Insertion Rate and Optimization of Redundancy of Constrained Systems with Unconstrained Positions

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Abstract— **Immink and Cai have shown that the maxentropic bound for the Wijngaarden-Immink coding scheme is not tight. They have given simple bit-stuffing coding schemes whose code rates are greater than the bound. We analyze this problem in terms of finite state transition diagrams. Then we give a better bound of the code rate of the coding scheme and prove that our bound is better than the maxentropic bound when the insertion rate is 1/2.**
Keywords: **Input constraint, Maxentropic coding, Reverse concatenation, Digital recording**

I. Introduction

The recording density of magnetic recording system is growing rapidly. The magnetic recording system consist of many advanced technologies, reading head (Giant Magneto-Resistive), servo motor, and magnetic media. Signal processing technologies, e.g., constrained coding, partial response equalization, error correcting coding etc., are also important technology of magnetic recording.

Since many long error correcting code have been developed recently, the code length for magnetic recording is also getting long. However we still have to impose input constraints because media and motor are incomplete. Hence we use both an error correcting code and a constrained code. A standard concatenation of these codes may induces long error propagations which might be beyond the error correcting capability of the error correcting code.

To overcome this problem, Bliss[1] and Immink[2] proposed a reverse concatenation system. In the system data sequences are first encoded by an encoder for input constraints and then encoded by another encoder for an error correcting code. Since outputs from the error correcting encoder may violate the input constraints, we must device a method by which output sequences from the reverse concatenation system always obey the input constraint.

There are several methods to solve the problem. Wijngaarden and Immink proposed a coding scheme for the reverse concatenation system [3]. Campello et al. analyzed the system in detail [4]. They proposed a bound on the redundancy of their coding scheme. Immink and Cai have shown the bound is not tight[5].

In this paper we introduce a better bound and prove that our bound is better than the maxentropic bound when the insertion rate is $1/2$.

II. Wijngaarden-Immink coding scheme

We consider an encoding scheme which encode a binary data of length m into a code word of length n. We assume that our input constraint is a $(0, k)$ constraint which requires that there should be at most k 0s between consecutive 1s. There are many code construction method by which we can construct an encoder which encodes binary unconstrained sequences into binary sequences satisfying the constraint. For example, see [6], [7].

Wijngaarden and Immink proposed a new method for constructing encoders for reverse concatenation systems. Their coding scheme consists of 2 steps. In the first step, a part of data sequences is encoded by an encoder for the input constraint but the result of the encoder contains 'holes,' that is, unconstrained positions. In the second step we put data at the unconstrained positions without any constraint. Therefore the unconstrained positions should satisfy the condition that even if we put any data pattern at the positions, the resulting code word should satisfy the given input constraint.

Let $\boldsymbol{\psi}^n = (\psi_1, \ldots, \psi_n)$ be a binary sequence where $\psi_i = 0$ means the i-th position is a unconstrained position, that is, we can put both 0 and 1 without any violation of the input constraint, and $\psi_i = 1$ means the i-th position is a constrained position, that is, we can put no data bit at the i-th position.

When we encode a data sequence, a part of the sequence is encoded by an encoder for the input constraint and the resulting bits are put at the constrained positions. Then some bits in the remaining part of the input sequence are put at the unconstrained positions. Or the unconstrained positions are used for parity bits of the bits at the constrained positions by an encoder for an error correcting code.

We define the insertion rate I by

$$I = \frac{n - w(\boldsymbol{\psi}^n)}{n} = 1 - \frac{w(\boldsymbol{\psi}^n)}{n} \quad (1)$$

where w means the Hamming weight. Since $w(\psi^n)$ depends on the code for the first step, I also depends on the code.

The code rate R of the coding scheme is given by

$$R = \frac{n - w(\boldsymbol{\psi}^n) + w(\boldsymbol{\psi}^n)R_0}{n} \quad (2)$$

where R_0 is the code rate of the encoder used in the first step.

An important problem of the coding scheme is to find $\boldsymbol{\psi}^n$ that maximizes the overall code rate of the coding scheme. Practical codes by this coding scheme have been reported.

III. MAXENTROPIC CODES AND INSERTION RATE

We give a simple coding scheme which implements the idea described in the previous section.

Let $\boldsymbol{x} = x_0 x_1 \cdots$ be a sequence satisfying a $(0, k')$ constraint. First we divide $\boldsymbol{x}$ into sub-blocks of length $k'+1$. Then we see that there is at least one 1 in each block. Blocks containing the longest sequence of 0 are

$$1\underbrace{0\cdots 0}_{k'} \text{ and } \underbrace{0\cdots 0}_{k'}1.$$

Next we insert (unconstrained) data sequences of length u at boundaries of sub-blocks. Then we can see that the resulting sequence satisfies a $(0, k'+u)$ constraint because the length of 0 symbol is expanded at least u by the insertion operation.

On the other hand we can construct another encoder for the $(0, k'+u)$ constraint by using a standard code construction techniques for constrained systems. One of advantages of the method described above is that we can put data bits at u unconstrained positions without constrained coding. This property may be useful in a reverse concatenation systems because in the first step we encode data bits by an encoder for the $(0, k')$ constraint and then parity bits of the encoded sequence by some error correcting code can be put at the unconstrained positions.

By the method we can encode data sequences as follows.

data sequences

⇓

constrained sequences

⇓

constrained coding

⇓

error correcting code

Therefore we can avoid the error propagation caused by a decoder for the input constraint.

The code rate $R_0(k', u)$ and the insertion rate of $I_0(k', u)$ of this coding scheme are given as follows

$$R_0(k', u) = \frac{u + (k'+1)R}{u + k' + 1}, \quad (3)$$

$$I_0(k', u) = \frac{u}{u + k' + 1}, \quad (4)$$

where R is the code rate of a code used in the first step. We put $k = k' + u$ and regard u as a parameter. Then we have

$$R_0^u(k, u) = \frac{u + (k - u + 1)R}{k + 1}, \quad (5)$$

$$I_0^u(k', u) = \frac{u}{k + 1}. \quad (6)$$

We suppose that we use a maxentropic code, a code whose code rate equals the capacity of the input constraint, in the first step. Then

$$\overline{R}_0^u(k, u) = \frac{u + (k - u + 1)C(k - u)}{k + 1} \quad (7)$$

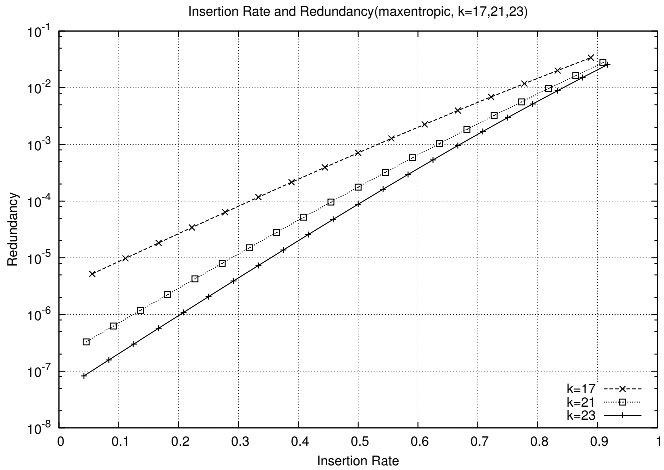

Fig. 1. Insertion Rate and Redundancy of Maxentropic Scheme for $k = 17, 21, 23$

where $C(\ell)$ is the capacity of a $(0, \ell)$ constraint. In Fig. 1 we show $I_0^u(k)$ and $1 - \overline{R}_0^u(k)$ for $k = 11, 17, 21$. In this paper we call the coding scheme the maxentropic scheme and also call these relations maxentropic bounds which mean attainable redundancies of the maxentropic coding scheme.

Immink and Cai have given a coding scheme which is better than the above coding scheme[5].

IV. IMMINK-CAI SCHEME I

In this section we give one of coding schemes introduced by Immink and Cai[5] that are called Immink-Cai(I-C) schemes in this paper. We will investigate other coding scheme in the next section.

Let r be a positive integer and let S be a set of words of length r excluding the all 0 sequence. We suppose that there is a code C_1 which encodes data sequences into sequences obtained by concatenating blocks in S. Let $k = u + 2(r-1)$. By interleaving data blocks of length u and blocks in S, we can obtain sequences satisfying a $(0, u + 2(r-1))$ constraint. The code rate R_1 of this coding scheme is

$$R_1 = \frac{u + \log(2^r - 1)}{u + r}$$

and the insertion rate I_1

$$I_1 = \frac{u}{u + r}.$$

Immink and Cai have shown that if the insertion rate is large this coding scheme is better than the maxentropic coding scheme. However we can give a coding scheme that is better than their coding scheme.

First we observe that the set of sequences obtained by concatenating sequences in S is a proper subset of a set of sequences satisfying the $(0, 2(r-1))$ constraint. We take $a, b, c \in S$ and put $abc = x_1 x_2 \cdots x_{3r}$. Then we can have $x_2 \cdots x_{2r-1} = 0^{2(r-1)}$. But if it happens then we must have $x_{2r} \cdots x_{3r} \neq p^{r+1}$.

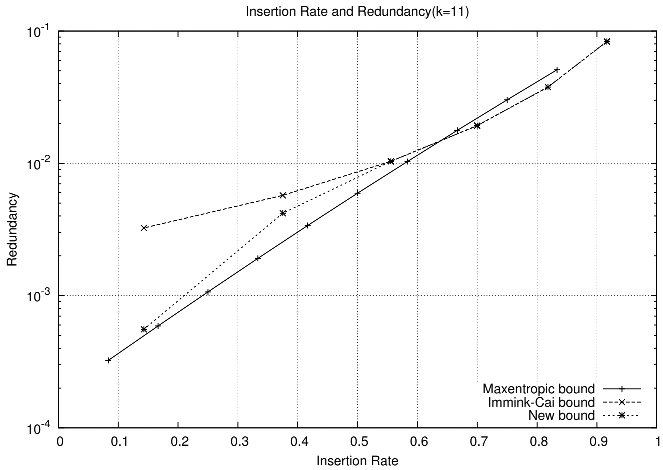

Fig. 2. Insertion Rate and Redundancy($k = 11$)

We consider a Finite State Transition Diagram(FSTD) T defined as follows: the state set of T is

$$V_T = \{(i, \ell) : 0 \le i \le 2(r-1) + u, 1 \le \ell \le r + u\}$$

and transition rules are given as follows.

1) if $1 \le \ell \le r$ then
   - if $i + 1 \le 2(r-1) + u$ then
   $$\begin{aligned}(i, \ell) &\to (i+1, \ell+1) : 1\\ (i, \ell) &\to (0, \ell+1) : 0\end{aligned}$$
   - if $i + 1 = 2(r-1) + u$ then
   $$(i, \ell) \to (0, \ell+1) : 0$$
2) if $r + 1 \le \ell r + u$ then
   - if $i + u + 1 \le 2(r-1) + u$ then
   $$\begin{aligned}(i, \ell) &\to (i+u+1, \ell') : 0\\ (i, \ell) &\to (u, \ell') : 1\end{aligned}$$
   where $\ell' = (l+1) \bmod (r+u)$.

The results are given in Fig. 2. '`New bound`' means our coding scheme. If the insertion rate is high then we obtain no improvement. If the insertion rate is low enough then we can improve the IC bound. However our bound is not better than the maxentropic bound.

V. IMMINK-CAI SCHEME II

In this section we investigate the second example given in [5] and give a coding scheme which is better than the example.

Let D_k be the $(k+1) \times (k+1)$ adjacency matrix of the graph G_k that represents the $(0, k)$-constraint. The entries of D_k are given by

$$\begin{aligned}d_{i1} &= 1, \quad i \ge 1,\\ d_{ij} &= 1, \quad j = i+1,\\ d_{ij} &= 0, \quad \text{otherwise.}\end{aligned} \tag{8}$$

Let D_k^0 be the $(k+1) \times (k+1)$ matrix given by

$$\begin{aligned}d_{ij}^0 &= 1, \quad j = i+1\\ d_{ij}^0 &= 0, \quad \text{otherwise.}\end{aligned} \tag{9}$$

Let G_k^0 be a graph represented by D_k^0. Then they defined a labeled graph whose adjacency matrix is given by

$$D_k^r (D_k^0)^u$$

with edge label of length $r + u$. Each edge in the graph corresponds to a concatenation of a path of length r in G_k and a path of length u in G_k^0. The path of length u in G_k^0 corresponds to arbitrary data sequence inserted in the second coding step. By $G_k^{(u,r)}$ we mean the graph we have just defined.

The constrained system represented by $G_k^{(u,r)}$ can also be represented by a graph H_k defined as follows: the state set of the graph is

$$\{(d, \ell) : 0 \le d \le k, 0 \le \ell \le r + u - 1\}$$

and state transition rules are defined as follows;

1) if $\ell \le r - 1$ and $d < k$ then
$$\begin{aligned}(d, \ell) &\to (d+1, \ell+1) : 0\\ (d, \ell) &\to (0, \ell+1) : 1\end{aligned}$$
2) if $\ell \le r - 1$ and $d = k$ then
$$(d, \ell) \to (0, \ell+1) : 1$$
3) if $r \le \ell < r + u - 1$ and $d < k$ then
$$(d, \ell) \to (d+1, \ell+1) : 0$$
4) if $\ell = r + u - 1$ and $d < k$ then
$$(d, \ell) \to (d+1, 0) : 0$$

where $(i, j) \to (i', j') : a$ means a state transition from state (i, j) to (i', j') with label a. A sequence of 0 of length u periodically appears in every sequence of labels obtained by reading edge labels along a path of H_k and corresponds to positions for data bits inserted in the second coding step. A similar graph was introduced in [8].

We define another periodic graph $H_k^{(u,r)}$ as follows: the state set of $H_k^{(u,r)}$ is

$$\{(d, \ell) : 0 \le d \le k, 0 \le \ell \le r - 1\} \tag{10}$$

and its transition rules are given by

- if $d < k - 1$ and $\ell < r - 1$ then
$$\begin{aligned}(d, \ell) &\to (d+1, \ell+1) : 0\\ (d, \ell) &\to (0, \ell+1) : 1\end{aligned}$$
- if $d = k$ and $\ell < r - 1$ then
$$(d, \ell) \to (0, \ell+1) : 1$$
- if $d + u + 1 \le k$ and $\ell = r - 1$ then
$$\begin{aligned}(d, \ell) &\to (d+u+1, 0) : 0\\ (d, \ell) &\to (0, 0) : 1,\end{aligned}$$

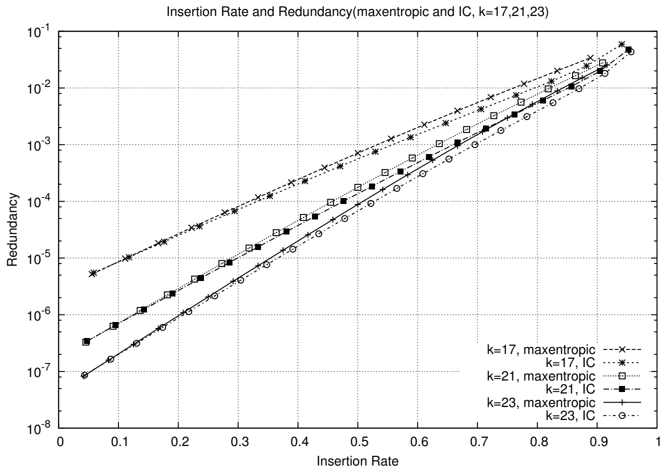

Fig. 3. Maxentropic and I-C scheme

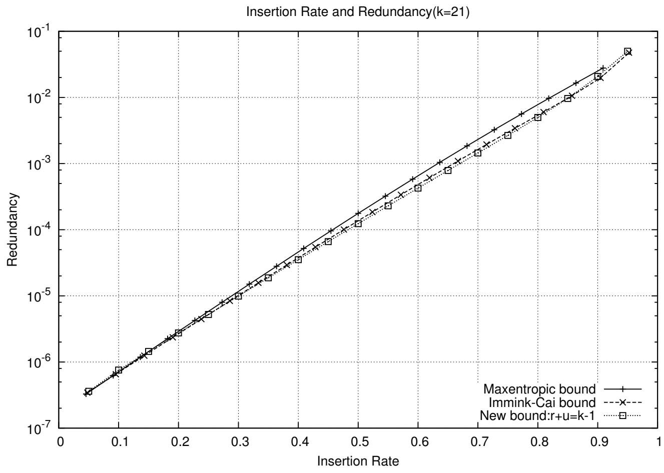

Fig. 4. Redundancy and Insertions Rate for 3 schemes

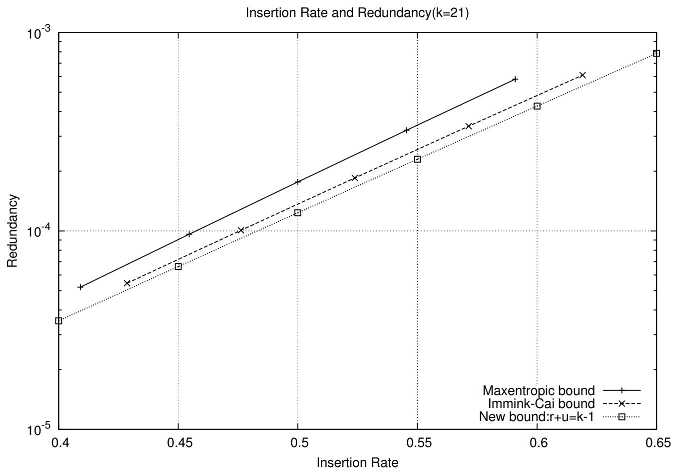

Fig. 5. Closeup of the center part of Fig. 4

- if $d+u+1 > k$ and $\ell = r-1$ then

$$(d,\ell) \quad \rightarrow \quad (0,0):1.$$

Roughly speaking we can obtain $H_k^{(u,r)}$ by skipping transitions for data bits from $G_k^{(u,r)}$.

Let $\boldsymbol{x}$ be a sequence of labels obtained by reading labels along a path in $H_k^{(u,r)}$ starting from a state $(d,0)$ for some d with $0 \le d \le k$. By inserting data sequences of length u after positions $r, 2r, 3r, \dots$ we can obtain a sequence satisfying the $(0,k)$ constraint. We can make a one to one correspondence between paths in $G_k^{(u,r)}$ and paths in $H_k^{(u,r)}$. Therefore we can use $H_k^{(u,r)}$ in stead of $G_k^{(u,r)}$ in analyzing the W-I coding schemes. However the correspondence is not shift invariant, i.e., position dependent. Hence their capacities are different.

A W-I coding scheme uses a constrained code in the first step. The maxentropic scheme and the I-C scheme use the following graphs in the first step,

- the maxentropic scheme : $H_k^{(u,k+1-u)}$,
- the I-C scheme : $H_k^{(u,k-u)}$.

The insertion rate and code rate are given as follows.

$$I_k^{(u,r)} \quad = \quad \frac{u}{u+r}, \tag{11}$$

$$R_k^{(u,r)} \quad = \quad \frac{u+rC(H_k^{(u,r)})}{u+r} \tag{12}$$

where $C(H_k^{(u,r)})$ is the channel capacity of $H_k^{(u,r)}$.

In Fig. 3 we show the relation ship between the insertion rate and the redundancy for $k=17,21,23$ and for the maxentropic and I-C schemes.

VI. Our coding scheme

Our scheme use a graph $H_k^{(u,k-1-u)}$ in the first coding step. This graph has the following advantage. Let J_{k+1} be the set of values of insertion rate for the maxentropic scheme and J_k that of the I-C scheme. It is not easy to compare curves in Fig. 3 because $J_{k+1} \cap J_k = \emptyset$. But we can compare our coding scheme and the maxentropic scheme when the insertion rate is $1/2$.

In Fig. 4 we show that redundancy of the maxentropic scheme, the I-C scheme and our scheme. We can see that our scheme seems to be better than other schemes. Fig. 5 is a closeup of the center part of Fig. 4.

We will prove that the observation is true for all odd k at the insertion rate $1/2$ in the next section.

VII. The maxentropic bound and our bound

In this section we assume that k is odd. We put

$$u = \frac{k+1}{2}, \quad \text{and} \quad u' = \frac{k-1}{2} = u-1. \tag{13}$$

Then the maxentropic scheme can be represented by $H_k^{(u,u)}$ and our scheme is represented by $H_k^{(u',u')} = H_k^{(u-1,u-1)}$.

For a labeled graph T, let $X(T)$ be a set of all bi-infinite sequnces of labels obtained by reading labels along bi-infinite paths in T. We show that

Lemma 1:

$$X(H_k^{(u,u)}) = X(G_{k-u})$$

where k is an odd positive integer and $u = (k+1)/2$.

This lemma says $H_k^{(u,u)}$ and G_{k-u} represents the same constrained system although the structure of $H_k^{(u,u)}$ is very different from that of G_{k-u}.

Proof: We consider a periodic version of G_{k-u}, say $\widetilde{G}_{k-u}^{(u)}$, as follows: the state set of $\widetilde{G}_{k-u}^{(u)}$ is

$$\{(d,\ell) : 0 \le d \le k-u, 0 \le \ell \le u-1\}, \qquad (14)$$

and state transition rules are

- if $d < k-u$ then

$$(d,\ell) \rightarrow (d+1, \ell+1 \bmod u) : 0, \qquad (15)$$
$$(d,\ell) \rightarrow (0, \ell+1 \bmod u) : 1; \qquad (16)$$

- if $d = k-u$ then

$$(d,\ell) \rightarrow (0, \ell+1 \bmod u) : 1; \qquad (17)$$

where a mod b means a reminder when we divide a by b.

It is obvious that $X(G_{k-u}) = X(\widetilde{G}_{k-u}^{(u)})$.

For every state $(d, u-1)$ in $H_k^{(u,u)}$ there are state transitions from the state to $(d+u, 0)$ and $(d+u+1, 0)$. Let $(d'+u, 0)$ be a state in $H_k^{(u,u)}$ with $d'+u \le k$. Let $\boldsymbol{x} = x_1 \cdots x_u$ be a path starting from the state and generating sequence $0^m 1 \boldsymbol{y}$ and $\boldsymbol{y}$ is an appropriate sequence where m should satisfy $m \le k-u-d'$ because $(d'+u, 0)$ is the state in $H_k^{(u,u)}$. We assume that paths are represented by a sequence of edges, not states. Then the terminal state of x_{m+1} should be $(0, m+1)$.

Consider a state $(d', 0)$ in $\widetilde{G}_{k-u}^{(u)}$. There is a path starting from $(d', 0)$ and generating $0^m 1 \boldsymbol{y}$ because $m \le k-u-d'$.

Suppose that $\boldsymbol{y}$ is not the empty sequence. From the above discussion we can see that if a path of length u starting from $(d'+u, 0)$, terminating $(e, u-1)$ and generating $0^m 1 \boldsymbol{y}$ in $H_k^{(u,u)}$ then there is a unique path starting from $(d', 0)$, terminating $(e, u-1)$ and generating $0^m 1 \boldsymbol{y}$ in $\widetilde{G}_{k-u}^{(u)}$. There is only one exception: a path starting from $(u, 0)$ and generating $0^{u-1}1$ in $H_k^{(u,u)}$. This path must terminate $(k, u-1)$. We regard this path as a path starting $(0,0)$ and terminating $(u-1, u-1)$ in $\widetilde{G}_{k-u}^{(u)}$. Note that we have $k-u = u-1$ from our assumptions. Therefore this path actually exists in $\widetilde{G}_{k-u}^{(u)}$. This correspondence is invertible. Therefore we can conclude that $X(H_k^{(u,u)}) = X(\widetilde{G}_{k-u}^{(u)})$. ■

Remark 2: It is easy to understand this lemma if we notice that we use a maxentropic code for G_{k-u} in the first step of the maxentropic coding scheme.

Next we consider a graph for our scheme. We define the following graph $S_k^{(u')}$: the state set of $S_k^{(u')}$ is

$$\{(d,\ell) : 0 \le d \le k-u', 0 \le \ell \le u'-1\}, \qquad (18)$$

and the transition rules are

- if $d < k-u'$ and $\ell < u'-1$ then

$$(d,\ell) \rightarrow (d+1, \ell+1) : 0,$$
$$(d,\ell) \rightarrow (0, \ell+1) : 1;$$

- if $a = k-u'$ and $\ell < u'-1$ then

$$(d,\ell) \rightarrow (0, \ell+1) : 0;$$

- if $d < k-u'-1$ and $\ell = u'-1$ then

$$(d,\ell) \rightarrow (d+1, 0) : 0,$$
$$(d,\ell) \rightarrow (0, 0) : 1;$$

- if $d = k-u'-2$ and $\ell = u'-1$ then

$$(d,\ell) \rightarrow (0, 0) : 1;$$

- if $d = k-u'-1$ and $\ell = u'-1$ then

$$(d,\ell) \rightarrow (0, 0) : 1;$$

- if $d = k-u'$ and $\ell = u'-1$ then

$$(d,\ell) \rightarrow (0, 0) : 1.$$

Lemma 3:

$$X(H_k^{(u',u')}) = X(S_k^{(u')}).$$

Proof: A proof of this lemma is similar to that of the previous lemma. But there are two exceptions: each of state $(k-u'-1, u'-1)$ and $(k-u'-2, u'-1)$ in $H_k^{(u',u')}$ has no outgoing edge with label 0. However we defined $S_k^{(u')}$ so that there is no edge with label 0 from state $(u'-1, u'-1)$ and $(u'-2, u'-1)$ to $(u', 0)$. ■

Lemma 4: If $k-u'-1 \ge 7$ then

$$C(H_k^{(u,u)}) \le C(H_k^{(u',u')}). \qquad (19)$$

Proof: Since $u' = u-1$, from the definitions we can see that $H_k^{(u,u)} = \widetilde{G}_k^{(u)}$ is almost a proper subgraph of $S_k^{(u')}$. The only exception is the nonexistence of edge from state $(k-u'-2, u'-1)$ with label 0. This means that any path $x_1 \cdots x_{k-u-1}$ starting from $(0,0)$ in $S_k^{(u')}$ can not generate sequence 0^{k-u-1}. However we establish an injective map from the set of all paths in $\widetilde{G}_k^{(u)}$ to the set of all paths in $S_k^{(u')}$ and this proves the theorem.

Let $x_0 x_1 \cdots x_{k-u'}$ be a path generating sequence $10^{k-u'1}1$ with $i(x_1) = (0,0)$ in $\widetilde{G}_k^{(u)}$ where $i(x)$ means the initial state of edge x.

Let $r = k-u'-1$. Consider a pattern

$$\boldsymbol{z} = 1\ 0^r\ 1\ a_1\ a_2\ a_3\ a_4\ a_5\ ,$$

where $a_1 \cdots a_5$ means any binary sequence of length 5. Hence the pattern matches 32 binary sequences. Next we consider the following patterns

$$\begin{aligned}
\boldsymbol{y}_1 &= 1\ 1\ 0^r\ 0\ 1\ b_1\ b_2\ b_3, \\
\boldsymbol{y}_2 &= 1\ b_1\ 1\ 0^r\ 0\ 1\ \ b_2\ b_3, \\
\boldsymbol{y}_3 &= 1\ b_1\ b_2\ 1\ 0^r\ 0\ 1\ \ b_3, \\
\boldsymbol{y}_4 &= 1\ b_1\ b_2\ b_3\ 1\ 0^r\ 0\ 1.
\end{aligned}$$

These patterns correspond to 32 binary patterns which can be generated by a path starting from $(0,0)$ in $S_k^{(u')}$.

Our injective map translate paths in $\widetilde{G}_k^{(u)}$ generating sequences containing $\boldsymbol{z}$ to paths in $S_k^{(u')}$ generating sequences containing $\boldsymbol{y}_1, \boldsymbol{y}_2, \boldsymbol{y}_3$ and $\boldsymbol{y}_4$. If $k - u' - 1 > 7$ then we can prove that the map is well defined and injective but we omit the proof here. Therefore we have

$$C(\widetilde{G}_k^{(u')}) \leq C(S_k^{(u')}).$$

From the above lemmas we have

$$C(H_k^{(u,u)}) = C(\widetilde{G}_k^{(u)}) \leq C(S_k^{(u)}) = C(H_k^{(u',u')}).$$

■

Theorem 5: When the insertion rate is $1/2$, we have

$$R_{max} \leq R_{new}, \tag{20}$$

where R_{max} and R_{new} are code rates of the maxentropic scheme and our scheme, respectively.

Proof: The code rates are give by

$$R_{max} = \frac{u + uC(H_k^{(u,u)})}{k+1} = \frac{1}{2}(1 + C(H_k^{(u,u)})),$$

$$R_{new} = \frac{u' + u'C(H_k^{(u',u')})}{k-1} = \frac{1}{2}(1 + C(H_k^{(u',u')})).$$

From Lemma 4 we have $C(H_k^{(u,u)}) \leq C(H_k^{(u',u')})$. Using $u' = u - 1$ we can show that $R_{new} - R_{max} \geq 0$ by a straight forward calculation. ■

In Fig. 5 we can observe that for almost all insertion rates the I-C bound can be obtained by translating points on the maxentropic bound to the upper-right. We can also give a theoretical explanation of this observation.

We define $I_{max}(d)$ and $I_{new}(d)$ as follows

$$I_{max}(d) = \frac{u+d}{k}, \tag{21}$$

$$I_{new}(d) = \frac{u'+d}{k+1}, \tag{22}$$

where $I_{max}(d)$ and $I_{new}(d)$ are the insertion rates of the maxentropic scheme and that of our scheme, respectively. We also define $R_{max}(d)$ and $R_{our}(d)$ as follows

$$R_{max} = \frac{u + d + (k+1-u-d)C(H_k^{(u,k+1-u)})}{k+1} \tag{23}$$

$$R_{new} = \frac{u' + d + (k-1-u'-d)C(H_k^{(u',k-1-u'-d)})}{k-1}. \tag{24}$$

We note that for positive d

$$I_{new}(d) > I_{max}(d) \tag{25}$$

and for negative d

$$I_{new}(d) < I_{max}(d). \tag{26}$$

Furthermore we can show that if $k - (u+d) - 1 > 7$ then we have

$$R_{new}(d) \geq R_{max}(d). \tag{27}$$

These inequalities can be observed in Fig. 5.

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