Markov Chain Monte Carlo Detection for Underwater Acoustic Channels

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***Abstract*—In this work, we develop novel statistical detectors to combat intersymbol interference for frequency selective channels based on Markov Chain Monte Carlo (MCMC) techniques. While the optimal maximum a posteriori (MAP) detector has a complexity that grows exponentially with the constellation size and the memory of the channel, the MCMC detector can achieve near optimal performance with a complexity that grows linearly. This makes the MCMC detector particularly attractive for underwater acoustic channels with long delay spread. We examine the effectiveness of the MCMC detector using actual data collected from underwater experiments. When combined with adaptive least mean square (LMS) channel estimation, the MCMC detector achieves superior performance over the direct adaptation LMS turbo equalizers (LMS-TEQ) for a majority of data sets transmitted over distances from 60 meters to 1000 meters.**

I. Introduction

Underwater acoustic (UWA) channels pose unique challenges due to the low speed of sound, limited communication bandwidth, and the intrinsic motion due to waves and currents [1]. Such channels feature large delay spread, frequency-dependent Doppler shift, and high time variability. To overcome these challenges, turbo equalization techniques have been applied to UWA channels [2], which demonstrate improved performance over the phase-coherent receivers in [3]. The LMS-TEQ turbo equalizer developed in [2] are based on the least mean square (LMS) algorithm and it is shown to converge rapidly to the optimal equalizer without the need for channel estimation.

In this work, we study the application of statistical detectors based on Markov Chain Monte Carlo (MCMC) techniques to UWA channels. Such detectors offer a low-complexity approximation to the maximum-likelihood sequence estimation (MLSE) and are used to directly perform data detection without channel equalization. The MCMC detectors have been studied previously in [4]–[6] for both multiple-input multiple-output (MIMO) frequency-flat channels and for frequency selective channels with inter-symbol interference [7]. Under the assumption of perfect channel state information (CSI) at the receiver, the MCMC detectors developed in these work demonstrate excellent performance at low-complexity. In this paper, we apply MCMC detectors to UWA channels and show that when combined with adaptive channel estimation, the MCMC detectors demonstrate excellent performance when compared with the adaptive LMS-TEQ equalizer of [2]. For UWA channels with sparse channel impulse response, we demonstrate that the variable step size LMS (VSLMS) algorithm provides superior channel tracking and hence yields improved performance than that of the standard LMS algorithm.

Experimental results are provided for multiple data sets measured over 60 meter to 1000 meter distances from the recent SPACE'08 experiment conducted off the coast of Martha's Vinyard, MA.

II. Channel Model and System Setup

We first describe the structure of the transmitter, the UWA channel model, and the structure of the receiver.

At the transmitter side, we pass every group of N_b information bits to the channel encoder to generate a sequence of N_c coded bits. The coded bits are interleaved and mapped into a sequence of complex symbols. We then insert pilot symbols and the resulting symbol sequence $\{x_n\}$ is divided into multiple packets, each containing N_p pilot symbols and N_d data symbols. Assuming that channel coding is performed across every I packets, we must have $N_b = IN_d(\log_2 M_c)R$, where R is the code rate and M_c is the size of the constellation. The sequence $\{x_n\}$ is passed through a pulse shaping filter and transmitted by a transducer through the UWA channel.

Assume that a total of K receiving hydrophones are used. The received signal at time n at the k-th receive hydrophone can be expressed as

$$y_n^{(k)} = \sum_{l=0}^{L} h_{n,l}^{(k)} x_{n-l} + v_n^{(k)}, \tag{1}$$

where $l = 0, \cdots, L$ is the index of l-th path, $h_{n,l}^{(k)}$ denotes the channel gain of the l-th path at time n, between the transducer and the k-th receive hydrophone.

The receiver structure is illustrated in Fig.1, which employs iterative joint channel estimation, data detection, and channel decoding over every groups of I packets. We divide each packet into multiple blocks and perform channel estimation and MCMC detection block-wise. Within each packet, we first estimate the channel from the pilot block. These channel estimates are then passed to the MCMC detector for data detection over the first data block. We then refine the channel estimates based on the output of the MCMC detector, and use these to perform data detection for the second block. The soft outputs of the MCMC detector from all the blocks within the group of I packets are passed to the channel decoder for decoding. For the next iteration, soft information from the channel decoder is fed back to the MCMC detector for data detection.

Details of the channel estimation and MCMC detection will be presented in the following sections.

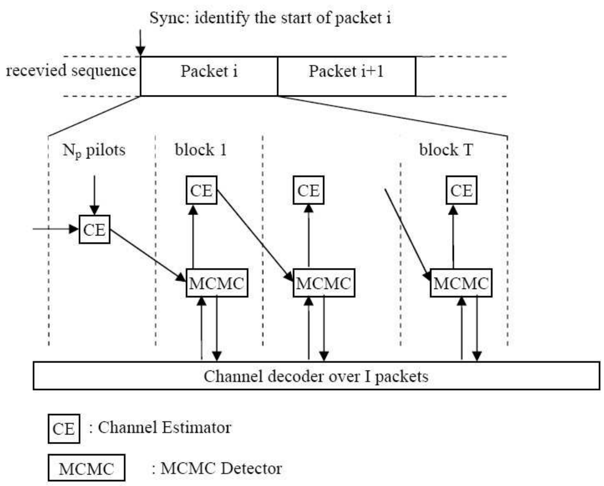

Fig. 1. Flowchart of the receiver

III. Channel estimation

We consider two adaptive channel estimation algorithms: the LMS and the VSLMS. The latter algorithm is shown to outperform the LMS when the channel impulse response is sparse.

A. Least mean square (LMS) channel estimation

For the LMS channel estimation, the channel gains are updated according to

$$\hat{h}^{(k)}_{n+1,l} = \hat{h}^{(k)}_{n,l} + \frac{\mu}{\sum_{i=0}^{L} |x_{n-i}|^2} e^{(k)}_n \hat{x}^*_{n-l}. \tag{2}$$

For the pilot block, each $\hat{x}_n$ equals the pilot symbol. For each data block within a packet, $\hat{x}_n$ denotes the soft value at the output of the MCMC detector, e.g., $\hat{x}_n = \sum_q qP(x_n = q)$, where $q = 0, 1, \cdots, M_c - 1$ is an arbitrary constellation point. The residual error $e^{(k)}_n$ in (2) is computed as $e^{(k)}_n = y^{(k)}_n - \sum_{l=0}^{L} \hat{h}^{(k)}_{n,l} \hat{x}_{n-l}$. We run the LMS algorithm for multiple passes within each block of symbols to ensure convergence and the step-size μ is set to be 0.02.

B. Variable step size LMS (VSLMS)channel estimation

For the VSLMS, instead of using the same step size μ to update all L channel taps, we allow the step sizes to vary for different taps. Let $\mu^{(k)}_{n,l}$ denote the step size for the l-th channel tap. We update the channel estimates as follows [8]

$$\hat{h}^{(k)}_{n+1,l} = \hat{h}^{(k)}_{n,l} + \mu^{(k)}_{n,l} e^{(k)}_n \hat{x}^*_{n-l}. \tag{3}$$

At the time n, $\mu^{(k)}_{n,l}$ can be updated from $\mu^{(k)}_{n-1,l}$ as

$$\mu^{(k)}_{n,l} = \mu^{(k)}_{n-1,l} + \rho \, \mathrm{Re}\{e^{(k)}_n \hat{x}^*_{n-l} \phi^{(k)}_{n,l}\}, \tag{4}$$

$$\phi^{(k)}_{n,l} = \alpha \phi^{(k)}_{n-1,l} + e^*_{n-1} \hat{x}_{n-l-1}, \tag{5}$$

where we set $\rho = 0.005, \alpha = 0.95$. Then the vector of step sizes is scaled such that $\frac{1}{L} \sum_{l=0}^{L} \mu^{(k)}_{n,l} = \mu = 0.02$ to guarantee stability. This will guarantee the same misadjustment for both LMS and VSLMS algorithms [9].

To give some insights to the mechanism of the VSLMS algorithm, we note that the sign of $\phi^{(k)}_{n,l}$ indicates the average direction of the stochastic gradient in the past. Accordingly, if the present gradient, $e^*_{n-1} \hat{x}_{n-l-1} \phi^{(k)}_{n,l}$, has the same sign as $\phi^{(k)}_{n,l}$, then we assume that the VSLMS algorithm has not converged yet. Thus, we should increase the step-size parameter $\mu^{(k)}_{n,l}$. Otherwise, it should be decreased.

IV. MCMC Detection

The basic principles of the MCMC detector for ISI channels with perfect CSI are presented in [7]. The main idea is to use the Gibbs sampler to generate a small set of most likely transmitted sequences, based on which the log-likelihood-ratio (LLR) of each transmitted bit is computed. Assume that each data block within a packet includes n_d data symbols, corresponding to $B = (\log_2 M_c) \cdot n_d$ transmitted bits $\mathbf{b} = (b_0, \cdots, b_{B-1})$. The Gibbs sampler is a statistical procedure used to draw one bit at a time. Consider bit b_m, where $0 \le m \le B - 1$. Let λ_m denote the LLR of b_m, provided by the channel decoder. Let $\mathbf{y} = (\mathbf{y}^{(1)}, \cdots, \mathbf{y}^{(K)})$, where $\mathbf{y}^{(k)}$ denotes the received signal sequence from the k-th hydrophone. We run the Gibbs sampler over I iterations to generate a set of I most likely transmitted sequences, denoted by $\{\mathbf{b}^{(1)}, \cdots, \mathbf{b}^{(I)}\}$. Details of the Gibbs sampler are described in Algorithm 1.

Algorithm 1: Gibbs sampler

generate an initial $\mathbf{b}^{(\mathbf{0})}$
for $n = 1$ to I
generate $b^{(n)}_0$ from distribution
$P(b_0 = a | b^{(n-1)}_1, b^{(n-1)}_2, \cdots, b^{(n-1)}_{B-1}, \mathbf{y}, \lambda_0)$
generate $b^{(n)}_1$ from distribution
$P(b_1 = a | b^{(n-1)}_0, b^{(n-1)}_2, \cdots, b^{(n-1)}_{B-1}, \mathbf{y}, \lambda_1)$
$\vdots$
generate $b^{(n)}_{B-1}$ from distribution
$P(b_{B-1} = a | b^{(n-1)}_0, b^{(n-1)}_1, \cdots, b^{(n-1)}_{B-2}, \mathbf{y}, \lambda_{B-1})$
end for

Note that when updating $b^{(n)}_m$ during the n-th iteration, we condition upon updated samples $(b^{(n)}_0, \cdots b^{(n)}_{m-1})$ obtained during the same iteration, and samples $(b^{(n-1)}_{m+1}, \cdots b^{(n-1)}_{B-1})$ obtained from the previous iteration. Let

$$\bar{\mathbf{b}}_m = (b^{(n)}_0, \cdots, b^{(n)}_{m-1}, b^{(n-1)}_{m+1}, \cdots, b^{(n-1)}_{B-1}).$$

We draw sample $b_m^{(n)}$ based on the conditional probability distribution

$$P(b_m = a|\bar{\mathbf{b}}_m, \mathbf{y}, \lambda_m), \text{ where } a = 0, 1.$$

For each a, we define

$$\mathbf{b}^a = \{b_0^{(n)}, \cdots, b_{m-1}^{(n)}, a, b_{m+1}^{(n-1)}, \cdots, b_{B-1}^{(n-1)}\},$$

and let $\mathbf{x}^a$ denote the symbol vector corresponding to $\mathbf{b}^a$. Also, let $\mathbf{x}_{j-L:j} = (x_{j-L}, x_{j-L+1}, \cdots, x_j)$. First, we assume that the channel gains $\{h_{j,l}^{(k)}\}$ are perfectly known, and the noise $\{v_n^{(k)}\}$ is white with a complex Gaussian distribution of zero mean and variance of σ_k^2. Assume that bit b_m is mapped to symbol x_i. Then we obtain

$$\begin{aligned} & P(b_m = a|\bar{\mathbf{b}}_m, \mathbf{y}) \propto \prod_{k=1}^{K} p(\mathbf{y}^{(k)}|\mathbf{x}^a) P(\mathbf{x}^a) \\ \propto \quad & \prod_{k=1}^{K} \prod_{j=i}^{i+L} p(y_j^{(k)}|\mathbf{x}_{j-L:j}^a) P(b_m = a) \\ = \quad & C \cdot \exp\left\{ \sum_{k=1}^{K} \sum_{j=i}^{i+L} \Big(-\frac{1}{\sigma_k^2} |y_j^{(k)} - \sum_{l=0}^{L} h_{j,l}^{(k)} x_{j-l}^a|^2 \Big) \right\} \\ & P(b_m = a), \end{aligned} \tag{6}$$

where $P(b_m = a)$ can be computed from λ_m and C is a scaling constant to ensure that $P(b_m = 0|\bar{\mathbf{b}}_m, \mathbf{y}) + P(b_m = 1|\bar{\mathbf{b}}_m, \mathbf{y}) = 1$. For channels with imperfect CSI, we replace $h_{j,l}^{(k)}$ in (6) by the estimated channel $\hat{h}_{j,l}^{(k)}$. We will also replace σ_k^2 in (6) by $\hat{\sigma}_k^2$ to take into account both channel estimation error and the variance of channel noise. To be specific, assume that

$$h_{i,l}^{(k)} = \hat{h}_{i,l}^{(k)} + e_{i,l}^{(k)}. \tag{7}$$

The received signal can be written as

$$\begin{aligned} y_i^{(k)} &= \sum_{l=0}^{L} h_{i,l}^{(k)} x_{i-l} + v_i^{(k)} \\ &= \sum_{l=0}^{L} \hat{h}_{i,l}^{(k)} x_{i-l} + v_i^{(k)} + \sum_{l=0}^{L} e_{i,l}^{(k)} x_{i-l}. \end{aligned} \tag{8}$$

Let $\tilde{n}_i^{(k)} = v_i^{(k)} + \sum_{l=0}^{L} e_{i,l}^{(k)} x_{i-l}$ and $\tilde{\sigma}_k^2 = \text{Var}(\tilde{n}_i^{(k)})$. We then estimate $\tilde{\sigma}_k^2$ from the pilot block such that

$$\tilde{\sigma}_k^2 \approx \frac{1}{N_p} \sum_{i=1}^{N_p} \left| y_i^{(k)} - \sum_{l=0}^{L} \hat{h}_{i,l}^{(k)} x_{i-l} \right|^2 \tag{9}$$

where $x_i, i = 1, \cdots, N_p$ are pilot symbols. Finally, we rewrite (6) as

$$\begin{aligned} P(b_m \quad &= \quad a|\bar{\mathbf{b}}_m, \mathbf{y}) & (10) \\ &\propto \quad \exp\left\{ \sum_{k=1}^{K} \sum_{j=i}^{i+L} \Big(-\frac{1}{\hat{\sigma}_k^2} |y_j^{(k)} - \sum_{l=0}^{L} \hat{h}_{j,l}^{(k)} x_{j-l}^a|^2 \Big) \right\} \\ & \quad\quad P(b_m = a). & (11) \end{aligned}$$

To obtain better performance, we run Q Gibbs samplers in parallel with I iterations each. Hence, a maximum of $Q \cdot I$ most likely transmitted sequences are generated by the MCMC, which are used to compute the output LLRs following the procedure given in [7].

V. Numerical Results

A. Experiment setup

The experiment was conducted off the coast of Martha's Vinyard, MA during Oct. 14th - Nov. 2nd, 2008. During the experiment, there is no movement of the transmitter and receiver. There is a single transducer, and a vertical hydrophone array deployed at 60, 200, and 1000 meters away from the source. The hydrophone array contains 12 elements spaced apart by 12cm. Epochs of data, each containing multiple data files for various modulation schemes, are transmitted every two hours. Every data file within an epoch contains 42 data packets with the same modulation scheme (e.g. 4QAM, 16QAM, or 64QAM). Each packet consists of $N_p = 400$ training symbols and $N_d = 1200$ data symbols. The data symbols within each packet are divided into $T = 3$ blocks, and each block contains $n_d = 1200/3 = 400$ symbols. The channel coding is across every $I = 6$ packets. The carrier frequency is 13 kHz, and the symbol rate is 9.77k sym/sec. The data bits are encoded by a rate $1/2$ recursive systematic convolutional (RSC) encoder with the generator polynomial $(23, 35)$. A square-root raised cosine filter with a roll-off factor 0.2 is used at both the transmitter and the receiver. For each data set, a preamble of 1000 symbols is inserted before data transmission to facilitate data synchronization. Estimation of the channel length L is performed after the synchronization process is complete. For the data sets considered here, we find L to be in the range of 60-80.

B. Experimental results

We first compare performance of the MCMC detector with the LMS-TEQ [2] over a set of 22 data files for the 1000 meter distance. Each file is from a different epoch and thus is transmitted two hours apart. For the 1000 meter distance, the estimated channel is not very sparse, as shown in Fig. 2, and hence the advantage of the VSLMS over the LMS is not evident. For this setting we use the LMS for channel estimation.

First, we note that with 4QAM modulation, due to lower date rates, both detectors obtain excellent performance, achieving error-free decoding results for almost all 22 data sets. Hence, we present our results only for the higher order modulations 16QAM and 64QAM. In Fig. 4 (for 16 QAM) and Fig 5 (for 64QAM), the x-axis represents a total of 22 files. The y-axis represents the average number of errors in information bits per packet. Fig. 4 shows that for 16QAM (each packet has $1200 * 4/2 = 2400$ information bits), the MCMC is much better than the LMS-TEQ after one iteration of detection/equalization and decoding. After seven iterations, the LMS-TEQ still has more than 35 bit errors per packet for files 7-12. In comparison, the MCMC has fewer errors and only files 10 and 18 have more than 35 bit errors per packet. The overall number of bit errors using MCMC is only one third of LMS-TEQ. For the same symbol rate of 9.77k sym/sec, the bit rate for 64QAM is higher (each packet has $1200 * 6/2 = 3600$ information bits), which yields more bit errors than that of the 16QAM. We observe from Fig. 5 that the MCMC performs better than the LMS-TEQ for all the files after the first iteration. After seven iterations, even though the number of errors is still high for most files, it is clear that the performance of MCMC is either much better than LMS-TEQ, e.g., files 1-8, 12-18, 21,22, or comparable to LMS-TEQ, e.g., files 9,10,11,19,20.

We also compare performance of the MCMC detector with the LMS-TEQ for the 60 meter distance. The channel estimation for MCMC is done using either LMS or VSLMS. Here we use a total of 4 receive hydrophones. As in the case of 1000 meters, the MCMC performs better than the LMS-TEQ for most cases after one iteration. After seven iterations, it is clear that the MCMC detector with either LMS or VSLMS outperforms the LMS-TEQ for the majority of data sets, e.g, files 5-9, 15-22. Also, since the UWA channel is more sparse for the 60 meter distance, as shown in Fig. 3, the VSLMS outperforms the LMS for the majority of data sets, e.g., 1, 5-9, and 17-22.

VI. Conclusion

In this paper, we studied MCMC detection for UWA channels. Through actual experimental data we have demonstrated the effectiveness of the MCMC detectors for both 60 meter and 1000 meter transmissions. Using LMS or VSLMS channel estimation, the MCMC detector achieves superior performance to the LMS-TEQ for the majority of the data sets that we have examined. The VSLMS algorithm is shown to provide better channel estimation than the LMS algorithm for sparse UWA channels.

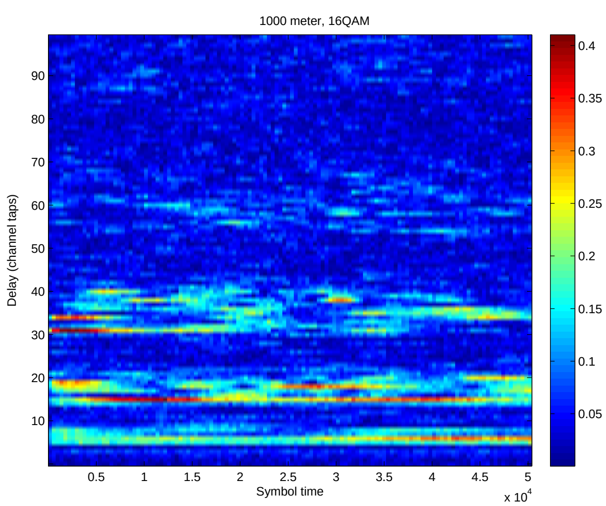

Fig. 2. Estimated channel impulse response for the 1000 meter distance

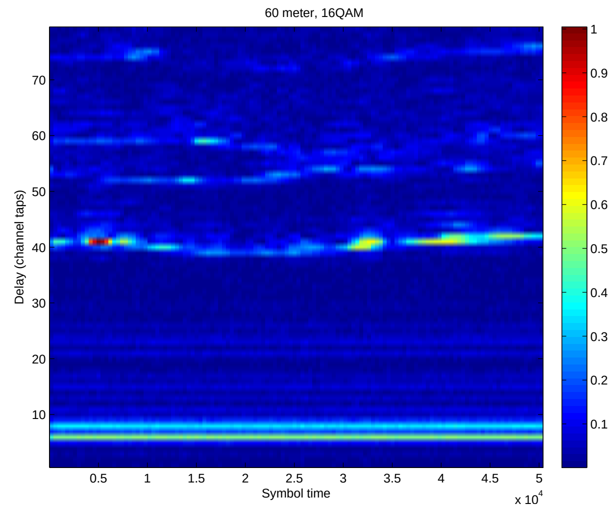

Fig. 3. Estimated channel impulse response for the 60 meter distance

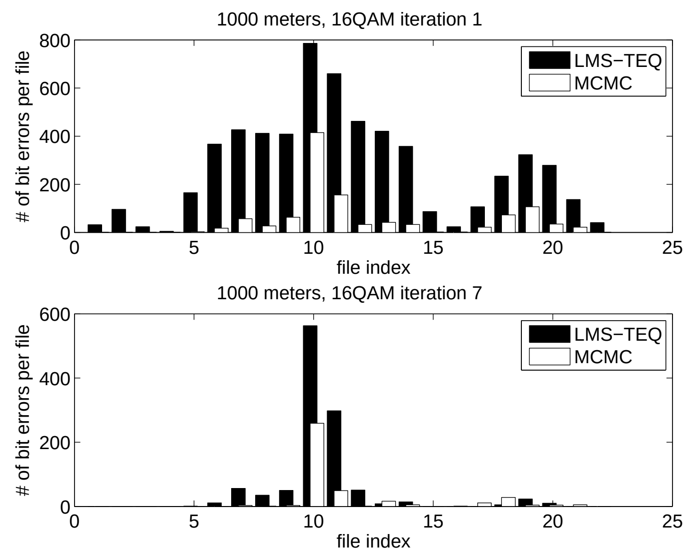

Fig. 4. Performance comparisons between LMS-TEQ and MCMC over 22 data sets for the 1000 meter distance. Assume 16QAM constellation and LMS channel estimation. $K = 10$ receive hydrophones are used.

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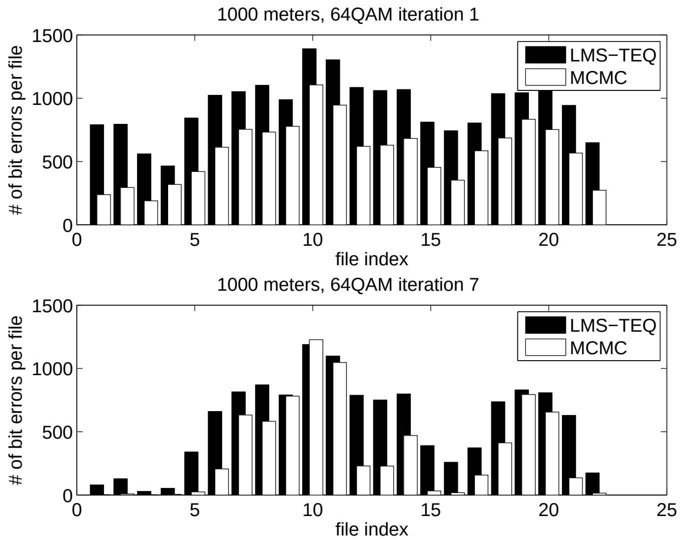

Fig. 5. Performance comparisons between LMS-TEQ and MCMC over 22 data sets for the 1000 meter distance. Assume 64QAM constellation and LMS channel estimation. $K = 10$ receive hydrophones are used.

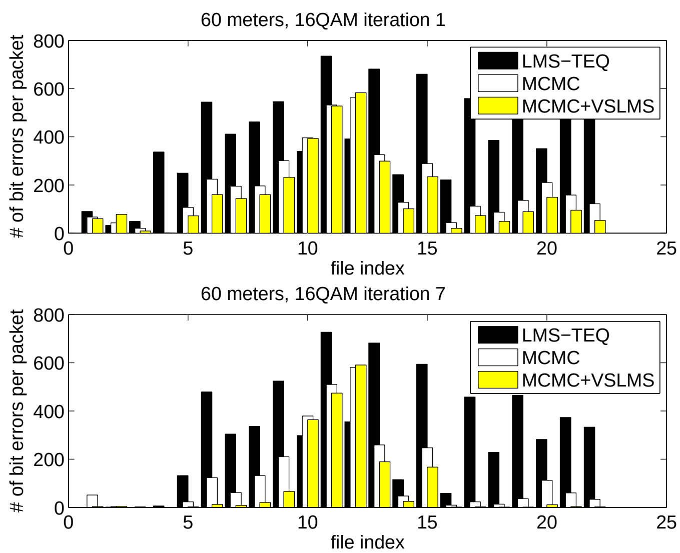

Fig. 6. Performance comparisons between LMS-TEQ and MCMC over 22 data sets for the 60 meter distance. Assume 16QAM constellation. Assume LMS or VSLMS channel estimation. $K = 4$ receive hydrophones are used.

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