Oblivious and Out-of-Band Relaying for Interference Networks

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Motivation

- Standard assumptions in network-information theory:
  - Design of encoding/decoding functions (e.g., codebooks) is performed jointly
  - All nodes are potentially aware at all times of the operations carried out by any other node
Motivation: Examples

Enc Dec
Relay
Relay aware of the codebook shared by enc and dec

Enc

Relay

Dec
Motivation: Examples

Decoders aware of the codebooks of both intended and interfering encoders

Enc 1

Dec 1

Enc 2

Dec 2

Simeone et al ( ) Oblivious Relaying 2/1/10 4 / 29
Motivation: Oblivious Processing

- Such full coordination may be impractical in decentralized scenarios.
- ... **Ultimate performance limits in the absence of full codebook information (oblivious processing)?**
  [Sanderovich et al 08]
Example: Primitive Relay Channel (PRC)

- Out-of-band relay-to-destination link of capacity $C$ [Zhang 88] [Kim 08]
- Relay unaware of encoder’s codebook (oblivious relaying)
Relay unaware of encoder’s codebook (oblivious relaying)
Decoders unaware of the interferer’s codebook (interference-oblivious decoding)
Overview

- Review and extend definition of oblivious processing of [Sanderovich et al 08]

- Capacity results:
  - capacity for discrete memoryless PRC with oblivious relaying
  - 1/2-bit capacity approximation for Gaussian memoryless PRC with oblivious relaying
  - capacity region of PIRC with oblivious relaying and decoding
  - sum-capacity of symmetric PIRC with oblivious relaying only

- Application to femtocells
[Sanderovich et al 08] Upper and lower bounds on capacity of primitive multirelay channel *without direct link* and with oblivious relaying.

Further studied by [Simeone et al 09]
System Model: Primitive Relay Channel (PRC)

- Discrete memoryless PRC \( (\mathcal{X}_1, p(y_1, y_2, y_3|x_1, x_2), \mathcal{Y}, \mathcal{Y}_3, C) \)

Gaussian PRC:

\[
Y_{3i} = \sqrt{\alpha} X_i + N_{3i},
Y_i = X_i + N_i,
\]

with \( N_{3i}, N_i \) independent zero-mean unit-power Gaussian, and \( \frac{1}{n} \sum_{i=1}^{n} E[X_i^2] \leq P \).
System Model: Primitive Interference Relay Channel (PIRC)

- Discrete memoryless PIRC
  \((\mathcal{X}_1, \mathcal{X}_2, p(y_1, y_2, y_3| x_1, x_2), \mathcal{Y}_1, \mathcal{Y}_2, \mathcal{Y}_3, C_1, C_2)\)
System Model: Oblivious Processing

- Following [Sanderovich et al 08]:
  - Encoding $x^n(F, W)$ dependent on both message $W$ and index $F$
  - The index $F \in [1, |X|^{2nR}]$ identifies the used codebook of rate $R$ [bits/ channel use]

**Definition**

With oblivious processing, when not conditioning on $F$, the codeword $x^n(F, W)$ "looks" i.i.d. $\sim p_X(\cdot)$:

$$p^n_X(x^n(F, W)) = \prod_{i=1}^{n} p_X(x_i)$$

(... but $p^n_X(x^n(F = f, W)) \neq \prod_{i=1}^{n} p_X(x_i)$ [Shamai Verdú 97]!)}
1. **Oblivious relaying**: The relay is not aware of both indices $F_1$ and $F_2$.

2. **Interference-oblivious decoding**: Destination $j$ only knows index $F_j$ and not $F_i$, $i \neq j$. 

Simeone et al. (2010) - Oblivious Relaying
This definition rules out:

- general multiletter input distributions
- time-sharing

... but allows all standard "single-letter" coding schemes (superposition coding, rate-splitting, ...)

Simeone et al ( )

Oblivious Relaying
In some scenarios, it may be reasonable to assume that all nodes share a time-sharing sequence $q^n \in Q^n$

Encoding/decoding functions depend on $q^n$

**Definition**

Oblivious processing *with enabled time-sharing* requires conditional independence

$$p^n_{X|Q}(x^n(F, W)|q^n) = \prod_{i=1}^{n} p_{X|Q}(x_i|q_i)$$
Achievable Rates

Definition

A rate pair \((R_1, R_2)\) is said to be achievable if there exists a sequence of codes such that \(\Pr[(\hat{W}_1, \hat{W}_2) \neq (W_1, W_2)] \to 0\), where the probability is taken with respect to \(W_1, W_2\) and \(F_1, F_2\). The capacity region \(C\) is the closure of the union of all achievable rates.
Capacity of PRC with Oblivious Relaying

Simeone et al.
The capacity of a PRC with oblivious relaying and enabled time-sharing is given by

$$ C = \max I(X; Y\hat{Y}_3|Q) $$

s.t. $$ C \geq I(Y_3; \hat{Y}_3|YQ) $$

where maximization is taken with respect to the distribution

$$ p(q)p(x|q)p(\hat{y}_3|y_3, q) $$

and the mutual informations are evaluated with respect to

$$ p(q)p(x|q)p(\hat{y}_3|y_3, q)p(y, y_3|x). $$

If time-sharing is not allowed, the above is still an upper bound on the capacity, and setting $Q = \text{const}$ leads to an achievable rate.
Achievability: CF with time-sharing [El Gamal et al 06]

Converse:
- The destination knows $F$ and $\tilde{Q} = q^n$: $H(W|Y^nS\tilde{F}\tilde{Q}) \leq n\epsilon_n$ with $\epsilon_n \to 0$
- The relay does not know $F$: $nC \geq H(\text{relay-to-destination message}(S)|\tilde{Q})$...
- Proof based on conditional independence of $X^n$ given $\tilde{Q}$
- $\hat{Y}_{3i} = [SX^{i-1}Y_3^{i-1}Y^{i-1}Y_i^n]$ (compare with Wyner-Ziv: $[SY^{i-1}Y^n_{i+1}]$)
Optimality of CF follows from the assumption of oblivious relaying

In [Sanderovich et al 08] (multirelay channel without direct link) optimality of (distributed) CF strategies remains elusive (as for the corresponding source coding problem, CEO problem)

Time-sharing in general necessary to achieve capacity
Gaussian PRC

- Optimization of input distribution $p(q)p(x|q)p(\hat{y}_3|y_3, q)$ in Proposition 1 open problem
  - Gaussian input distribution is generally not optimal [Sanderovich et al 08]
  - Non-Gaussian test channels may be advantageous [Dabora Servetto 08]

**Theorem**

The rate achievable via CF (and hence oblivious relaying)

$$R_{CF} = \frac{1}{2} \log_2 \left( 1 + P + \frac{\alpha P}{1 + P + \alpha P} \right)$$

on the Gaussian PRC, by employing Gaussian channel inputs, Gaussian test channel and no time-sharing, is at most half bit away from the capacity of the PRC with codebook-aware (and thus also oblivious) relaying.
Proof: The proof is obtained by comparing the achievable rate above with the cut-set bound upper bound (which holds even with non-oblivious relaying)

\[ R_{UB} = \min \left\{ \frac{1}{2} \log_2 (1 + P) + C, \frac{1}{2} \log_2 (1 + \alpha P + P) \right\}. \]
Primitive Interference Relay Channel

Enc 1

Relay

Enc 2

Dec 1

Dec 2

$C_1$

$C_2$
The capacity region of the PIRC with oblivious relaying, interference-oblivious decoding and enabled time-sharing is given by the set of all non-negative pairs \((R_1, R_2)\) that satisfy

\[
R_j \leq I(X_j; Y_j \hat{Y}_3^{(j)} | Q), \text{ for } j = 1, 2,
\]

for some distribution \(p(q) \prod_{j=1}^2 p(x_j | q) p(\hat{Y}_3^{(j)} | y_3, q) p(y_1, y_2 | x_1, x_2)\) that satisfy

\[
C_j \geq I(Y_3; \hat{Y}_3^{(j)} | Y_j Q) \text{ for } j = 1, 2.
\]

If time-sharing is not enabled, the above is an outer bound and setting \(Q = \text{const}\) leads to an achievable rate region.
Achievability: Relay employs CF and the destinations treat the interfering signal as noise.

Converse: Similar to Proposition 1.
**Definition:** A symmetric PIRC is defined by (i) 
\[ p_{Y_1, Y_3 | X_1 X_2}(\cdot, y_3 | x_1, x_2) = p_{Y_2, Y_3 | X_1 X_2}(\cdot, y_3 | x_1, x_2) \] [Carleial 78]; (ii) The relay is constrained to send \( S_1 = S_2 \).

**Theorem**

The sum-capacity of the symmetric PIRC and of the PMARC with oblivious relaying, interference-aware decoding and enabled time-sharing is given by

\[
C_{sum} = \max I(X_1 X_2; Y_1 \hat{Y}_3 | Q) \\
s.t. C \geq I(Y_3; \hat{Y}_3 | Y_1 Q)
\]

where maximization is taken with respect to the distribution

\[ p(q)p(x_1 | q)p(x_2 | q)p(\hat{y}_3 | y_3, q) \]

and the mutual informations are evaluated with respect to

\[ p(q)p(x_1 | q)p(x_2 | q)p(\hat{y}_3 | y_3, q)p(y_1, y_3 | x_1, x_2). \]
Proof: The model is equivalent to a primitive multiple access relay channel (PMARC)...
Closed-Access femtocells: Relay only for home user
Open-Access femtocells: Relay for both outdoor and home user
Conclusions

- Oblivious Processing: Constraint on codebook state information
- Primitive Relaying: Out-of-band relay-to-destination link
- Derived capacity results for primitive relay and interference channels
- Application to femtocell
- Future work: capacity region of PMARC, in-band relaying,...