Grassmannian Predictive Frequency Domain Compression for Limited Feedback Beamforming

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Abstract— **Frequency domain channel correlation can be exploited to reduce feedback in limited feedback beamforming multiple-input multiple-output orthogonal frequency division multiplexing wireless systems. Prior methods rely on downsampling, interpolation, or clustering the channel state information in the frequency domain. The resulting compressed samples are quantized using one-shot quantization on the Grassmann manifold. The resolution, unfortunately, is limited. We propose a new frequency domain compression technique to obtain high resolution channel state information. The key idea is to use predictive coding on the Grassmann manifold, exploiting the correlation between adjacent subcarriers.**

I. Introduction

Multiuser multiple-input multiple-output orthogonal frequency division multiplexing (MIMO-OFDM) wireless systems are expected to deliver rich broadband content over wireless networks with high reliability by exploiting multiuser, spatial, and spectral diversity [1]. Unfortunately, the achievable throughput and link reliability of multiuser MIMO-OFDM wireless systems critically depend on the accuracy of channel state information (CSI) at the transmitter. Limited feedback is a popular approach to quantize and convey CSI to the transmitter via a finite rate link [2]. In multiuser MIMO-OFDM wireless systems, limited feedback is challenging because of the CSI accuracy needed to mitigate multiuser interference and the increased feedback requirement as the number of subcarriers are increased [3]–[8].

A number of approaches have been proposed towards reducing the feedback overhead in multiuser MIMO-OFDM wireless systems. In [3], interpolation of precoding matrices with parameter optimization was proposed for MIMO-OFDM systems. The idea was to feedback the CSI for few subcarriers and interpolate the remaining CSI at the transmitter, hence reducing the feedback requirement. Alternatively, a clustering approach was proposed where the codeword selection is optimized over a group of subcarriers in MIMO-OFDM systems [4]. For multiuser systems, sum rate performance and impact of quantization in limited feedback multiuser MIMO system were characterized using random vector quantization (RVQ) argument in [5]. Using limited feedback techniques such as RVQ, time domain quantization, and analog feedback, sum rate loss with respect to perfect CSI at the transmitter was studied in [7]. With the goal of reducing the number of feedback bits, interpolation technique was extended to multiuser MIMO-OFDM [6] and a quantized antenna combining with clustering approach was shown to dramatically reduce the feedback overhead in [8]. Despite these efforts to minimize feedback requirement, these methods, except time domain approach, used one-shot quantization on the Grassmann manifold which is limited in resolution [9].

In this paper, we propose to exploit CSI correlation in the frequency domain to encode the CSI for each OFDM symbol using Grassmannian predictive coding [9], [10] in multiuser MIMO-OFDM systems. The proposed technique is shown to provide higher resolution CSI compared to conventional one-shot quantization approach with the same number of feedback bits. Simulation results illustrate improved sum rate performance for the proposed technique which can be matched with twice the number of feedback bits with one-shot quantization.

Notation: We use lower case bold letters (e.g., $\mathbf{v}$) to denote vectors and upper case bold letters (e.g., $\mathbf{H}$) to denote matrices. We use * and ^{T} to denote the Hermitian and the matrix transpose, respectively.

II. System Description

Consider a zero-forcing multiuser MIMO-OFDM system [7]. We assume that the base station is equipped

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with N_t transmit antennas and that $U = N_t$ mobile users are each equipped with single receive antenna. To isolate the impact of limited feedback, we assume that U users are already scheduled and we do not consider the scheduling. At the transmitter, the k-th subcarrier, of N total subcarriers, transmits a complex symbol $\mathsf{s}_u[k]$ for each user u. The transmit symbol is multiplied by a unit norm $N_t \times 1$ beamforming vector $\mathsf{v}_u[k]$. We assume that the transmit power is identically assigned to all subcarriers. We also assume that the sampled impulse response of the channel is shorter than the cyclic prefix. Then, the channel for the k-th subcarrier of u-th user after the discrete Fourier transform (DFT) can be described by an $N_t \times 1$ channel vector $\mathsf{h}_u[k]$. Assuming perfect sampling and synchronization, the received signal for u-th user on k-th subcarrier may be written as

$$\begin{aligned} \mathsf{y}_u[k] &= \mathsf{h}_u^*[k]\mathsf{v}_u[k]\mathsf{s}_u[k] \\ &+ \mathsf{h}_u^*[k] \sum_{n=1, n\neq u}^{U} \mathsf{v}_n[k]\mathsf{s}_n[k] + \mathsf{n}_u[k] \end{aligned} \quad (1)$$

where $\mathsf{n}_u[k]$ is assumed to be an i.i.d. complex Gaussian noise with zero mean and unit variance. In this regime, the signal to interference plus noise ratio (SINR) for the u-th user on k-th subcarrier can be written as

$$\text{SINR}_u = \frac{\frac{P}{N_t}|\mathsf{h}_u^*\mathsf{v}_u|^2}{1 + \frac{P}{N_t}\sum_{n\neq u}|\mathsf{h}_u^*\mathsf{v}_n|^2} \quad (2)$$

assuming Gaussian signaling and treating the interference as Gaussian noise. The achievable rate per subcarrier for user u is given by

$$\mathcal{R}_u = \log_2(1 + \text{SINR}_u) \quad (3)$$

and the sum rate by $\mathcal{R} = \sum_{u=1}^{U} \mathcal{R}_u$. The input-output model in (1) implies that each subcarrier may be treated separately as a narrowband beamforming system.

III. Grassmannian Predictive Frequency Domain Compression

In this section, we provide a brief background on the Grassmannian predictive coding and describe the proposed algorithm.

A. Preliminaries

Under the sum rate performance metric (3) and (2), the normalized channel vector for user u on subcarrier k can be identified as the Grassmann manifold $\mathcal{G}_{N_t,1}$; a set of one dimensional subspaces in N_t dimensional Euclidean space [11]. Because of its non-Euclidean structure, quantization and signal processing on the Grassmann manifold is challenging. For instance, the Euclidean distance metric is not well defined. Instead, the chordal distance is more common on $\mathcal{G}_{N_t,1}$ given by the sine of the subspace angle $d(\mathsf{x}, \mathsf{y}) = \sin(\theta) = \sqrt{1 - |\mathsf{x}^*\mathsf{y}|^2}$ for $\mathsf{x}, \mathsf{y} \in \mathcal{G}(N_t, 1)$.

A brief description of the basic operations used in Grassmannian predictive coding is provided here [9], [10]. Based on the smooth manifold structure of the Grassmann manifold, it is possible to relate two points $\mathsf{x}[k], \mathsf{x}[k+1] \in \mathcal{G}_{N_t,1}$ by considering the tangent vector emanating from $\mathsf{x}[k]$ to $\mathsf{x}[k+1]$. Let $\rho = \mathsf{x}^*[k]\mathsf{x}[k+1]$ denote the inner product. The instantaneous subspace angle is $d = d(\mathsf{x}[k], \mathsf{x}[k+1]) = \sqrt{1 - |\rho|^2}$. If $\mathsf{x}[k],\ \mathsf{x}[k+1] \in \mathcal{G}_{N_t,1}$, then the tangent vector emanating from $\mathsf{x}[k]$ to $\mathsf{x}[k+1]$ is

$$\mathbf{e}[k+1] = \tan^{-1}\left(\frac{d}{|\rho|}\right) \frac{\mathsf{x}[k+1]/\rho - \mathsf{x}[k]}{\|\mathsf{x}[k+1]/\rho - \mathsf{x}[k]\|_2} \quad (4)$$

such that $\|\mathbf{e}[k]\|_2 = \tan^{-1}(d/|\rho|)$ is the arc length between $\mathsf{x}[k]$ and $\mathsf{x}[k+1]$ and $\vec{\mathbf{e}}[k] = (\mathsf{x}[k+1]/\rho - \mathsf{x}[k])/(d/|\rho|)$ is the unit tangent direction vector. In this paper, we propose a new tangent quantization technique inspired by the rotation technique used in [12]. This is described in detail in Section III-B.

Given a tangent vector $\mathbf{e}[k+1]$ with respect to $\mathsf{x}[k]$, the tangent can be mapped onto the manifold using a one parameter map. If $\mathsf{x}[k],\ \mathsf{x}[k+1] \in \mathcal{G}_{N_t,1}$ and $\mathbf{e}[k+1]$ is the tangent vector emanating from $\mathsf{x}[k]$ to $\mathsf{x}[k+1]$, then the geodesic path between $\mathsf{x}[k]$ and $\mathsf{x}[k+1]$ is

$$\begin{aligned} G(\mathsf{x}[k], \mathbf{e}[k+1], t) &= \mathsf{x}[k]\cos(\|\mathbf{e}[k+1]\|_2 t) \quad (5) \\ &+ \frac{\mathbf{e}[k+1]}{\|\mathbf{e}[k+1]\|_2}\sin(\|\mathbf{e}[k+1]\|_2 t) \end{aligned}$$

for $t \in [0, 1]$ such that $G(\mathsf{x}[k], \mathbf{e}[k+1], 0) = \mathsf{x}[k]$ and $G(\mathsf{x}[k], \mathbf{e}[k+1], 1) = \mathsf{x}[k+1]$. One method to introduce a notion of prediction is by considering a tangent expression with respect to $\mathsf{x}[k+1]$ such that it follows the path between $\mathsf{x}[k]$ and $\mathsf{x}[k+1]$. This is called the predicted tangent direction computed using *parallel transport*. Let $\mathsf{x}[k],\ \mathsf{x}[k+1] \in \mathcal{G}_{N_t,1}$ and $\mathbf{e}[k+1]$ be the tangent vector emanating from $\mathsf{x}[k]$ to $\mathsf{x}[k+1]$. Then, the parallel transported tangent vector emanating from $\mathsf{x}[k+1]$ along the geodesic direction $\mathbf{e}[k+1]$ is $\mathbf{e}[k+2] = \tan^{-1}(d/|\rho|)\,[(\mathsf{x}[k+1]\rho^* - \mathsf{x}[k])/d]$. It can be interpreted as transforming the tangent vector onto another tangent space connected by the geodesic. Combining parallel transport and geodesic operation, the one step predicted vector $\tilde{\mathsf{x}}[k+1] \in \mathcal{G}_{N_t,1}$ along the geodesic direction from $\mathsf{x}[k]$ to $\mathsf{x}[k+1]$ is $\tilde{\mathsf{x}}[k+1] =$

$|\rho|\mathsf{x}[k]+\rho^*\mathsf{x}[k]-\mathsf{x}[k-1]$ such that $d(\mathsf{x}[k],\tilde{\mathsf{x}}[k+1]) = d(\mathsf{x}[k-1],\mathsf{x}[k])$. For notational brevity, we denote the prediction operation by a map $P : \mathcal{G}_{N_t,1} \times \mathcal{G}_{N_t,1} \to \mathcal{G}_{N_t,1}$ which takes current and previous estimate vectors and outputs the predicted vector, i.e., $\tilde{\mathsf{x}}[k+1] = P(\hat{\mathsf{x}}[k-1],\hat{\mathsf{x}}[k])$.

B. Tangent Quantization

Let $\mathsf{x}[k]$ be a point on $\mathcal{G}_{N_t,1}$ and $\mathbf{e}[k]$ the tangent vector associated with $\mathsf{x}[k]$. That is, $\mathbf{e}^*[k]\mathsf{x}[k] = 0$. Unfortunately, the tangent vector $\mathbf{e}[k]$ lives in the N_t dimensional complex vector space. Furthermore, we note that the tangent space is not the same for different $\mathsf{x}[k]$ on $\mathcal{G}_{N_t,1}$ and that there are no further constraints. To aid in the quantization operation, we build a canonical codebook around the special $N_t \times 1$ root vector $\mathsf{x}_1 := \begin{bmatrix}1 & 0 & \cdots & 0\end{bmatrix}^T$. A similar concept was used to derive local codebooks in [12]. The main difference is that the codebook in [12] consists of points on the Grassmann manifold, whereas in this case, the codebook will consist of vectors in Euclidean space that are orthogonal to a given vector $\mathsf{x}[k]$.

To exploit the canonical codebook, a unitary rotation matrix will be constructed such that a rotation from $\mathsf{x}[k]$ to x_1 and back can be performed. One way to construct such unitary matrices is to use the complex Householder matrix [13]. Let $\mathsf{u}[k] = \mathsf{x}_1 - \mathsf{x}[k]$. Then, the complex Householder is given by $\mathsf{H}_{\text{ouse}} = \mathsf{I} - \mathsf{u}\mathsf{u}^*/\mathsf{u}^*\mathsf{x}_1$ [13]. Note that H_{ouse} is a unitary matrix and the first column of H_{ouse} contains the entries of $\mathsf{x}[k]$ while the remaining columns are orthogonal to $\mathsf{x}[k]$. Then, $\mathsf{x}[k] = \mathsf{H}_{\text{ouse}}\mathsf{x}_1$ and $\mathsf{x}_1 = \mathsf{H}^*_{\text{ouse}}\mathsf{x}[k]$ as desired.

Now we define a B-bit canonical tangent vector codebook, $\chi = \{\mathsf{v}_1, \mathsf{v}_2, \ldots, \mathsf{v}_{2^B}\}$, with respect to x_1. Due to the structure of x_1, each entry of χ, can be written $\mathsf{v} = \begin{bmatrix}0 & \tilde{\mathsf{v}}^*\end{bmatrix}^*$ where $\tilde{\mathsf{v}}$ is a $N_t - 1 \times 1$ vector. Then, using the canonical tangent codebook and the Householder transform, local tangent codebook can be constructed which satisfies the orthogonality requirement. The main benefit is that the dimension of the codebook is reduced by one and that the orthogonality is preserved. In this paper, we construct the reduced dimension canonical codebook using the LBG algorithm [14]. To quantize the error tangent vector in the predictive coding algorithm, the Householder transform which transforms the canonical tangent vector to the base of error tangent vector is computed. Then, the codeword that yields the geodesic estimate with smallest chordal distance is chosen as the codeword. Finally, the selected codeword index is used to represent the quantized error tangent.

C. Proposed Algorithm

Let $\{\mathsf{x}[k]\}_{k=1,\ldots,N} \in \mathcal{G}_{N_t,1}$ be the input sequence, i.e., normalized channel vectors across the subcarriers, with subcarrier index k. The main idea is to quantize the tangential error $\mathbf{e}[k]$ between the predicted vector $\tilde{\mathsf{x}}[k]$ and the current observed vector $\mathsf{x}[k]$. Then, the quantized tangent error is used to construct the estimate $\hat{\mathsf{x}}[k]$ of the current observed vector. The current and previous estimated vectors, $\hat{\mathsf{x}}[k]$ and $\hat{\mathsf{x}}[k-1]$, are used to compute the predict vector $\tilde{\mathsf{x}}[k+1]$. To initialize the prediction, the CSI for the first two subcarriers, i.e., $\mathsf{x}[1]$ and $\mathsf{x}[2]$, are quantized using one-shot quantization.

The pseudo code for the encoder at the receiver side is shown in Algorithm 1. At time k, using (4), the error tangent vector $\mathbf{e}[k]$ emanating from $\tilde{\mathsf{x}}[k]$ to $\mathsf{x}[k]$ is computed. Then, the quantization is performed using the canonical tangent codebook in Section III-C. We denote the quantization by $Q : \mathbb{C}^n \to \mathbb{N} \times \mathbb{N}$ which takes the error tangent vector and outputs the codeword index. The estimated vector is

$$\hat{\mathsf{x}}[k] = G(\tilde{\mathsf{x}}[k], i[k], 1). \tag{6}$$

Finally, the prediction is performed to obtain $\tilde{\mathsf{x}}[k+1]$ for the next sample. The encoding procedure is repeated up to subcarrier N. At subcarrier N, the algorithm terminates with $N\log_2(|\chi|)$ bits representing the CSI across the OFDM subcarriers. The decoding is performed similarly to [9], [10].

Algorithm 1 GPC encoder algorithm

Input: $\mathsf{x}[k]$
Initialize $\hat{\mathsf{x}}[1]$ and $\hat{\mathsf{x}}[2]$
for all k=1,2,…,N **do**
  $\mathbf{e}[k] = L(\tilde{\mathsf{x}}[k], \mathsf{x}[k])$
  $i[k] = Q(\mathbf{e}[k])$
  $\hat{\mathsf{x}}[k] = G(\tilde{\mathsf{x}}[k], i[k], 1)$
  $\tilde{\mathsf{x}}[k+1] = P(\hat{\mathsf{x}}[k-1], \hat{\mathsf{x}}[k])$
end for
Output: $i[k]$

IV. Simulation Results

In the following, sum rate simulation results for a multiuser MIMO-OFDM system with $N_t = 4$ transmit antennas, $U = 4$ scheduled users, and $N = 64$ subcarriers are presented. A discrete channel model with 6 taps that are i.i.d. and zero mean unit variance

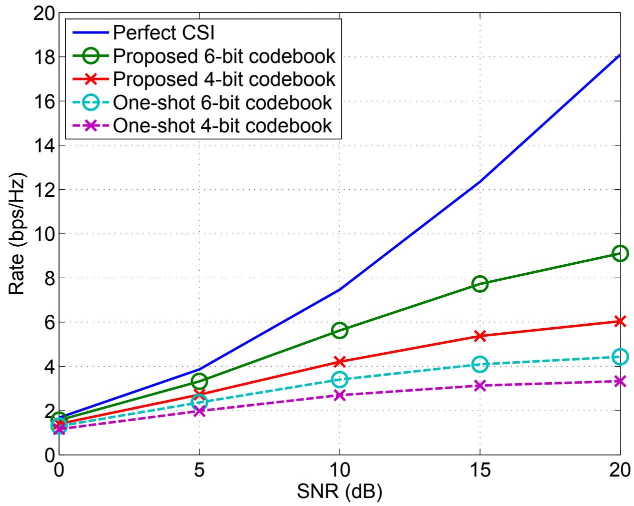

Fig. 1: Comparison of averaged sum throughput per subcarrier for $N_t = U = 4$, $N = 64$, and $K = 1$.

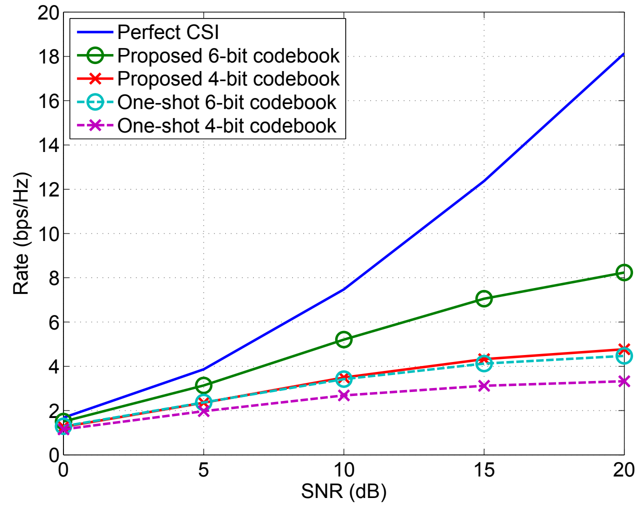

Fig. 2: Comparison of averaged sum throughput per subcarrier for $N_t = U = 4$, $N = 64$, and $K = 2$.

Gaussian is used. The receiver is assumed to have perfect channel estimate for all the subcarriers.

In Fig. 1, average achievable sum throughput per subcarrier for the cases of perfect CSI at the transmitter, one-shot quantization using 4 and 6-bit codebooks, and the proposed Grassmannian predictive coding using 4 and 6-bit codebooks are compared. Channel vectors for every subcarrier, i.e., downsampling rate of $K = 1$, are encoded resulting in 4 and 6 bits of feedback per subcarrier. The comparison shows that the proposed Grassmannian predictive coding yields over twice the sum rate of one-shot approach. This is because Grassmannian predictive coding provides higher resolution coding compared to the one-shot approach. We have found through simulation that 13-bit codebook for one-shot approach is required to achieve the sum rate of the proposed approach using 6-bit codebook.

In Fig. 2, average achievable sum throughput per subcarrier for the same codebooks as Fig. 1 are shown but now with the CSI downsampled by a factor of $K = 2$. That is, only the channel vectors for one in two subcarriers are used resulting in effective feedback rate of 3 and 2-bit per subcarrier. At the transmitter side, the beamformer for the skipped subcarrier is interpolated by simple replication from adjacent subcarrier. The plot shows that slight degradation of sum rate is experienced compared to Fig. 1, but the proposed approach still significantly outperforms the one-shot approach.

V. Conclusion

In this paper, we proposed Grassmannian predictive compression technique to encode frequency domain CSI in multiuser MIMO-OFDM systems. Frequency domain correlation of the CSI is exploited to perform high resolution encoding. Furthermore, we proposed an improved tangent space quantization technique using Householder transform. Simulation results show improved sum rate over one-shot coding approach. Our future work will consider joint clustering and coding to further reduce the feedback requirement and study the sum rate performance tradeoff.

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