Interference Channel with a Relay: Models, Relaying Strategies, Bounds

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***Abstract*—This paper discusses the impact of relaying in interference-limited networks by studying a two-user Gaussian interference channel with a relay (ICR). Various models for relay reception and transmission are possible: The relay can receive and transmit in the same band as the sources (i.e., *in-band relay reception/ transmission*), or else relay reception, or both reception and transmission, can take place over orthogonal links of limited capacity (i.e., *out-of-band relay reception or/ and transmission*). In all scenarios, the selection of either *signal relaying*, *interference forwarding* or a combination thereof for the relay operation is identified as a common design choice. While signal relaying enables the destination to obtain a stronger desired signal or additional information, interference forwarding helps removing part of the interference. Important scenarios under which signal relaying and interference forwarding are optimal are identified, including cognitive relaying where the relay is unaware of the source codebooks and out-of band relaying, both under certain modified strong interference conditions and the latter under additional capacity constraints on the orthogonal relay links.**

I. INTRODUCTION

In a communication network consisting of independent transmitters and receivers, interference between different communication sessions is unavoidable. Examples include both traditional cellular systems and ad-hoc or wireless sensor networks. A simple network model that accounts for mutual interference between simultaneous communications is the two-user interference channel (IC), which has attracted considerable interest over the past thirty years, since the original work of [1]. Despite the amount of research activity, the capacity region of the IC is not fully known except for some special cases. For instance, in [3], the capacity region of the Gaussian IC is obtained for the case where the interfering links between the sources and destinations can support higher rates than the direct links. Under this "strong interference" condition, the capacity region is equal to the capacity of the compound multiple access channel. Also, recently, the sum capacity of the Gaussian IC is obtained in the "noisy" or "low" interference regime [18] [19] [20]. In this regime, the optimal strategy for maximum sum-rate is shown to be simply treating the interference as noise.

Beside interference, another basic element of complex wireless networks is the possibility for different nodes to cooperate. A basic network model that accounts for this possibility is the relay channel (RC), in which a single point-to-point channel is aided by a relay. The RC has been widely studied since [4], and the capacity is known in some special cases, including the physically-degraded RC studied in [4] (see [6] for some recent results).

Recently, there has been interest in characterizing the possible advantages of cooperation (relaying) in interference-limited systems, rather than in point-to-point scenarios. A natural model to address this issue is an IC aided by a relay (ICR). The ICR has been first studied in [7], where a Gaussian model is considered and an achievable rate region is obtained via rate splitting into common and private messages at the sources [2], decode-and-forward (DF) at the relay and joint decoding at the destinations [2]. It is shown via numerical results that the sum-rate in a symmetric IC is maximized when the relay only forwards common messages, which need to be decoded at both destinations. The discrete memoryless and Gaussian IC with a relay is further investigated in [9] [10], in which simplified channel models are considered where the relay only receives from one source. DF-based strategies at the relay with joint decoding at the destinations are proposed without rate splitting and shown to exhaust the capacity region under some conditions. The papers [9] [10] emphasize the fact that forwarding the interference of even a single source may improve the rates of both users. In the related works [8] [11] [12] and [14], the relay is assumed to be aware of the users' messages a priori (*message cognitive* relay) and sophisticated achievable strategies are investigated. In [12] [14], an alternative model, termed *signal cognitive*, is introduced, in which the relay observes the signals transmitted by the sources in a non-causal fashion, but is unaware of the codebooks. For this model, strong relay-interference conditions, under which decoding the interference in full is optimal, are provided in [12] [14]. Further investigation of interference forwarding is provided in [13], where the authors modify the classical relay channel model by considering an additional source of fixed rate interference. However, this model does not include a destination for the interferer.

In this paper, we review previous works on the Gaussian ICR and outline some of the recent results. We classify ICR models depending on whether the channel over which the relay receives or transmits is the same as (in-band) or orthogonal to (out-of-band) the channel used by the underlying IC. In-band relaying models a scenario in which the relay operates in the same network as the IC (for example both

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cellular), out-of-band relaying is applicable when transmission to and/or from the relay takes place via an orthogonal wireless interface (such as cellular for the IC and WiFi for the relay). According to this classification, the models of [7] [9] [10] fall in the category of *in-band relay reception/ transmission* (see Fig. 1), while [8] [11] [12] [14] consider *out-of-band relay reception/ in-band relay transmission* (which include cognitive models, see Fig. 2), and [15] studies ICR with *out-of-band relay reception/ transmission* (see Fig. 3). Details of these models are presented in Section II. In all these models, we identify as a common design choice the selection of either *signal relaying*, *interference forwarding* or a combination thereof for the relay operation. While signal relaying enables the destination to obtain a stronger desired signal or additional information, interference forwarding helps removing part of the interference. However, different models also support different information/signal processing options. In-band transmission enables the relay to transmit signals that combine with the source transmissions either in a coherent or incoherent way, the former being useful for signal relaying as well as interference forwarding, while the latter serves the purpose of partially canceling the interference. When the relay receives out-of-band signals, it can obtain and forward extra information that is not transmitted over the IC. Section III describes the transmission strategies in detail. In Section IV we discuss some special cases where different design choices in terms of signal relaying or interference forwarding/cancelation are optimal.

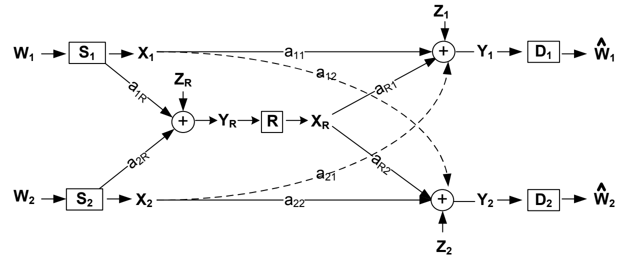

Fig. 1. ICR with in-band relay reception/ transmission [7] [9] [10].

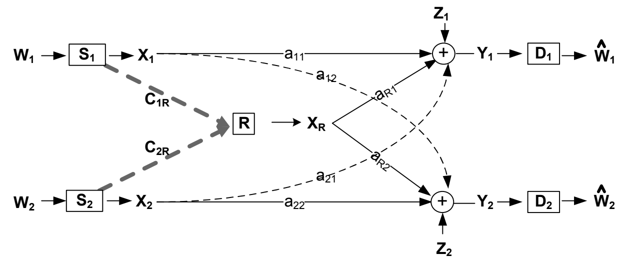

Fig. 2. ICR with out-of-band relay reception/ in-band relay transmission (cognitive relay for $C_{iR} \to \infty$, $i = 1, 2$) [8] [12] [11] [14].

II. System Models

We focus on different classes of Gaussian ICRs, namely ICR with *in-band relay reception/ transmission* (Fig. 1), ICR with *out-of-band relay reception/in-band relay transmission* (Fig. 2) and ICR with *out-of-band relay reception/ transmission* (Fig. 3). In all models, each source S_i, $i = 1, 2$, wishes to send a message index W_i, uniformly drawn from the set $[1, 2^{nR_i}]$, to its destination D_i, with the help of the relay R. The sources S_1 and S_2 communicate simultaneously to their respective destinations D_1 and D_2 via a Gaussian IC. Moreover, $X_{i,t} \in \mathbb{R}$ represents the (real) input symbols of source S_i, on which we enforce the power constraint $1/n \sum_{t=1}^{n} x_{i,t}^2 \leq P_i$ for each codeword, and $\{Z_{i,t}\}$ is an independent identically distributed (i.i.d.) Gaussian noise process with unit power. Depending on the relay's in-band/ out-of-band reception/ transmission operation, we give the details of each system model in the following.

A. In-band Relay Reception/ Transmission

For the ICR with in-band relay reception/ transmission studied in [7] [9] [10], the relay receives and transmits simultaneously in the same band where the sources communicate with the destinations, as depicted in Fig. 1. The signals received at time $t = 1, ..., n$ on the Gaussian channel by R, D_1 and D_2 are:

$$Y_{R,t} = a_{1R}X_{1,t} + a_{2R}X_{2,t} + Z_{R,t} \quad \text{(1a)}$$

$$Y_{1,t} = a_{11}X_{1,t} + a_{21}X_{2,t} + a_{R1}X_{R,t} + Z_{1,t} \quad \text{(1b)}$$

$$Y_{2,t} = a_{12}X_{1,t} + a_{22}X_{2,t} + a_{R2}X_{R,t} + Z_{2,t}, \quad \text{(1c)}$$

respectively, where $X_{R,t}$ is the transmitted signal by the relay with the power constraint $1/n \sum_{t=1}^{n} x_{R,t}^2 \leq P_R$ and $Z_{R,t}$ is the unit-power Gaussian noise at the relay. A $(2^{nR_1}, 2^{nR_2}, n)$ code for the ICR at hand is defined by the encoding function at the source S_i, $i = 1, 2$:

$$f_i\colon [1, 2^{nR_i}] \to \mathbb{R}^n \quad (2)$$

which maps a message $W_i \in [1, 2^{nR_i}]$ into a codeword $X_i^n \in \mathbb{R}^n$; the encoding function, $f_{r,t}\colon \mathbb{R}^{t-1} \to \mathbb{R}$, $t = 1, ..., n$, at the relay R with $x_{r,i} = f_{r,i}(y_r^{i-1})$, and by the decoding function at the destination D_i, $i = 1, 2$,

$$g_i : \mathbb{R}^n \to [1, 2^{nR_i}], \quad (3)$$

which maps the received signal over the IC, Y_i^n, into the estimated message $\hat{W}_i$.

B. Out-of-band Relay Reception/ In-band Relay Transmission

With out-of-band relay reception, the relay receives the source signals over links that are orthogonal to each other and to the underlying IC (see Fig. 2). The orthogonal link from S_i to the relay has capacity C_{iR}, $i = 1, 2$. When $C_{iR} \to \infty$, $i = 1, 2$, the model reduces to the *message cognitive* scenario of [8] [11] [12] [14], where the messages W_i, $i = 1, 2$, are known at the relay non-causally. An alternative model introduced in [12] [14] assumes that the relay knows the transmitted codewords X_1^n and X_2^n non-causally, rather than the messages W_1 and W_2 (*signal cognitive* relay), and is unaware of the codebooks. In both cases, we assume in-band relay transmission so that the relay transmits its codeword X_R^n to the destinations simultaneously with the sources. Hence, the received signals at the destinations at time $t = 1, ..., n$ for message/signal cognitive relay are given by (1b)-(1c).

A $(2^{nR_1}, 2^{nR_2}, n)$ code for the ICR with both message and signal cognitive relay is defined by the encoding function at the

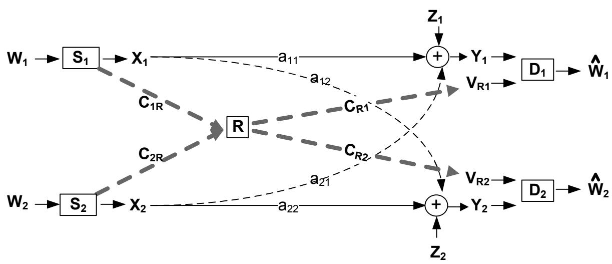

Fig. 3. ICR with out-of-band relay reception/ transmission [15].

source S_i, $i = 1, 2$ and decoding function at the destinations D_i, $i = 1, 2$, discussed in Section II-A. However, unlike the previous model, the relay encoding function for the message cognitive relay model is given by

$$f_r : [1, 2^{nR_1}] \times [1, 2^{nR_2}] \rightarrow \mathbb{R}^n, \tag{4}$$

which maps the messages W_1 and W_2 into a relay codeword x_r^n, and for the signal cognitive relay model by $f_r : \mathbb{R}^n \times \mathbb{R}^n \rightarrow \mathbb{R}^n$, which maps the codewords x_1^n and x_2^n into a relay codeword x_r^n.

C. Out-of-band Relay Reception/Transmission

Finally, with out-of-band relay reception/ transmission, as sketched in Fig. 3 and studied in [15], the relay is assumed to be connected to each source and each destination via a finite-capacity link. All the four links at hand are orthogonal to each other and to the Gaussian IC. Moreover, the links from S_1, S_2 to the relays have capacities C_{1R}, C_{2R} in bits/channel use (of the IC), respectively, and the links from the relay to the destinations D_1, D_2 have capacity of C_{R1}, C_{R2} [bits/channel use], respectively. The signals received at time $t = 1, ..., n$ on the Gaussian channel by D_1 and D_2 are given by

$$Y_{1,t} = a_{11}X_{1,t} + a_{21}X_{2,t} + Z_{1,t} \tag{5a}$$
$$Y_{2,t} = a_{12}X_{1,t} + a_{22}X_{2,t} + Z_{2,t}, \tag{5b}$$

A $(2^{nR_1}, 2^{nR_2}, n)$ code for the ICR with out-of-band relay reception/ transmission is defined by the encoding function at the source S_i, $i = 1, 2$:

$$f_i\text{: } [1, 2^{nR_i}] \rightarrow \mathbb{R}^n \times [1, 2^{nC_{iR}}], \tag{6}$$

which maps a message $W_i \in [1, 2^{nR_i}]$ into a codeword $X^n \in \mathbb{R}^n$ and a message to the relay $V_{iR} \in [1, 2^{nC_{iR}}]$; the encoding function at the relay R

$$f_r : [1, ...2^{nC_{1R}}] \times [1, ...2^{nC_{2R}}] \rightarrow [1, ...2^{nC_{R1}}] \times [1, ...2^{nC_{R2}}] \tag{7}$$

which maps the received messages (V_{1R}, V_{2R}) into messages (V_{R1}, V_{R2}); and by the decoding function at the destination D_i, $i = 1, 2$,

$$g_i : \mathbb{R}^n \times [1, ...2^{nC_{Ri}}] \rightarrow [1, ...2^{nR_i}], \tag{8}$$

which maps the received signal over the IC, Y_i^n, and from the relay, V_{Ri}, into an estimated message $\hat{W}_i$.

III. Transmission Strategies

Relaying in interference-limited systems calls for novel transmission strategies that optimally balance the needs to enhance the useful signal and mitigate interference. The two basic options in this context are *signal relaying*, whereby the relay transmits the desired source signal to its intended destination, and *interference forwarding,* whereby the relay forwards the interfering source signal to the interfered destination [10] [16]. As it will explained below, these two basic approaches are not mutually exclusive and take different forms depending on the specific model under study (see taxonomy in the previous section). In general, either approach or a combination thereof may be optimal in different scenarios, as discussed in the next section.

Signal relaying and interference forwarding are better understood when seen in conjunction with the standard coding approach over ICs, namely the *message-splitting* technique of Han and Kobayashi (HK) [2]. The HK coding approach, which provides the largest known achievable regions for ICs (see, e.g., [17]), prescribes splitting the message W_i (and thus the rate R_i) of any ith user into independent *private* and *common* components, so that the private part is decoded only at the intended destination, while the common part is possibly decoded not only by the intended destination, but also at the interfered receiver. In other words, private parts are treated as noise (i.e., as unstructured signals) by the corresponding interfered receiver, while the structure of the codebook of common part is exploited by the interfered destination when decoding. As an example, under some channel conditions, interfering common parts may be decoded and stripped off by a given decoder, thus obtaining an interference-free received signal.

In the presence of a relay, as detailed below, more general message-splitting coding schemes may be advantageous, depending on the specific ICR model. However, in all ICR models, the relay has the choice to forward either private and/ or common message parts and, in so doing, it may privilege signal relaying and/ or interference forwarding. With *signal relaying*, the relay attempts to boost the useful signal at the intended destination, be it for a private or a common message splits, as for regular point-to-point relay channels. Instead, *interference forwarding* may translate into different operations: The relay can forward a common message split to the interfered destination so as to help the latter decode, and thus strip off, the signal carrying such message (see, e.g., [9] [10] [12]. Alternatively, the relay may forward private-message parts with a negative correlation in order to reduce the corresponding residual interference power at the interfered destination [12].

A. In-band Relay Reception/ Transmission

The system model for an ICR with in-band relay reception/ transmission is given in Section II-A. Here, we briefly recall the transmission scheme proposed in [7], which is based on DF relaying. Other similar techniques are proposed in [9] [10]. The sources perform standard HK coding by splitting each message W_i into a common W_{ic} and private part W_{ip}

as $W_i = (W_{ic}, W_{ip})$ with index sets $W_{ic} \in [1, \ldots, 2^{NR_{ic}}]$, $W_{ip} \in [1, \ldots, 2^{NR_{ip}}]$, $i = 1, 2$. We recall that W_{ic} is decoded at both destinations, whereas W_{ip} is decoded only at the intended destination D_i. The relay decodes both *common* and *private* message indices from the sources and transmits them after properly allocating its power among the common and private messages of S_1 and S_2. Standard block-Markov coding is employed by dividing the messages into B equal blocks (i.e., $W_{ic} = (W_{ic}^{(1)}, W_{ic}^{(2)}, ..., W_{ic}^{(B)})$ and similarly for W_{ip}), and performing transmission over $B+1$ blocks.

To elaborate, the ith source transmits in the bth block the superposition $X_i^{n,(b)} = X_{ic}^{n,(b)} + X_{ip}^{n,(b)}$, where $X_\ell^{n,(b)} = \tilde{X}_\ell^n(W_\ell^{(b-1)}, W_\ell^{(b)}) + U_\ell^n(W_\ell^{(b-1)})$, $\ell \in [ic, ip]$. Codewords $\tilde{X}_\ell^n$ and U_ℓ^n are generated independently (with given power allocation), and the latter is used to cooperate with the relay. In fact, at block b, the relay transmits the superposition $X_R^{n,(b)} = \beta_{1c} U_{1c}^n(\widehat{W}_{1c}^{(b-1)}) + \beta_{2c} U_{2c}^n(\widehat{W}_{2c}^{(b-1)}) + \beta_{1p} U_{1p}^n(\widehat{W}_{1p}^{(b-1)}) + \beta_{2p} U_{2p}(\widehat{W}_{2p}^{(b-1)})$, where $\widehat{W}_\ell^{(b)}$ is the estimate of $W_\ell^{(b)}$ at the relay and β_ℓ are appropriate scaling factors.

According to the discussion above, the relay helps the source-destination pairs by relaying the codewords U_ℓ^n. Forwarding the common-message codewords (U_{1c}^n, U_{2c}^n) can be seen as both *signal relaying* and *interference forwarding*, where the latter aims at improving decodability at the interfered destination. Forwarding the private-message codewords (U_{1p}^n, U_{2p}^n) can also be seen as both *signal relaying* and *interference forwarding*. However, here the latter is designed to possibly reduce the equivalent channel gain seen by the private message at the interfered destination by choosing a negative β_ℓ [12]. It is shown in [12] that in the moderate-interference regime, the relay should allocate most of its power to the latter type of interference forwarding, whereas as the interfering link gets stronger, it opts for the former type, thus aiding the interfered destination to decode and remove the interference.

B. Out-of-band Relay Reception/ In-band Relay Transmission

In the presence of out-of-band relay reception/ in-band relay transmission with message cognition (recall Sec. II-B with $C_{iR} \to \infty, i = 1, 2$), the message-splitting strategy used at the sources can be more general than the conventional scheme discussed above. In particular, the individual messages are split as $W_i = (W_{ic}, W_{ip})$ where $W_{ic} \in [1, \ldots, 2^{NR_{ic}}]$ is the common message to be decoded at both destinations and $W_{ip} \in [1, \ldots, 2^{NR_{ip}}]$ is the private message to be decoded at D_i, $i = 1, 2$ only. However, since the relay has out-of-band links from both sources, in the message cognitive set-up investigated in [8] [11], one can further split the private messages as $W_{ip} = (W_{ip'}, W_{i\widehat{p}})$ $i = 1, 2$: The splits $(W_{ic}, W_{ip'})$ are jointly encoded by each source and the relay, similar to the discussion above, while encoding of $W_{i\widehat{p}}$, $i = 1, 2$, is performed *by the relay only.* We emphasize that this is possible since the relay has a priori knowledge of the source messages. In the encoding of $W_{ip'}$, the knowledge of the private message $W_{jp'}$, which causes interference at D_i, $i, j = 1, 2, j \neq i$, is exploited via Dirty Paper Coding (DPC) similar to [5]. Moreover, forwarding of $W_{i\widehat{p}}$ can be seen as a form of *signal relaying*, in which, unlike the schemes discussed above, new information is injected in the network by the relay operation.

In the signal-cognitive scenario of [12] [14] the relay observes the codewords (X_1^n, X_2^n) only and does not know the codebooks used. In this case, the relay can only forward some combination of the (unstructured) signals X_1^n and X_2^n, rather than being able to operate directly with the messages. Hence, the relay is forced to beamform both signals, and thus both common and private parts, using the same correlation sign. Using positive correlation, this leads to both signal relaying and interference forwarding, whereby the latter aims at improve the decodability of interference.

C. Out-of-band Relay Reception/ Transmission

In this section, we address the set-up investigated in [15] as described in Sec. II-C. In this setting, since the relay has orthogonal links towards the destination, joint encoding of some message splits is not of interest (as coherent operation is not an option). Moreover, the following extension of the HK coding scheme is tailored to the model at hand: Each message W_i is split as $W_i = (W_{iR}, W_{ip}, W_{ic'}, W_{ic''})$, $i = 1, 2$, where: (*i*) $W_{iR} \in [1, ...2^{nR_{iR}}]$ is a private message that is transmitted by the source directly to the relay and from there to the intended destination D_i (*signal relaying*). Notice the since the relay has orthogonal channels to the IC, this message is conveyed interference-free to D_i; (*ii*) $W_{ip} \in [1, ...2^{nR_{ip}}]$ is a private message that is transmitted over the IC, decoded at D_i and treated as noise at D_j, $j \neq i$; (*iii*) $W_{ic'} \in [1, 2^{nR_{ic'}}]$ is a common message that is transmitted over the IC *and also to the relay*. Specifically, the relay conveys $W_{ic'}$ to D_j only, $j \neq i$, to enable interference cancellation (*interference forwarding*); (*iv*) $W_{ic''} \in [1, ...2^{nR_{ic''}}]$ is a common message that is transmitted over IC only and decoded at both destinations. Notice that as a result of (*i*) and (*iii*), the messages sent over the links are $V_{1R} = (W_{1c'}, W_{1R})$, $V_{2R} = (W_{2c'}, W_{2R})$ and $V_{R1} = (W_{2c'}, W_{1R})$, $V_{R2} = (W_{1c'}, W_{2R})$ (recall Sec. II-C). Overall, it is noted that the relay conveys both messages *independent* of the transmission on the IC (W_{iR}), which bring additional information bits directly to the destinations and can be seen as *signal relaying*, and messages that are *correlated* with the transmission over the IC and enable interference cancellation ($W_{ic'}$), which can be seen as *interference forwarding*. In the next section, we discuss a few special cases where either signal relaying and/ or interference forwarding are optimal.

IV. Capacity Regions

In this section, we provide some special cases of the ICR models discussed above for which optimality of signal relaying and/or interference forwarding is known. The results herein are selected from [12] [14] [15]. The following theorem demonstrates the optimal relaying scheme for an out-of-band relay reception/in-band relay transmission model with a *signal cognitive* relay.

Theorem 1: (Theorem 3 [14]) For an ICR with a signal cognitive relay satisfying the *strong relay-interference* conditions

$$a_{ij} \geq a_{ii} + (a_{Ri} + a_{Rj})\sqrt{\frac{P_R}{P_i}} \text{ for } i, j = 1, 2, i \neq j,$$

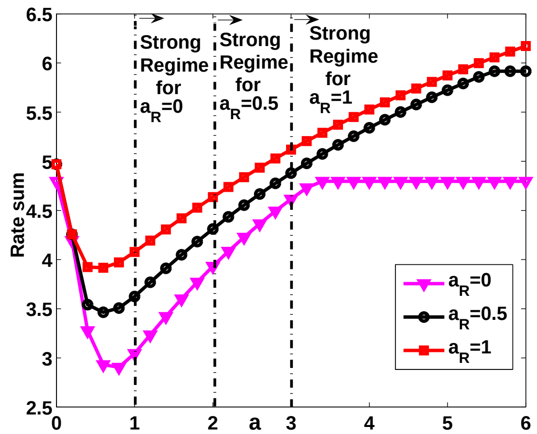

Fig. 4. Out-of-band relay reception/ in-band relay transmission with a signal cognitive relay [14]: Maximum sum-rate versus $a = a_{12} = a_{21}$ for different a_R: $a_{11} = a_{22} = 1$, $P_1 = P_2 = P_R = 10$. In the strong relay-interference regime, the shown sum-rate is the sum-capacity (Theorem 1).

the capacity region is given by the convex hull of the union of all rates (R_1, R_2) satisfying

$$R_i \leq C(\Psi_i), i = 1, 2 \quad (9)$$

$$R_1 + R_2 \leq \min\{C(\Psi_{t_1}), C(\Psi_{t_2})\} \quad (10)$$

where $\Psi_i = (a_{ii} + a_{Ri}\sqrt{\beta_{it}})^2 P_i$, $\Psi_{t_i} = (a_{ii} + a_{Ri}\sqrt{\beta_{it}})^2 P_i + (a_{ji} + a_{Ri}\sqrt{\beta_{jt}})^2 P_j$, $\beta_{1t}P_1 + \beta_{2t}P_2 \leq P_R$, for $i, j = 1, 2$, $i \neq j$.

Achievability of the capacity region (9)-(10) is obtained by having the receivers decode both the desired and interfering signals. Moreover, the relay beamforms with both codewords, so that the optimal relaying involves both *signal relaying* and *interference forwarding* (recall Sec. III-B). Fig. 4 shows an achievable sum-rate of a symmetric signal cognitive system for $a_{21} = a_{12} = a$, $a_{11} = a_{22} = 1$, $P_1 = P_2 = P_R = 10$, and $a_{R1} = a_{R2} \in \{0, 0.5, 1\}$. In the strong relay-interference regime, the shown sum-rate is the sum-capacity by Theorem 1.

A capacity region similar to Theorem 1 can be obtained for the one-sided IC with a one-sided signal cognitive relay for which $a_{12} = a_{R2} = 0$ (Theorem 3 [12]). Since there is no interference at D_2, the strong relay-interference condition and the rate sum constraint at D_2 (from Theorem 1) can be relaxed for the one-sided scenario. The one-sided case further highlights the role of the relay in managing the interference. The relay spends most of its power in interference forwarding: When the interference is low (small a_{21}), the relay negatively correlates its signal with the interfering source thus effectively reducing the power of the received interference. When the interference gets stronger (large a_{21}), the relay beamforms with S_2, helping D_1 decode the interference (Figure 5 [12]).

In the following theorems, we consider the out-of-band relay reception/ transmission of [15] and demonstrate optimal relaying operations.

Theorem 2: (Theorem 3 [15]) For an ICR with out-of-band relay reception/ transmission with $C_{R1} \leq C_{1R}$ and $C_{R2} \leq C_{2R}$, the capacity region is given by the capacity region $\mathcal{C}_{IC}$ of the standard IC, enhanced by (C_{R1}, C_{R2}) along the individual rates, i.e., by the set of rates

$$\{(R_1, R_2)\colon (R_1 - R_1', R_2 - R_2') \in \mathcal{C}_{IC}\}.$$

for some $0 \leq R_1' \leq C_{R1}$ and $0 \leq R_2' \leq C_{R2}$. Equivalently, the capacity region is given by the union over the sets of rates (R_1, R_2) that satisfy

$$R_1 \leq \frac{1}{n} I(X_1^n; Y_1^n) + C_{R1} \quad (11)$$

$$R_2 \leq \frac{1}{n} I(X_2^n; Y_2^n) + C_{R2}, \quad (12)$$

for some input distributions $p(x_1^n)p(x_2^n)$ that comply with the power constraints.

Achievability of the capacity region of Theorem 2 is obtained by letting every source transmit independent information towards the intended destination over the out-of-band links to and from the relay. Given the condition $C_{R1} \leq C_{1R}$ and $C_{R2} \leq C_{2R}$, such additional (private) information can have rate up to C_{R1} and C_{R2} for the first and second links, respectively, and enhances accordingly the capacity of the underlying IC. In other words, the capacity region is achieved under the conditions of Theorem 2 by *signal relaying* (recall Sec. III-C).

While Theorem 2 provides a general capacity result for the case where the relay-to-destination links set the performance bottleneck, i.e., $C_{R1} \leq C_{1R}$ and $C_{R2} \leq C_{2R}$, we next investigate the capacity region for the complementary scenario in which such condition is not satisfied. We focus specifically on the case characterized by $C_{1R} \geq C_{R1}$ and $C_{R2} \geq C_{2R}$, where the extension to the *dual* scenario $C_{R1} \geq C_{1R}$ and $C_{2R} \geq C_{R2}$ will be straightforward (and not explicitly stated) by appropriately switching indices. Under the assumption at hand, the following rate

$$R_{ex_{12}} = \min\{C_{1R} - C_{R1},\ C_{R2} - C_{2R},\}$$

plays a key role. This can be interpreted as the *excess rate from S_1 to D_2 on the relay links*, once user 1 and user 2 have allocated the maximum possible rate on the relay links for *signal relaying*, namely $R_{1R} = \min\{C_{R1}, C_{1R}\} = C_{1R}$ and $R_{2R} = \min\{C_{R2}, C_{2R}\} = C_{R2}$.

Theorem 3: (Theorem 4 [15]) In a ICR with out-of-band relay reception/ transmission and channel conditions $a_{21} \geq a_{22}$ and $R_{ex_{12}} \geq \max\{0, \frac{1}{2}\log(1 + a_{11}^2 P_1 + a_{21}^2 P_2) - \frac{1}{2}\log(1 + a_{12}^2 P_1 + a_{22}^2 P_2)\}$, the following gives the capacity region

$$R_1 \leq \frac{1}{2}\log\left(1 + a_{11}^2 P_1\right) + C_{R1} \quad (13)$$

$$R_2 \leq \frac{1}{2}\log\left(1 + a_{22}^2 P_2\right) + C_{2R} \quad (14)$$

$$R_1 + R_2 \leq \frac{1}{2}\log\left(1 + a_{11}^2 P_1 + a_{21}^2 P_2\right) + C_{R1} + C_{2R}. \quad (15)$$

Achievability of the capacity region in Theorem 3 calls for two different strategies depending on the channel conditions. In both cases, the receivers decode both message from the

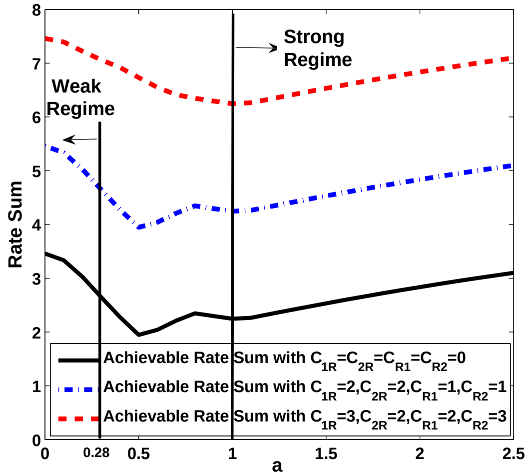

Fig. 5. ICR with out-of-band relay reception/ transmission [15]: Achievable sum-rate versus $a = a_{12} = a_{21}$ for various relay link capacities, $a_{11} = a_{22} = 1$, $P_1 = P_2 = 10$. $C_{1R} = C_{R1} = C_{2R} = C_{R2} = 0$ corresponds to the maximum sum-rate with no relay, $C_{1R} = C_{2R} = 2, C_{R1} = C_{R2} = 1$ satisfies the conditions in Theorem 2, and $C_{1R} = C_{R2} = 3, C_{2R} = C_{R1} = 2$, satisfies the conditions in Theorem 3.

two sources. In the first scenario, we have $(a_{11}^2 - a_{12}^2)P_1 + (a_{21}^2 - a_{22}^2)P_2 \leq 0$, so that the sum-rate bound due to receiver D_1 sets the performance bottleneck irrespective of a positive excess rate $R_{ex_{12}}$ (which clearly increases the sum-rate at D_2). Therefore, it can be seen that the capacity region of Theorem 3 is attained by *signal-relaying* only. In the second scenario, we have $(a_{11}^2 - a_{12}^2)P_1 + (a_{21}^2 - a_{22}^2)P_2 > 0$, so that, conversely, the sum-rate bound at D_2 may be more restrictive than the sum-rate bound at D_1. In this scenario, it can be seen that the optimal relay operation is to perform *interference forwarding* from S_1 to D_2 with rate equal to $\frac{1}{2}\log(1 + P_1 + a_{21}^2 P_2) - \frac{1}{2}\log(1 + a_{12}^2 P_1 + P_2)$ (recall Sec. III-C).

In Fig. 5, we show an achievable sum-rate for the ICR with out-of-band relay reception/ transmission [15] for different configurations of the relay link capacities and with $P_1 = P_2 = 10$, $a_{11} = a_{22} = 1$ and $a_{21} = a_{12} = a$. For comparison, we show the case $C_{1R} = C_{2R} = C_{R1} = C_{R2} = 0$. Moreover, we first consider a scenario where relay-to-destination links have smaller capacities than the source-to-relay links, $C_{1R} = C_{2R} = 2, C_{R1} = C_{R2} = 1$, thus falling within the assumptions of Theorem 2. It can be seen that the sum-rate increases by $C_{R1} + C_{R2} = 2$ for all values of a which is achieved by the relay transmitting additional source information, via *signal relaying*. Also, from Theorem 2, we can conclude that in the noisy $a < 0.28$ [19] and strong $a > 1$ [3] interference regimes, the sum-rate shown is the sum-capacity. Finally, we consider a situation with $C_{1R} = C_{R2} = 3, C_{2R} = C_{R1} = 2$, which falls under the conditions of Theorem 3 for $a \geq 1$. As stated in the Theorem, for $a \geq 1$, the sum-rate shown in the sum-capacity and is $C_{R1} + C_{2R} = 4$ bits/channel use larger than the reference case of zero relay capacities.

V. Conclusion

The potential benefits of cooperation over wireless networks must be reconsidered in order to account for the role of interference. In this paper, a brief review of current research activity in this context has been discussed focusing on a Gaussian two-user interference model with a relay (ICR). A taxonomy of ICR models has been presented, distinguishing scenarios with in-band or out-band relay reception/ transmission, along with corresponding transmission strategies. Some illustrative results have been presented that show the need in different scenarios not only for standard signal relaying, but also for interference forwarding.

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