version of the first scheme. Each component of $\mathbf{w} = [w_1, \ldots, w_M]$ is still selected from a QPSK constellation. We take care of the phase invariant property of $\mathbf{w}$ by setting $w_1 = 1/\sqrt{M}$ and select $w_k, k = 2, \ldots, M$ by

$$w_k = \operatorname*{argmax}_{v \in \{\pm 1/\sqrt{M}, \pm j/\sqrt{M}\}} \left| h_k^H v + \sum_{\ell=1}^{k-1} h_\ell^H w_\ell \right|^2. \tag{7}$$

This scheme is essentially sucessively selecting the entries to maximize correlation between channel vector and the first k entries of the quantized vector. As such, we expect this to outperform the first scheme.

We can further improve upon the second scheme by studying the structure of (7). Expectedly, the cumulative metric is dominated by components of $\mathbf{h}$ that have bigger magnitudes. Hence we improve the performance upon the second scheme by sorting the elements of $\mathbf{h}$ by descending magnitude prior to invoking (7). We expect this efficient scheme to have better performance than the second scheme at the expense of a slight increase in complexity (during channel element sorting). We shall numerically compare the performance of these schemes in Section V.

C. Proposed Algorithm

We now demonstrate how to leverage existing work on noncoherent communication systems in the beamforming vector quantization problem. Our feedback quantizer is a joint source-channel coder for a given beamforming vector. Once properly coded, feedback information is transmitted using N_f feedback slots at an average rate of B/N_f bits per feedback channel use. The feedback decoder decodes a beamforming vector for transmission use during the rest of the coherence time. Fig. 2 shows the structure of the feedback communication system.

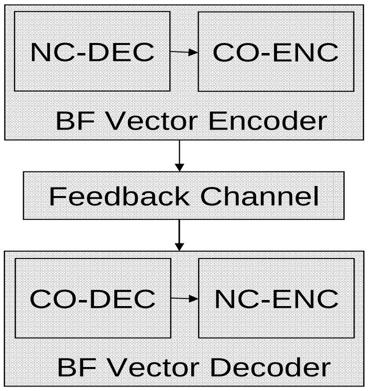

Fig. 2. Block diagram for the transmitting and receiving a beamforming vector in the feedback channel.

The vector encoder consists of a noncoherent decoder followed by a coherent encoder. Correspondingly, the vector decoder consists of a coherent decoder followed by a noncoherent encoder. The noncoherent codec functions as a source coder for beamforming vectors, and its suitability derives from the connection identified. The coherent codec is a channel coder that modulates a sequence of information symbols to protect against channel effects.

Recall that one key goal is to generate a low complexity codec with decent performance. As trellis codes have significantly lower complexity than spherical codes, we implement both the coherent and noncoherent codecs as trellis codes. Hence we have a joint trellis coded quantization/modulation system similar to [5]. For simplicity, we use the Ungerboeck 8-state trellis [9] and the modified version in [6] to implement the coherent and noncoherent codecs, respectively.

Note that to decode a $2^{B/N_f}$-state trellis for N_f stages incur a complexity of $O(2^{B/N_f} N_f)$. Encoding on the same trellis incurs slightly smaller complexity. Hence the vector encoder and vector decoder both incur a complexity of $O(2^{B/N_f} N_f)$. This is a signficant reduction from the $O(2^B)$ processing complexity for spherical code.

V. SIMULATION

In this section, we compare the performance of the proposed trellis quantization against four other quantization methods. The performance metric we use is the average squared chordal distance between the channel vector $\mathbf{h}$ and the quantized beamforming unit vector $\mathbf{w}$

$$E\{d(\bar{\mathbf{h}}, \mathbf{w})\} = 1 - E\left\{ \frac{|\mathbf{h}^H \mathbf{w}|^2}{\|\mathbf{h}\|^2} \right\}, \tag{8}$$

where the expectation is taken across the channel distribution. The first scheme for comparison is the random vector quantization (RVQ) scheme [12], where a codebook of 2^B unit vectors are randomly and uniformly generated on the unit sphere. The average distortion for RVQ is [13]

$$E\{d_{\text{RVQ}}(\bar{\mathbf{h}}, \mathbf{w})\} = 2^B \beta \left(2^B, \frac{M}{M-1} \right), \tag{9}$$

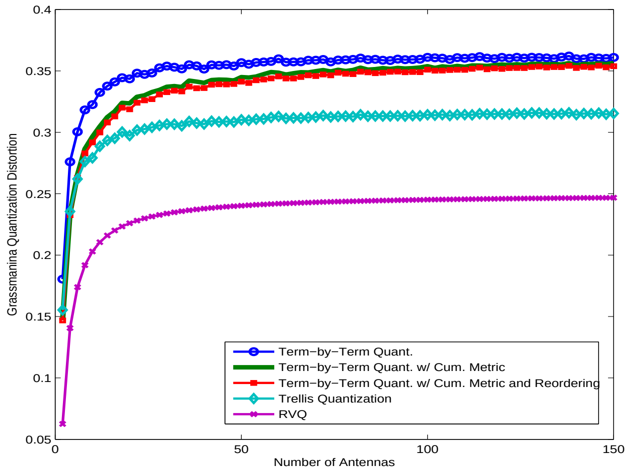

Fig. 3. Comparison of average quantization distortion.

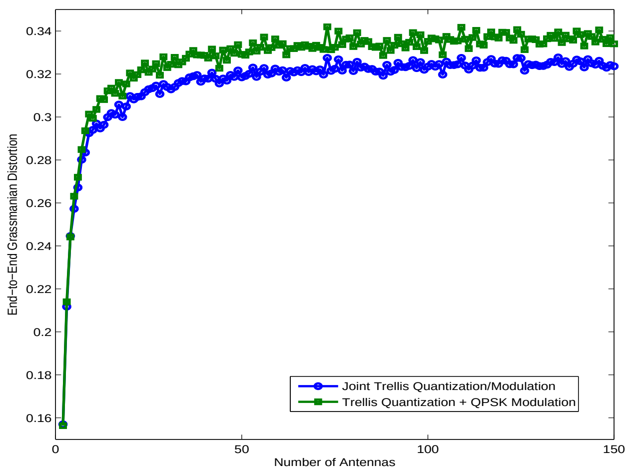

Fig. 4. End-toend Grassmanian distortion of joint trellis quantization/modulation and trellis quantization with QPSK encoder/decoder

where $\beta(\cdot,\cdot)$ is the beta function. We also compare against the three schemes presented in Section IV-B.

Fig. 3 illustrates the comparison. As we can see, the trellis quantization has less distortion compared to all variants of the element-by-element methods. Observe also that the distortion reduction is significant by using the cummulative metric (7). On the other hand, the improvement from reordering the channel magnitudes is trivial. Essentially, the entry-by-entry selection method using metric (7) is a greedy method to select each entry of the beamforming vector $\mathbf{w}$. Fundamentally, the proposed trellis based method keeps track of 8 possible paths (vectors) and select the best path (vector) from the trellis. This selection diversity gave rise to the significant performance boost of the trellis-based scheme over the entry-by-entry schemes. Meanwhile, the trellis-based scheme has more distortion than RVQ, which is the tradeoff obtained for complexity reduction.

Next, we study the performance when the feedback channel is an Rayleigh fading channel. Here we compare the performance of our proposed method against a scheme where the coherent codecs in Fig. 2 is simply the uncoded QPSK constellation. Fig. 4 shows the comparison, and expectedly, TCQ/TCM performs better than TCQ/uncoded QPSK.

VI. CONCLUSION

As multi-site cooperative transmission schemes gain importance in the latest standard developments, the ability to quantize channels efficiently becomes more important. Inspired by the connection between noncoherent communication and phase invariant property for beamforming vectors, we proposed an efficient trellis based quantization technique that outputs a quantized vector with complexity growing linearly with total number of antennas. Initial performance plots show decent quantization performance. Potential future research directions include designing better trellises and designing better constellation set.

VII. ACKNOWLEDGEMENT

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