Transmission capacity of wireless ad hoc network with non-parametric linear MMSE receiver

Byungju Lee, Sunho Park and Byonghyo Shim
School of Information and Communication, Korea University
Anam-dong, Sungbuk-gu, Seoul, Korea, 136-713
Email: \{bjlee, shpark\}@isl.korea.ac.kr, bshim@korea.ac.kr

Abstract—Recent works on ad hoc network study have shown that achievable throughput can be made to scale linearly with the number of receive antennas even if the transmitter has only a single antenna. In this paper, we put forth an approach achieving linear scaling of the transmission capacity in the practical ad hoc network without loss of transmission efficiency. The key feature to make our approach effective is to exploit the autocorrelation of the received signal for the MMSE receiver. From the transmission capacity analysis as well as numerical simulations, we show that the proposed non-parametric linear MMSE receiver brings substantial performance gain over existing multiple receive antenna algorithms.

I. INTRODUCTION

In the ad hoc network, multiple transmit-receive pairs communicate simultaneously without the benefit of fixed infrastructure. Due to the uncoordinated nature of communication, multiple transmitters attempt to communicate simultaneously, creating substantial inter-user interference. As an effective way to mitigate the interference, multiple receive antenna techniques have received much attention recently [1]. There have been several prior works investigating the transmission capacity of single-stream transmission with multiple receive antennas [2]–[4]. In [2], the maximal ratio combining (MRC) is employed for achieving the spatial diversity. In [3], the full zero forcing (ZF) is used to remove interference from the strong interferers. Recently, it is shown that the network-wide throughput is scaled linearly with the number of receive antennas even for MMSE linear receiver [4]. All these promising gains are achieved when the channel state information at the receiver (CSIR) is fully available. Recently, transmission capacity is characterized under the impact of inaccurate CSIR [5]. In this approach, spatial covariance matrix of the MMSE receiver is obtained by listening to interferer transmissions. When the number of interferer observations is large enough, this method achieves large fraction of optimal transmission capacity. However, since the desired transmitter should remain inactive when the spatial covariance matrix information is estimated, it brings an overhead that reduces the transmission efficiency.

In this paper, we put forth an approach achieving linear scaling of the transmission capacity in the practical ad hoc network without loss of transmission efficiency. Our approach is based on a non-parametric linear MMSE receiver which employs the autocorrelation of the received signal. We demonstrate that the maximum SINR of the proposed method is equivalent to that of the genie MMSE receiver even with the inclusion of the desired channel information in the spatial covariance matrix. This enables the desired transmitter to deliver the information seamlessly without bothering to consider the covariance estimation. From the analysis, we show that network-wide throughput is characterized by the number of receive antennas and observations. The derived transmission capacity scaling suggests that multiple receive antennas can significantly improve the performance of the ad hoc network, even in the imperfect CSIR condition.

The rest of this paper is organized as follows. After a brief summary of the system model and conventional MMSE technique in Section II, we present the proposed method in Section III. The simulation results and discussions are provided in Section IV.

II. AD HOC NETWORK

A. System Model

In the ad hoc network, the active transmitters are placed according to a 2-D homogeneous Poisson point process (PPP) of density \( \lambda \) (transmitters/m\(^2\)) [6]. Each transmit antenna communicates with a receiver equipped with \( N \) antennas and each receiver is randomly located at \( d \) meters away from the corresponding transmitter (see Fig. 1). Under the assumption that transmit nodes are mutually independent, we can focus on the performance of a typical transmit-receive pair (denoted by \( \text{Tx}_d \) and \( \text{Rx}_d \)). In the \( \text{Rx}_d \)'s viewpoint, the set of interferers except \( \text{Tx}_d \) also forms a homogeneous PPP [6]. The set of all the active transmitters is denoted by \( \mathcal{A} = \{(d, h_d), (X_i, h_i), \lambda, i \in \mathbb{N}\} \) where \( h_d \) is the channel vector between the typical transmitter and receiver and \( X_i \) and \( h_i \) are the location and channel vector of the \( i \)th transmitting node with respect to the typical receiver, respectively. Under the frequency-flat channel assumption, the \( N \)-dimensional received signal \( y \) is expressed as

\[
y = d^{-\alpha/2}h_d s_d + \sum_{i \in \mathcal{A}\setminus\{\text{Tx}_d\}} |X_i|^{-\alpha/2}h_i s_i + w
\]  

where \( \alpha \) is the path-loss exponent (\( \alpha > 2 \)), \( w \) is the \( N \times 1 \) complex Gaussian noise vector (\( w \sim \mathcal{CN}(0, \sigma^2 \mathbf{I}) \)), and \( s_d \) and \( s_i \) are the symbol transmitted by the desired and \( i \)th transmitter (\( E[|s_d|^2] = E[|s_i|^2] = \rho \)), respectively.
As a means to address this issue, Jindal et al. proposed the method to blindly estimate the sample covariance matrix for MMSE operation [5]. In this approach, the sample covariance matrix is estimated by listening to interferer transmissions in the absence of desired signal. The sample covariance, when the desired transmitter remains inactive for $K$ symbol durations, is given by

$$\hat{\Sigma} = \frac{1}{K} \sum_{i=1}^{K} r_i r_i^H$$

(5)

where $r_i$ represents the $i$th observation including interference and noise. By replacing $\hat{\Sigma}$ with $\Sigma$, i.e., $\hat{\nu}_d = \hat{\Sigma}^{-1} h_d$, the resulting SINR becomes $\text{SINR} = \frac{\nu_d^H \hat{\Sigma}^{-1} h_d}{h_d^H \hat{\Sigma}^{-1} \hat{\Sigma}^{-1} h_d}$. Under the assumption that all the interferers send independent Gaussian symbols, the expected SINR using $\hat{\nu}_d$ becomes [9]

$$E_{\hat{\Sigma}} \left[ \frac{\nu_d^H \hat{\Sigma}^{-1} h_d}{h_d^H \hat{\Sigma}^{-1} \hat{\Sigma}^{-1} h_d} \right] = \left( 1 - \frac{N-1}{K+1} \right) h_d^H \Sigma^{-1} h_d.$$  

(6)

When compared to (4), the expected SINR in (6) contains an additional scaling factor $1 - \frac{N-1}{K+1}$. 

### III. NON-PARAMETRIC LINEAR MMSE RECEIVER

#### A. MOTIVATION

The MMSE receiver technique estimating the outcome of interference plus noise covariance has following drawbacks. First, the desired transmitter should be inactive when the covariance matrix is estimated (say $K$ symbol period). Although the receiver can provide fairly good covariance matrix for large $K$, the sampling time overhead will bring the significant transmission rate loss. Second, if the channel state is changed, the covariance matrix information should also be updated. For example, if the channel (under block fading) is changing per $T$ symbols period, then the effective data rate is reduced by the factor of $\frac{T}{T+\frac{K}{K+1}}$.

#### B. NON-PARAMETRIC LINEAR MMSE

An idea motivated by this observation is to estimate the sample covariance matrix from observations including desired channel information as well as the interference plus noise covariance. From [10], the estimated desired symbol of the linear MMSE is given by

$$\nu_d = \arg\max_{\nu_d} \left( \frac{\nu_d^H h_d h_d^H \nu_d}{\nu_d^H \Sigma^{-1} \nu_d} \right) = \Sigma^{-1} h_d.$$  

(3)

Plugging (3) into (2), the resulting SINR of the MMSE filter becomes

$$\text{SINR}_{\text{MMSE}} = \frac{\rho d^{-\alpha} \nu_d^H h_d ^H \nu_d}{\nu_d^H \Sigma^{-1} \nu_d} = \frac{\rho d^{-\alpha} (\nu_d^H \Sigma^{-1} h_d)^2}{\nu_d^H \Sigma^{-1} \nu_d} = h_d^H \Sigma^{-1} h_d.$$  

(4)

In order to ensure that the MMSE is operating properly, the desired channel state and the interference plus noise covariance should be obtained at the receiver. The desired channel can be estimated accurately via pilot symbols. However, it is not clear how the desired receiver estimate the spatial covariance matrix and what effect estimation error has on transmission capacity. 

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**Fig. 1.** A typical transmitter-receiver pair subject to interference (dashed lines).

**B. PERFORMANCE METRICS AND CONVENTIONAL MMSE RECEIVER**

If a unit norm receive filter $v_d$ is applied to the received signal vector, the estimate of desired symbol becomes $\hat{s}_d = v_d^H y$ and the resulting signal-to-interference-and-noise (SINR) ratio becomes

$$\text{SINR} = \frac{\rho d^{-\alpha} v_d^H h_d h_d^H v_d}{\nu_d^H \Sigma^{-1} \nu_d}.$$  

(2)

When the transmission rate for all nodes is equal, say $R = \log_2 (1 + \beta)$, we say a communication is successful if the received SINR is larger than $\beta$. Then the outage probability at SINR threshold $\beta$ is $P_{\text{out}} = P(\text{SINR} \leq \beta)$, which is an increasing function of $\lambda$. The maximum interference density such that the outage does not exceed $\epsilon$ is $\lambda = \max\{\lambda : P_{\text{out}}(\lambda) \leq \epsilon\}$. The transmission capacity of the ad hoc network, a measure of the number of successful transmissions per unit area, can be expressed as $C(\epsilon) = \lambda (1 - \epsilon) \log_2 (1 + \beta)$ bps/Hz/meters$^2$ [2], [7]. Clearly, SINR is a key factor determining the transmission capacity and controlling interference is an important issue to be addressed. As a means to mitigate interference, multiple receive antenna technique has been proposed in recent years [5], [8]. Among many receiver techniques, the genie MMSE receiver is well known since it maximizes the SINR in (2) [8].

Denoting the spatial covariance of the interference plus noise as $\Sigma = \frac{1}{N} I + d^\alpha \sum_{i \in A(\lambda) \setminus \{Tx_d\}} |X_i|^{-\alpha} h_i h_i^H$ and

$$\text{SNR} = \frac{\rho d^{-\alpha}}{\sigma^2},$$

the genie MMSE receive filter is given by

$$\nu_d = \arg\max_{\nu_d} \left( \frac{\nu_d^H h_d h_d^H \nu_d}{\nu_d^H \Sigma^{-1} \nu_d} \right) = \Sigma^{-1} h_d.$$  

(3)

Plugging (3) into (2), the resulting SINR of the MMSE filter becomes

$$\text{SINR}_{\text{MMSE}} = \frac{\rho d^{-\alpha} (\nu_d^H \Sigma^{-1} h_d)^2}{\nu_d^H \Sigma^{-1} \nu_d} = \frac{\rho d^{-\alpha} (\nu_d^H \Sigma^{-1} h_d)^2}{\nu_d^H \Sigma^{-1} \nu_d} = h_d^H \Sigma^{-1} h_d.$$  

(4)

In order to ensure that the MMSE is operating properly, the desired channel state and the interference plus noise covariance should be obtained at the receiver. The desired channel can be estimated accurately via pilot symbols. However, it is not clear how the desired receiver estimate the spatial covariance matrix and what effect estimation error has on transmission capacity.
Using the Sherman-Morrison formula \([10]\), \((\Sigma + h_d h_d^H)^{-1} = \Sigma^{-1} - \frac{h_d h_d^H \Sigma^{-1}}{1 + h_d^H \Sigma^{-1} h_d}\), the numerator of (9) becomes
\[
\left( h_d^H (\Sigma + h_d h_d^H)^{-1} h_d \right)^2 = \left( h_d^H \Sigma^{-1} h_d - \frac{h_d^H \Sigma^{-1} h_d}{1 + h_d^H \Sigma^{-1} h_d} \right)^2
\]
\[
= \frac{(h_d^H \Sigma^{-1} h_d)^2}{(1 + h_d^H \Sigma^{-1} h_d)^2}. \tag{10}
\]
In a similar way, one can show that
\[
h_d^H (\Sigma + h_d h_d^H)^{-1} \Sigma (\Sigma + h_d h_d^H)^{-1} h_d = \frac{h_d^H \Sigma^{-1} h_d}{(1 + h_d^H \Sigma^{-1} h_d)}. \tag{11}
\]
Hence, (9) becomes
\[
\text{SINR} = \frac{(h_d^H \Sigma^{-1} h_d)^2}{h_d^H \Sigma^{-1} h_d} = h_d^H \Sigma^{-1} h_d. \tag{12}
\]
Interestingly, even with the inclusion of the desired channel information in the covariance matrix, the maximum transmission capacity can be achieved by the linear MMSE receiver.

By a simple modification of Theorem 3.1, we can analyze the SINR of the proposed algorithm under the imperfect CSIR condition. The sample covariance of the proposed receiver is \(\hat{\Sigma}_d = \Sigma + h_d h_d^H\) where \(\Sigma\) represents the observations of the noise plus interference. With the knowledge of \(h_d\), the receiver can compute the filter \(\hat{v}_d = \hat{\Sigma}_d^{-1} h_d\) and the resulting SINR becomes
\[
\text{SINR}_{\text{prop}} = \frac{(h_d^H \hat{\Sigma}_d^{-1} h_d)^2}{(h_d^H \hat{\Sigma}_d^{-1} \Sigma \hat{\Sigma}_d^{-1} h_d)}. \tag{13}
\]
One can show that the expected SINR is
\[
E[\text{SINR}_{\text{prop}}] = \left( 1 - \frac{N - 1}{M + 1} \right) h_d^H \Sigma^{-1} h_d. \tag{14}
\]
In this paper, we skip the details and briefly describe the key feature of the proposed expected SINR. Note that the expected SINR using an MMSE receiver based upon \(\hat{\Sigma}_d\) is smaller than that with perfect knowledge of \(R_{yy}\) and they differ by a factor of \(1 - \frac{N - 1}{M + 1}\). This factor is an increasing function of the number of observations \(M\) and converges to one when \(M\) goes to infinity because \(\hat{\Sigma}_d\) converges to \(\Sigma\) for large \(M\). Due to the fact that the length of the packet is typically on the order of hundreds of symbols, the expected SINR and the transmission capacity using \(\hat{\Sigma}_d\) is larger than the method employing \(\hat{\Sigma}\).

**C. The Scaling Law of the Non-parametric Linear MMSE**

The outage probability under the perfect CSIR assumption in a Poisson filed of interferers is \([4]\)
\[
P_{\text{out}}(\gamma, \lambda) = 1 - \sum_{i=0}^{N-1} \frac{(\lambda \gamma \frac{\lambda}{2} \frac{\lambda - 1}{2})^i}{i!} \exp(-\lambda \gamma). \tag{15}
\]
where \(\gamma = \beta d^a\) and \(\lambda = \frac{G}{d^a} \Gamma \left( \frac{d}{a} \right) \Gamma \left( 1 - \frac{d}{a} \right)\). For analytical tractability, we focus on the interference-limited regime where the noise is negligible. Under this assumption, (15) becomes
\[
P_{\text{out}}(\gamma, \lambda) = 1 - \sum_{i=0}^{N-1} \frac{(\lambda \gamma \frac{\lambda}{2} \frac{\lambda - 1}{2})^i}{i!} \exp(-\lambda \gamma). \tag{16}
\]
and the outage probability of the non-parametric linear MMSE under the imperfect CSIR assumption is
\[
P_{\text{out}}(\gamma', \lambda) = 1 - \sum_{i=0}^{N-1} \frac{\lambda (\gamma' \frac{\gamma'}{2} \frac{\gamma' - 1}{2})^i}{i!} \exp(-\lambda \gamma \theta \frac{\gamma}{2 \gamma}). \tag{17}
\]
where \(\gamma' = \frac{\beta d^a}{\alpha d^a}\) and \(\theta = \left( 1 - \frac{\gamma - 1}{M + 1} \right)\). Since the mean number of successful transmissions can be expressed as \(\lambda(1 - P_{\text{out}}(\gamma', \lambda))\), the optimization problem of the maximum interferer density becomes
\[
\lambda_{\text{max}} = \arg \max_{0 \leq \lambda < \infty} \lambda(1 - P_{\text{out}}(\gamma', \lambda)). \tag{18}
\]
Following theorem states that the linear scaling law of proposed filter is also achieved under the imperfect CSIR assumption.

**Theorem 3.2:** The optimum density in the interference-limited regime is
\[
\lambda_{\text{max}} = f(N, M) \triangleq \frac{\alpha}{\gamma \frac{\gamma}{2}} \tag{19}
\]
and \(f(N, M)\) is a function satisfying \(\frac{N}{2} \leq f(N, M) \leq N\) for \(M > N - 2\).

**Proof:** The parameter \(f(N, M)\) is a function of the number of receive antennas and the number of sample observations. By equating the derivative of \(\lambda(1 - P_{\text{out}}(\gamma', \lambda))\) with respect to \(\lambda\) to zero, the \(f(N, M)\) corresponds to the positive root of the polynomial \(Q(x)\) given by
\[
Q(x) = \sum_{i=0}^{N-1} \frac{(\theta \frac{\gamma}{2} x)^i}{i!} - \left( \theta \frac{\gamma}{2} x \right)^N \frac{N - 1)!}{(N - 1)!}. \tag{20}
\]
This polynomial has at most one real positive root by Descartes’ rule of signs \([4]\). The value of the polynomial at \(\frac{N}{2}\) is lower bounded as
\[
Q \left( \frac{N}{2} \right) \geq \sum_{i=0}^{N-1} \frac{\theta \frac{\gamma}{2} x^i}{i! 2^i} - \sum_{i=0}^{N-1} \frac{\theta \frac{\gamma}{2} x^i}{i! 2^i} \frac{N - 1)!}{(N - 1)!} \tag{21}
\]
where \((a)\) follows from the \((1 + x)^n = \sum_{k=0}^{n} \binom{n}{k} x^k\). The right-hand side of inequality (21) is positive for all values of \(N\) and \(M > N - 2\). On the other hand, the value of the polynomial at \(N\) is upper bounded as
\[
Q(N) = \sum_{i=0}^{N-1} \frac{\theta \frac{\gamma}{2} x^i}{i!} - \sum_{i=0}^{N-1} \frac{\theta \frac{\gamma}{2} x^i}{i! N!} \frac{(N - 1)!}{(N - 1)!} \tag{22}
\]
The left-hand side of inequality (22) is negative for \( M > N - 2 \). Thus, in the interval \([\frac{N}{2}, N]\), the polynomial has one positive root.

Theorem 3.2 indicates that the non-parametric linear MMSE receiver under sufficient sample observations provides a linear scaling of the optimum density with the number of receive antennas. Note also that using the autocorrelation of the received signal \( R_{yy} \) instead of the true covariance, the maximum interferer density can be reduced by a factor of \( \left(1 - \frac{N-1}{M+1}\right) \).

IV. SIMULATION RESULTS AND DISCUSSIONS

In this section, we compare the performance of the proposed non-parametric linear MMSE method with the genie MMSE, MMSE with imperfect CSIR, as well as MRC and ZF schemes (full ZF and partial ZF). The simulation setup is based on the 2-D PP transmitters which are realized on the square distances. We assume that the elements of each transmitter’s channel vector are i.i.d zero mean complex Gaussian random variables with unit variance. In Fig. 2, we plot the transmission capacity as a function of \( N \). Note that \( K = 10 \) (10% of packet length) and \( M \) is the packet length. Generally, we observe that the transmission capacity of the proposed non-parametric linear MMSE method and the proposed receiver employing (14) is not much different from the transmission capacity of the MMSE with perfect CSIR. In particular, the transmission capacity of the proposed receiver is larger than that of the MMSE with imperfect CSIR and partial ZF, and the gain gets larger as \( N \) increases. In Fig. 3, we plot the transmission capacity as a function of the number of blocks. We observe that the transmission capacity of the MMSE and partial ZF are consistent due to perfect CSIR structure. On the contrary, the transmission capacity of the proposed method and MMSE with imperfect CSIR are an increasing function of the number of blocks since the accuracy of the covariance matrix increases when the number of blocks grows.

In this work, we investigated an approach achieving linear scaling of the transmission capacity in the practical ad hoc network without loss of transmission efficiency. Motivated by the fact that the MMSE with imperfect CSIR brings significant transmission rate loss due to the inactive mode of the desired transmitter, we employed the autocorrelation of the received signal for obtaining sample covariance matrix. This enables the desired transmitter to communicate without transmission latency. Future study needs to be directed towards the investigation of the performance when the interfering transmitters are heterogeneous.

REFERENCES