

# A Graph-Dynamics Interpretation of the Sum-Product Algorithm\*

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*Abstract*—A well-known result by Yedidia, Freeman, and Weiss (IEEE Trans. Inf. Theory, vol. 51, no. 7, pp. 2282–2312, July 2005) says that fixed points of the sum-product algorithm correspond to stationary points of the (variational) Bethe free energy.

This result can be given a reinterpretation in terms of graph covers. Namely, when running the sum-product algorithm on a Tanner graph then a fixed point corresponds to a certain pseudo-codeword of that Tanner graph: it is, after taking a biasing channel-output-dependent term properly into account, the pseudo-codeword that has locally the most (or the least) pre-images in all  $M$ -covers when  $M$  goes to infinity.

This observation can be suitably extended to the transient part of the sum-product algorithm by expressing the sum-product algorithm in terms of a graph-dynamical system involving graph covers, twisted graph covers, and valid configurations therein. This yields new insights into the sum-product algorithm and its behavior.

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\* The full version of this paper will be submitted to the IEEE Transactions on Information Theory.