Full Diversity Spatial Modulators

Oliver M. Collins, Sundeep Venkatraman and Krishnan Padmanabhan
Department of Electrical Engineering
University of Notre Dame, Notre Dame, Indiana 46556
Email: {ocollins,svenkatr,kpadmana}@nd.edu

***Abstract*— This talk presents a new approach for achieving both full rate and full transmit diversity. The techniques in this talk work on a symbol by symbol basis and so are called spatial modulators. When there is only a single receive antenna, the goal is to design the modulator at each transmit antenna so that(even for the worst case realization of the channel coefficients) the scheme achieves the capacity of the single transmitter multiple receiver system with the same channel coefficients. The talk presents two approaches to modulator design. The first expands the constellation by using a set of random phase sequences to modulate the transmitted symbol at each antenna. The receiver thus sees an equivalent single antenna system with an expanded and distorted constellation. The second approach does not increase the constellation size and transmits different permutations of the same constellation from different antennas. The permutations can be designed either algebraically or numerically and the receiver thus sees a distorted constellation that depends on the channel realization. In addition to being full rate and full diversity (which is impossible for coventional space-time codes except in special cases) these schemes are potentially very simple to implement.**

1. Introduction

This talk presents a new paradigm for designing full diversity, capacity achieving schemes for multi-antenna systems. The designs presented in this talk are not, strictly speaking, space-time codes. The techniques presented work on a symbol by symbol basis (as opposed to a space-time code, which works over several symbols), and are hence better referred to as spatial modulation schemes. This talk outlines the design of some simple spatial modulators which achieve performance comparable to or better than existing space-time codes.

Space-time codes promise to exploit the diversity advantage which is available in multi-antenna systems while keeping the receiver complexity low ([1]). Such schemes typically transmit copies or correlated versions of the data across both time and antennas. The fact that the transmitted signals from different antennas experience different amounts of scattering and attenuation means that certain copies of the received data are better than others. This redundancy in the received signals is exploited by a receiver which has knowledge of the channel state information to provide improved performance.

Space-time codes were first systematically studied in [2]. This paper presented a class of codes called space-time trellis codes due to their similarity to trellis codes used over Gaussian channels. These codes were proved to achieve full diversity but not necessarily the full coding gain for a multi-antenna system. There have been other advances in space-time trellis coding ([3]-[6]) but this technique is inherently limited by the fact that it requires an exponential growth in trellis complexity with the number of transmit antennas to achieve diversity and coding gains.

Space-time block codes (STBCs) are another class of space-time codes that have been extensively researched. The first STBC in the literature is the Alamouti code ([7]). The Alamouti code is a scheme for a 2×1 system which produces exactly the same performance as a 1×2 system with the same transmit power. This was the first example of a full rate, capacity achieving STBC that also achieved full diversity. The Alamouti code, being an orthogonal space-time code, also has a much simpler receiver structure compared to space-time trellis codes. These advantages of the Alamouti scheme led to the search for full rate orthogonal space-time codes for systems with greater numbers of transmit antennas. Orthogonal space-time block codes were subsequently studied in considerable detail in [8] using the theory of orthogonal designs. Tarokh et al ([8]) showed that, for complex signaling alphabets, the Alamouti scheme is the only orthogonal full rate STBC possible. For real alphabets, the paper also showed that it is possible to design orthogonal STBCs for 2,4 and 8 antennas. For any other number of antennas, no full rate orthogonal codes are possible. Their paper also deals with generalized orthogonal designs and provides a few examples, although relaxation of the orthogonality constraint this way doesn't provide any significant benefits. Another paper by Ganesan and Stoica ([9]) presented a different approach based on maximizing the SNR of the received signal. This paper shows that STBCs based on orthogonal designs maximize the received SNR. While this paper arrived at the same solutions as [8], the analysis provides a more intuitive understanding of orthogonal STBCs. Another paper by Wang and Xia([10]) proved that for complex orthogonal space-time block codes for systems with more than two transmit antennas, the rates are upper bounded by 3/5. In the case of generalized orthogonal designs, the rates are upper bounded by 4/5. These papers made it clear that in order to achieve the best possible performance (in terms of both diversity gain and capacity) for arbitrary numbers of transmit antennas, newer and more flexible approaches to space-time block coding had to be developed.

One novel approach was described in a paper by Jafarkhani([12]), which involves sacrificing orthogonality for a simple ML decoding receiver over pairs of transmitted symbols. These codes were named quasi-orthogonal STBCs and Jafarkhani's paper presents such codes for 4 and 8 transmit antennas and complex signaling alphabets. There have subsequently been many more papers investigating quasi-orthogonal STBCs ([13]-[16]). These codes have an advantage over orthogonal STBCs in that full rate and full diversity codes can be designed for arbitrary numbers of transmit antennas which is not possible if strict orthogonality must be maintained. However, Papadias and Foschini ([17]) show that the lack of orthogonality in Jafarkhani's designs results in a net capacity loss. Their arguments can easily be extended to general quasi-orthogonal designs.

From the point of view of designing capacity achieving codes, Hassibi and Hochwald ([11]) introduced a general class of codes called linear dispersion codes which are designed to maximize the average mutual information over the MIMO channel while allowing for a receiver structure similar to V-BLAST. These linear dispersion codes include orthogonal STBCs as a special case. While the ML decoding of these codes is exponentially complex, the linear structure of these codes makes linear decoding algorithms like successive nulling and cancelation and sphere decoding([11],[18], [19]) viable alternatives. However, these codes have a major disadvantage in that they do not usually achieve full diversity at high rates. These codes are also designed to achieve the average mutual information (mutual information averaged over channel states). Therefore, the design of these space-time codes is explicitly dependent on the distribution of the channel coefficients. This is unlike STBCs based on orthogonal designs, which are designed to work for any realization of the channel states.

This talk presents two simple high rate and full diversity spatial modulation schemes where the coding matrix is of size $1 \times N$ (N is the number of transmit antennas). The schemes are designed to perform as well as or better than the equivalent 1 transmitter, N receiver system for the worst case realization of channel coefficients. Section 2 formalizes the problem and motivates the spatial modulator as a solution. It also describes in detail, the first spatial modulation scheme which this talk explores. This scheme involves transmitting the same symbol from all antennas, where each antenna is modulated by a random phase. These phases modulating each antenna are randomly chosen from a finite number of random phase sequences generated at each time instant. Although the receiver knows the set of random phase sequences, it does not know which particular phase sequence was chosen at the transmitter. Thus, the transmitter uses additional bits to select a particular phase sequence. Since the receiver knows the channel state information and the set of modulating phases used at each time instant, it effectively sees an expanded constellation which is made up of a number of rotated and scaled versions of the constellation at the transmitter. The goal of this scheme is to use the expanded constellation seen at the receiver to provide a guaranteed worst case performance (i.e., for the worst-case channel realizations). Section 2 goes on to present numerical results of capacity for the spatial modulation scheme.

Section 3 presents the second spatial modulation scheme. In this scheme, each transmitter sends a different permutation of the same constellation. In contrast with the scheme described in Section 2, this is a pure modulation scheme since there is no expansion in the constellation size or reduction in the code rate. The constellation seen by the receiver is a linear combination of the different permutations of the transmitted constellation. Depending on the channel realizations and the choice of the permutations, the received constellation can be highly distorted and can have a significantly degraded capacity. This section explores different methods of designing the permutations in order to minimize the capacity loss for the worst case channel realizations. Simulation results show that an appropriate choice of the permutations does indeed result in a very small capacity loss. Finally, this section shows how to combine this spatial modulator with the scheme described in Section 2 to design hybrid schemes with potentially better performance.

2. Channel Model and Description of Coding Scheme

This section considers a system with multiple transmit antennas and a single receive antenna. The aim is to design a coding scheme that achieves the full diversity gain for an $N \times 1$ system. For an $N \times 1$ system with appropriate error correction coding, a scheme is said to have full diversity if it achieves the same capacity as an equivalent $1 \times N$ system. This definition is based on the idea behind Alamouti's code. The capacity of a 2×1 system using the Alamouti code is identical to that of a 1×2 system irrespective of the specific realization of the channel coefficients.

It is a well known result that for a single transmitter, multiple receiver system, the optimal approach is to combine the different received signals using Maximal Ratio Combining (MRC). When MRC is used, the effective SNR is proportional to $||\mathbf{H}||_2$, where $\mathbf{H}$ denotes the vector of channel coefficients. Consequently, the capacity of the system is also a function of $||\mathbf{H}||_2$. In order to get a fair comparison between the capacities for different realizations of the channel coefficients, the capacity is evaluated under the constraint that $||\mathbf{H}||_2 = 1$. The goal is to design a spatial modulation scheme whose performance will either equal or surpass the performance of the single transmitter multiple receiver case.

This section calculates the capacity achievable by the spatial modulation scheme for the worst case realization of the channel coefficients (under the constraint $||\mathbf{H}||_2 = 1$) and compares this with the performance of a single transmitter multiple receiver system for an identical realization. There are many reasons why this is a more meaningful metric of performance than average mutual information. Firstly, a simple channel model where the coefficients are independent and Rayleigh distributed may not be realistic. Secondly, the design of the spatial modulator should be independent of the channel statistics, which will not be true if the mutual information is averaged over the distribution of the channel states. Finally, this approach ensures that the spatial modulators have a guaranteed worst case performance.

The spatial modulation scheme in this talk can be described by the following equation,

$$Y = \mathbf{H}^T \Theta X + Z \tag{2.1}$$

where $\mathbf{H}^T$ is the row vector of channel state coefficients, and $\Theta = [e^{j\theta_1} \;\; e^{j\theta_2} \;\; \ldots \;\; e^{j\theta_N}]^T$. Θ is a column vector which represents the random phase shift sequence which is used to modulate X, which is the symbol transmitted by each antenna. The symbol X is selected from an alphabet of size n. The sequence Θ is selected from a set of possible phase sequences Ξ of size m. Each phase sequence Θ in Ξ is made up of i.i.d. realizations of a uniform random variable in $[0, 2\pi]$. This set Ξ which is generated at the transmitters at each time instant is known to the receiver as well. Since the set Ξ is freshly generated at each time instant, it is meaningful to compute the average capacity where the averaging is done over the phases Θ. The channel state vector $\mathbf{H}$ is known at the receiver. Therefore, the receiver sees

$$Y = AX + Z \tag{2.2}$$

where $A = \mathbf{H}^T\Theta$ is the effective channel coefficient. It must be noted however, that since the receiver knows only $\mathbf{H}$ and not Θ, both the amplitude and phase of the effective channel coefficient is determined by the choice of phase sequence, which can be decoded by the receiver. The receiver consequently sees a composite constellation which is a function of the channel coefficients, the set of phase sequences and the constellation of X. This composite constellation has mn points and their arrangement relative to each other depends on $\mathbf{H}$ and Ξ. This composite constellation also differs from one time instant to another because the set Ξ differs at each time instant. The information rate achievable by the system for a given realization of the channel coefficients is given by $I(X, \Theta; Y|\mathbf{H})$. By the chain rule of mutual information,

$$I(X, \Theta; Y|\mathbf{H}) = I(\Theta; Y|\mathbf{H}) + I(X; Y|\mathbf{H}, \Theta) \qquad (2.3)$$

The second term can be written as $E_\Theta\left[I(X; Y|\mathbf{H}, \Theta = \theta)\right]$. Due to the concavity of mutual information with respect to θ, this will be less than the capacity of a system without any modulating phases. Therefore, there is a loss in capacity with respect to the second term of (2.3). However, the first term of (2.3) is non zero because information is transmitted through Θ. Our goal is to choose sufficiently many sequences in Ξ so as to make the increase in the first term greater than the decrease in the second term.

A. Numerical Performance Results

For convenience, we can normalize the channel coefficients and X to have unity power. In other words, we can look at the system as being

$$Y = \sqrt{\rho}\ \mathbf{H}^T\Theta X + Z \qquad (2.4)$$

where ρ is the SNR of the system. The information rate achievable by the system for a given realization of the channel coefficients is given by $I(X, \Theta; Y|\mathbf{H})$. We need to find the capacity for the worst case realization of $\mathbf{H}$. We must note that the capacity is independent of the phases of the channel coefficients because the modulator multiplies the transmitted signal with random phases at each antenna. Therefore, we only need to find the worst case amplitudes of the channel coefficients. While it is easy to calculate the capacity for a given realization of $\mathbf{H}$, it is much harder to analytically obtain the worst case amplitudes of $\mathbf{H}$. Therefore we solve this problem numerically by quantizing the amplitudes of $\mathbf{H}$ finely and searching exhaustively for the worst case capacity. This is valid because the mutual information is a continuous and smooth function of $\mathbf{H}$ and hence, a sufficiently fine quantization will yield the true worst case capacity.

For the exhaustive search, we evaluate the capacity of the candidate modulation scheme under the constraint $||\mathbf{H}||_2 = 1$. We compare the performance of this modulator with that of the equivalent $1 \times N$ system(which is a constant under the constraint $||\mathbf{H}||_2 = 1$). Specifically, if $N - 1$ channel coefficients were zero and the last one had amplitude one, it would satisfy $||\mathbf{H}||_2 = 1$. Therefore, the $1 \times N$ capacity is the same as the capacity of a point to point Gaussian channel where the channel coefficient is 1 for the constellation under consideration.

The exhaustive search was carried out for QPSK, 8-PSK and 16-QAM. For each constellation, we varied the number of phase sequences (which is given by m) and the number of transmit antennas. The exhaustive search showed that the worst case realization of the channel coefficients occurred either in the case where all the coefficients had equal amplitude or where all coefficients except one were zero. In other words, the worst case for our coding scheme in terms of the achievable rate is either the degenerate case where only one antenna has a non zero coefficient (which makes it a point-to-point channel) or when all antenna coefficients have the same amplitude. Therefore, the worst case capacity is the minimum of the single antenna capacity and the capacity with all channels having equal amplitudes. Table I presents both the point-to-point capacity and the equal amplitude modulator capacities for these different constellations.

TABLE I
CAPACITIES OF SPACE-TIME CODING SCHEME FOR DIFFERENT SCENARIOS

Const	Capacity	m	n_{tx}	Modulator capacity (equal amplitudes)
16QAM (SNR 5.2dB)	2.02	2	4	1.83
		2	8	1.83
		4	4	1.96
		4	8	1.96
		8	4	2.03
		8	8	2.03
		16	4	2.07
		16	8	2.07
8PSK (SNR 3dB)	1.49	4	4	1.495
		8	4	1.53
QPSK (SNR 0dB)	0.97	4	4	0.95
		8	4	0.98

We first performed simulations for 16-QAM for 4 and 8 antennas at an SNR of 5.2 dB with only two and four phase sequences. For these cases, the capacity of the modulator with equal amplitudes was worse than the capacity of the point-to-point case. Therefore, the worst case capacity was always achieved when all the antenna coefficients were equal in magnitude. As shown Table I, in the case of 16-QAM, the capacity of our spatial modulator with 2 phase sequences is 1.83 bits/s/Hz which is 0.2 bits/s/Hz less than the capacity of 16-QAM alone. With four phase sequences, the spatial modulator capacity is 0.06 bits/s/Hz less than 16-QAM capacity. For smaller constellations like 8-PSK and QPSK, the same trends hold. For these constellations, having only two or four phase sequences results in a worst case capacity slightly lower than the constellation's capacity.

In the case where we use eight or more phase sequences, the worst case capacity is achieved when the amplitudes of all coefficients except one are zero, which is the opposite of the case of two phase sequences. Therefore, using more than eight phase sequences ensures that the modulator will not have capacity less than the equivalent $1 \times N$ channel. These results show that in the worst case, there is a loss of capacity if we use only two phase sequences. Increasing the number of phase sequences improves the capacity of the modulator. When the number of sequences is increased beyond 8, the worst case constellation seen by the receiver is that of a point-to-point channel with increasing phase resolution and so the capacity grows negligibly.

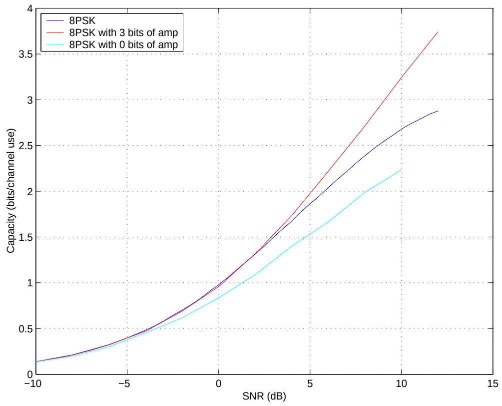

Fig. 1. Plot of capacity of 8-PSK with different numbers of phase sequences.

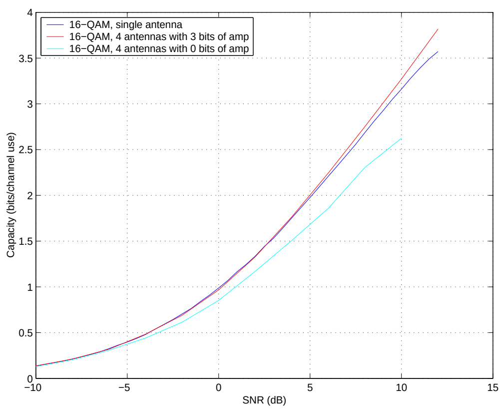

Fig. 2. Plot of capacity of 16QAM with different numbers of phase sequences.

Figures 1 and 2 show the variation in capacity with SNR for 8-PSK and 16-QAM schemes for four antenna systems. The blue curves are the capacities of 8-PSK and 16-QAM in the equivalent $1 \times N$ system. The cyan and red curves are for one and eight phase sequences respectively, where the channel amplitudes are equal. The worst case capacity of the scheme is the minimum of the blue curve and either the red curve (in the case of eight phase sequences) or the cyan curve (in the case of only one phase sequence). These curves show that if we do not have multiple phase sequences, the loss in capacity in the worst case is significant at SNRs above -5dB irrespective of the constellation used. However, if we use 8 phase sequences our scheme performs as well as the point-to-point case in the worst case.

To summarize, this section presents results that show how we can design codes where the worst case capacity of our scheme is equal to the achievable rates for the equivalent single transmitter multiple receiver case. The results also show that increasing the number of phase sequences for our coding system quickly reaches a point of diminishing returns. However, the improvement in capacity before these diminishing returns is sufficient to justify the use of our scheme. The talk will present simple extensions of this scheme to the multiple receive antenna case but we do not present it here for lack of space.

3. Transmitting Constellation Permutations

A. Description of Coding Scheme

In the previous section, we saw how transmitting the same symbol from the different antennas modulated by a random sequence of phases performed as well as the corresponding single transmitter multiple receiver case. In this section we consider an alternative approach to the problem of designing a spatial modulator. Instead of transmitting the same symbol from each transmitter, we consider a one-to-one mapping between the constellation symbols at one transmitter and the symbols at the other transmitters. In other words, there is a different permutation of the same constellation being used at different antennas. The equation describing this new scheme is

$$Y = \sqrt{\rho}\mathbf{H}^T \mathbf{X}_\theta + Z \qquad (3.1)$$

where, $\mathbf{X}_\theta = [X_1 e^{j\theta_1} \;\; X_2 e^{j\theta_2} \;\; \ldots \;\; X_N e^{j\theta_N}]$, ρ is the SNR, and $X_1, X_2, \ldots, X_N$ are the permutations of the same symbol which are transmitted from each antenna and $||\mathbf{H}||_2 = 1$. The vector of θ_is is the random phase sequence modulating the symbols from each transmit antenna. These phases are chosen from an i.i.d, uniform distribution. We first consider the case where there is only one vector of θ_is at each time instant. Consequently, what the receiver sees is an AWGN channel with a distorted constellation. The way the points are clustered depends on the specific realization of $\mathbf{H}$.

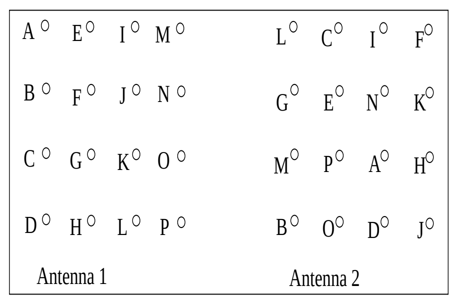

Fig. 3. Example of permutation of the same constellation used at the two transmitters.

As an example, Figure 3 illustrates how the coding scheme would work for 16-QAM in a two antenna system. The figure clearly shows how the symbols at different antennas are related. If the symbol labeled by A is selected at antenna 1 for transmission, then the symbol labeled A is also selected in antenna 2. Note that the two symbols are not the same.

Our aim is to design the permutations such that even in the worst case realization of $\mathbf{H}$, the capacity loss is minimized. In order to achieve this, it is necessary to design the permutations on each antenna such that the average power seen at the receiver is independent of the channel realizations $\mathbf{H}$. The conditions to be satisfied for the average power to be independent of $\mathbf{H}$ are

$$\begin{aligned} E[I_i I_j + Q_i Q_j] &= 0 \\ \text{and } E[I_j Q_i] - E[I_i Q_j] &= 0 \end{aligned} \qquad (3.2)$$

One way to satisfy (3.2) is when the components (I_i, Q_i) are pairwise uncorrelated with (I_j, Q_j) for $i \neq j$. In other words, if the in-phase and quadrature components across antennas are pairwise uncorrelated, the average power will be independent of $\mathbf{H}$. One easy way to satisfy these conditons is to make use of orthogonal Latin squares to construct the constellations at the two antennas. With orthogonal Latin squares, the components will actually be pairwse independent which is a stronger condition than required. Using these constructs, we can come up with permutation designs for which the average received power is independent of the realization of $\mathbf{H}$.

Although making the I and Q components pairwise is sufficient to satisfy the conditions in equation (3.2), it is not a necessary condition. Hence, there are additional permutations that satisfy (3.2) which cannot be derived from orthogonal Latin square constructs. For a small number of transmit antennas and small constellations, we can perform an exhaustive search to obtain the optimal permutation. In other cases, we will have to use algebraic techniques similar to Latin squares to get good solutions.

B. Numerical Results

Since we cannot analytically find out the permutation of symbols at the two antennas that give the best worst case performance, we perform an exhaustive search. In the case of QPSK and 8-PSK, it is computationally feasible for us to search exhaustively because we have to search through $4!$ and $8!$ possibilities respectively. However, for 16-QAM, there are $16!$ possibilities, which is computationally infeasible and therefore we use the conditions in Equation (3.2) to prune the search. For this permutation based scheme, the worst case realization of the channel coefficients is unlike that in the previous two sections where either all coefficients have equal amplitude or when all except one are zero. This is due to the asymmetry of the constellations transmitted by the two antennas.

We consider two different transmission schemes. In the first scheme, the transmitters do not use any random phase sequences to modulate the symbols. Therefore, the receiver only needs to know the permutations and the channel state. Also, the constellation seen by the receiver does not change with time. The worst case constellation seen by the receiver depends on both the amplitude and the phase of the channel coefficients. In the second scheme, the transmitters modulate the symbol with a different random phase sequence at each time instant. The receiver needs to know these phase sequences in addition to the permutations and the channel state. In this case, the constellation depends on the phase sequences, and hence the capacity needs to be averaged over multiple realizations of the phase sequence. The worst case constellation depends only on the amplitudes of the channel realizations.

For the first scheme with no phase sequences, a search for the permutation with the best worst case performance (over all amplitudes and phases of $\mathbf{H}$) for 16-QAM yielded a capacity of 1.9408 bits/s/Hz. From our exhaustive search, we find that the best permutation is the one given in Figure 3. Since the capacity of 16-QAM for a single transmitter system is 2.025 b/s/Hz, the best permutation of points corresponds to a loss of about 4%. For the second scheme where we average over the phases of the channel coefficients, the capacity is 1.9735 b/s/Hz, (which is a loss of 2.5%).

While these losses are not large, there is the possibility of recouping them by combining this scheme with the spatial modulator of Section 2 by using more sequences in the set Ξ. We used the same permutation as above, but now Ξ has two sequences, i.e., there is one bit of information being transmitted using Θ. In this case, the worst case capacity is 1.9853 (loss of $< 2\%$) and the phase averaged capacity is 2.0347 (which is a gain of 0.5%).

While the results presented look very promising, the design problem becomes more complicated in the case of more than two transmit antennas. This is because we need to consider permutations of the constellation at each antenna and search exhaustively for the worst case realization of $\mathbf{H}$ for each permutation and calculate its capacity. This is too cumbersome to be done computationally but future research will attempt algebraic solutions to this optimization problem which promise a great improvement in capacity for the N transmitter case, while maintaining low receiver complexity.

REFERENCES

[1] E. G. Larsson and P. Stoica, *Space-Time Block Coding for Wireless Communications*, Cambridge University Press, 2003

[2] V. Tarokh, N. Seshadri and A. R. Calderbank, “Space-time codes for high data rate wireless communication: performance criterion and code construction,” *IEEE Trans. on Inform. Theory,* vol. 44, no.2, pp. 744-765, March. 1998.

[3] V. Tarokh, A. Naguib, N. Seshadri and A. R. Calderbank, “Combined array processing and space-time coding,” *IEEE Trans. on Inform. Theory,* vol. 45, no. 4, pp. 1121-1128, May. 1999.

[4] Y. Liu, M. P. Fitz and O. Y. Takeshita, “Full rate space-time turbo codes,” *IEEE Journal on select. areas in Comm.*, vol. 19, no. 5, pp. 969-980, May. 2001.

[5] S. Bäro, G. Bauch and A. Hansmann, “Improved codes for space-time trellis-coded modulation,” *IEEE Communications Letters,* vol. 4, no.1, pp. 20-22, Jan. 2000.

[6] H. Jafarkhani and N. Seshadri, “Super-orthogonal space-time trellis codes,” *IEEE Trans. on Inform. Theory,* vol. 49, no. 5, pp. 937-950, Apr. 2003.

[7] S. M. Alamouti, “A simple transmitter diversity scheme for wireless communications,” *IEEE J. Select. Areas Commun.*, vol. 16, pp. 1451-1458, Oct. 1998.

[8] V. Tarokh, H. Jafarkhani and A. R. Calderbank, “ Space-time block codes from orthogonal designs,” *IEEE Trans. on Inform. Theory,* vol. 45, no. 4, pp. 1456-1467, May. 1999.

[9] G. Ganesan and P. Stoica, “ Space-time block codes: a maximum SNR approach,” *IEEE Trans. on Inform. Theory,* vol. 47, no. 4, pp.1650-1656, May. 2001.

[10] H. W. Wang and X.-G. Xia, “ Upper bounds of rates of complex orthogonal space-time block codes,” *IEEE Trans. on Inform. Theory,* vol. 49, no. 10, pp.2788-2796, Oct. 2003.

[11] B. Hassibi and B. M. Hochwald, “High rate codes that are linear in space and time,” *IEEE Trans. on Inform. Theory,* vol.48, pp.1804-1824, July. 2002.

[12] H. Jafarkhani, “A quasi-orthogonal space-time code,” *IEEE Trans. on Inform. Theory,* vol.48, pp.1804-1824, July. 2002.

[13] W. Su and X.-G. Xia, “Signal constellations for quasi-orthogonal space-time block codes with full diversity,” *IEEE Trans. on Inform. Theory,* vol.50, no. 10, pp.2331-2347, Oct. 2004.

[14] N. Sharma and C. B. Papadias, “Improved quasi-orthogonal codes through constellation rotation,” *IEEE. Trans. Commun.*, vol. 51, no. 3, pp.332-335, March 2003.

[15] C. Yuen, Y. L. Guan and T. T. Tjhung, “Quasi-orthogonal STBC with minimum decoding complexity,” *IEEE Trans. Wireless Commun.*, vol. 4, no. 5, pp. 2089-2094, Sept. 2005.

[16] N. Sharma and C. B. Papadias, “Full-rate full-diversity linear quasi-orthogonal space-time codes for any number of transmit antennas,” *EURASIP Journal on Applied Signal Processing,* Vol. 2004 (2004), issue 9 , pp. 1246-1256.

[17] C. B. Papadias and G. J. Foschini, “Capacity-approaching space-time codes for systems employing four transmitter antennas,” *IEEE Trans. on Inform. Theory,* vol. 49, no. 3, pp. 726-733, Mar. 2003.

[18] M. O. Damen, A. Chekeif and J.-C. Belfiore, “Lattice codes decoder for spacetime codes,” *IEEE Commun. Lett.*, vol.4, pp. 161-163, May 2000.

[19] M. O. Damen, K Abed-Meraim and J.-C. Belfiore, “ A generaized sphere decoder for asymmetrical space-time communication architecture,” *Electron. Lett.*, p. 166, Jan. 2000

[20] D. Guo, S. Shamai and S. Verdu, “Mutual information and minimum mean-square error in Gaussian channels,” *IEEE Trans. on Inform. Theory,* vol. 51, no. 4, pp. 1261-1282, Apr. 2005.

[21] S. Venkatraman and O. M. Collins, “The performance of BICM with a single decision feedback on the AWGN channel,” *Proc. IEEE Int. Symp. on Information Theory,* July6-11, 2008, Toronto, Canada.