Sub-modularity and Antenna Selection in MIMO systems

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Abstract—In this paper, we show that the optimal receive antenna subset selection problem for maximizing the mutual information in a point-to-point MIMO system is sub-modular. Consequently, a greedy step-wise optimization approach, where at each step a receive antenna that maximizes the incremental gain is added to the existing receive antenna subset, is guaranteed to be within a \((1 - 1/e)\) fraction of the global optimal value. For a multiple relay network with single antenna equipped source and destination, we show that the relay antenna selection problem to maximize the mutual information is modular, when complete channel state information is available at the relays. As a result, a greedy step-wise optimization approach leads to an optimal solution for the relay antenna selection problem with linear complexity in comparison to the brute force search that incurs exponential complexity.

I. INTRODUCTION

Transmit/receive antenna selection for point-to-point MIMO channels is a topic that has been extensively studied in literature, see [1]–[7] and references therein.\(^1\) With transmit (receive) antenna selection, a subset of the total number of transmit (receive) antennas is chosen for maximizing various performance metrics, such as capacity, reliability or diversity gain and several others. Antenna selection has numerous advantages, such as simplified circuitry, a fewer number of transmit chains/power amplifiers etc., and therefore has been an object of interest in theory as well as in practice. In the new paradigm of massive MIMO [8], [9], where hundreds of antennas are deployed at base-stations for achieving very high data rates, antenna selection is very useful for opportunistically exploiting the gains offered by large number of antennas, however, with very small number of RF chains, thereby simplifying the circuitry significantly. With the growing popularity of relay based communication, antenna selection at both the transmitter/receiver, as well as at multiple relays has also attracted a lot of attention [10]–[13]. In the relay paradigm, in addition to usual advantages of antenna selection, relay antenna selection allows the set of relays to serve multiple source-destination pairs at the same time, thereby providing large spectral efficiency/reliability gains.

For point-to-point MIMO channels, assuming that antenna selection has been done (either by brute force, or a greedy method, or some simple heuristic approach), capacity expressions have been derived in [2], [3], while diversity gain computations have been provided in [14]. Most of the analytical work in this area has concentrated on the evaluation of chosen metric given that the antenna selection has been done \textit{apriori}. Finding the optimal antenna subset, however, is a challenging problem in its own right. With the recent interest in sufficiently large number of transmit/receive antennas such as massive MIMO applications [8], a brute force search for antenna selection is too expensive, and it is compounded by the fact that it needs to be done periodically for every coherence interval. For example, in a typical massive MIMO application with 100 antennas at the base-station, and an antenna subset size of 4 or 8 to select for reception (4x/8x MIMO being considered in standards), a brute-force approach requires one million computations for optimal subset selection in every coherence interval. There are a large number of papers on reduced complexity antenna selection algorithms [1], [5]–[7], however, most of them do not provide any theoretical guarantees on the performance of the algorithm. The same holds true for the relay antenna selection algorithms.

In this paper, in a major departure from the previous heuristic approaches, we study the antenna selection problem more systematically by leveraging results from the area of approximation algorithms. Approximation algorithms for solving combinatorial optimization problems is a major field of study in computer science [15], where an approximate solution to an optimization problem is derived that has a fixed bounded distance from the optimal solution. One of the techniques used in approximation algorithms is to check if the objective function is sub-modular, since in discrete combinatorial optimization, sub-modular objective functions play a role that is akin to convex functions in the continuous domain. Sub-modular functions have been a topic of study ever since the celebrated result of [16], that showed that a greedy algorithm (maximize per-step reward) achieves a \((1 - 1/e)\) fraction of the optimal solution [16] if the objective function is sub-modular. A function is called sub-modular if it satisfies a diminishing returns property, i.e. the marginal gain from adding an element to a set \(S\) is at least as high as the marginal gain from adding the same element to a superset of \(S\). A special case of a sub-modular function is a modular function.

\(^{1}\)The literature in this area is quite extensive and we only provide an incomplete list.
(non-diminishing return) for which the value from adding an element \( a \) to a set \( S \) is equal to the sum of the value just using \( S \) and the value with using \( a \). For a modular function, it is well-known that a greedy algorithm achieves the optimal solution [17]–[19].

In this paper, for point-to-point MIMO channels, we study the receive antenna selection problem for maximizing the mutual information or achievable rate, where the goal is to select the \( L \) best antennas among the total \( N_r \) receive antennas. We assume that the number of transmit antennas \( N_t \leq N_r \), similar to the uplink in massive MIMO paradigm. We show that the objective function in the receive antenna selection problem is sub-modular, and hence the mutual information with the greedy algorithm is guaranteed to be within a \((1 - 1/e)\) fraction of the optimal mutual information value. The greedy algorithm at each step updates the receive antenna subset by adding that antenna to the existing subset that has the highest increment to the mutual information among the available antennas. Therefore, the complexity of the greedy algorithm is linear in the number of antennas. Thus, the greedy antenna selection policy not only has a guaranteed performance bound but is also computationally simple for practical implementation.

For the relay antenna selection problem, we consider a single antenna equipped source, several relays with total \( N_r \) antennas, and a destination with a single antenna. The problem we consider is to select \( L \) best antennas out of the available \( N_r \) antennas to maximize the mutual information between the source and the destination, when each relay has channel state information. For the relay antenna selection problem, we show that the mutual information expression is modular, i.e. the mutual information with using \( n \) relay antennas is equal to the sum of mutual information with using only \( n - 1 \) antennas and the mutual information with using only a single antenna. Hence, using the results from approximation algorithms, we show that a greedy algorithm achieves the optimal solution, however, with linear complexity compared to the exponential complexity of the brute force approach.

II. NOTATION

Let \( \mathbf{A} \) denote a matrix, \( \mathbf{a} \) a vector and \( a_i \) the \( i^{th} \) element of \( \mathbf{a} \). The transpose and conjugate transpose are denoted by \( ^T \) and \( ^\dagger \), respectively. \( \mathbf{I}_n \) denotes the \( n \times n \) identity matrix. The expectation of function \( f(x) \) with respect to \( x \) is denoted by \( \mathbb{E}(f(x)) \). \( (x)^+ \) denotes \( \max\{0, x\} \). Let \( S_1 \) be a set and \( S_2 \) be a subset of \( S_1 \). Then \( S_2 \setminus S_1 \) denotes the set of elements of \( S_1 \) that do not belong to \( S_2 \). The cardinality of set \( S \) is denoted by \( |S| \). We use the symbol := to define a variable.

III. ORGANIZATION

The rest of the paper is organized as follows. Section IV describes the system model and problem statement for the receive antenna selection problem in point-to-point MIMO channels. In Section V, we establish the sub-modularity of the antenna receive antenna selection problem in point-to-point MIMO channels. The relay selection problem is discussed and analyzed in Section VI, where the modularity of relay selection problem is derived. Section VII illustrates some numerical examples, and some final conclusions are made in Section VIII.

IV. SYSTEM MODEL FOR THE POINT-TO-POINT MIMO CHANNEL

Consider a MIMO wireless channel between a transmitter with \( N_t \) antennas and its receiver with \( N_r \) antennas. Assuming that the receiver uses \( L \leq N_r \) antennas (indexed by \( \mathcal{R}_L \)) to receive the \( N_t \) independent streams \( x = [x_1, \ldots, x_{N_t}]^T \), the received signal at the destination is given by

\[
y = \sqrt{\frac{P}{N_t}} \mathbf{H}_{\mathcal{R}_L} \mathbf{x} + \mathbf{n},
\]

where \( P \) is the average transmit power, \( \mathbf{H}_{\mathcal{R}_L} \in \mathbb{C}^{N_r \times L} \) is the channel coefficient matrix with entries \( \mathbf{H}_{\mathcal{R}_L}(i, j) \) corresponding to the channel coefficient between the \( i^{th} \) receive antenna of the selected set \( \mathcal{R}_L \) and the \( j^{th} \) transmit antenna. The results of this paper are applicable for any continuous distribution on the entries of \( \mathbf{H}_{\mathcal{R}_L} \). We assume that the receiver exactly knows the channel state information (CSI) \( \mathbf{H}_{\mathcal{R}_L}, \forall \mathcal{R}_L \subset \{1, \ldots, N_r\} \) and uses it for performing antenna selection, while the transmitter has no CSI and uses all its antennas with equal power allocation.

For the receive signal model (1), the mutual information using antenna subset \( \mathcal{R}_L \) [20], is given by

\[
C_{\mathcal{R}_L} := \log \det \left( \mathbf{I} + \frac{P}{N_t} \mathbf{H}_{\mathcal{R}_L} \mathbf{H}_{\mathcal{R}_L}^\dagger \right),
\]

and the antenna selection problem is to find the optimal set \( \mathcal{R}_L^* \) of \( L \) transmit antennas that maximizes the mutual information, i.e.

\[
\mathcal{R}_L^* = \arg\max_{\mathcal{R}_L \subset \{1,2,\ldots,N_t\}} C_{\mathcal{R}_L}. \tag{3}
\]

Remark 1: For MIMO channels, the tradeoff between rate of transmission and reliability (diversity gain) is captured through the diversity-multiplexing tradeoff (DMT) [21]. The DMT directly depends on the outage probability that is defined as

\[
P_{\text{out}}(R) := P(C_{\mathcal{R}_L} \leq R).
\]

Thus, maximizing \( C_{\mathcal{R}_L} \) over antenna subsets is equal to obtaining the optimal diversity-multiplexing tradeoff of MIMO channels with antenna selection.

The antenna selection problem (3) has no elegant solution, and with a brute force approach, one needs to make \( \binom{N_r}{L} \) computations to solve it. Since \( N_r \) can be large, e.g., in order of tens or hundreds for massive MIMO applications [8], the brute force search is too expensive. Moreover, the search needs to be carried out after each coherence interval since channel coefficients change independently across different coherence intervals. To simplify the complexity of antenna selection, many heuristic approaches have been proposed in literature, however, none of them provide any theoretical guarantees on their performance.
In this paper, we present a systematic study of receive antenna selection algorithm and provide theoretical guarantees on its performance by leveraging ideas from the field of approximation algorithms that are quite popular in the computer science community. Essentially, we make use of the well-known result that for a sub-modular objective function, a greedy solution approximates the optimal solution by a factor of \((1 - 1/e)\) [16]. We will show that the receive antenna selection problem is sub-modular in the number of antennas, and hence a greedy solution, which at each step maximizes the incremental gain in the mutual information achieves a \((1 - 1/e)\) fraction of the optimal solution. To begin with, we need the following definitions.

**Definition 1:** Let \(U\) be a set, and \(2^U\) denote the power set of \(U\). Let \(f\) be a function defined on \(f : 2^U \rightarrow \mathbb{R}^+\). Then \(f\) is called **monotone** if \(f(S \cup \{a\}) - f(S) \geq 0\), for all \(a \in U, S \subseteq U, a \notin S\), and \(f\) is called a **sub-modular** function if it satisfies

\[
f(S \cup \{a\}) - f(S) \geq f(T \cup \{a\}) - f(T),
\]

for all elements \(a \in U, a \notin T\) and all pairs of subsets \(S \subseteq T \subseteq U\). In particular, a function \(f\) is called **modular** if it satisfies

\[
f(S \cup \{a\}) - f(S) = f(T \cup \{a\}) - f(T),
\]

for all elements \(a \in U, a \notin T\) and all pairs of subsets \(S \subseteq T \subseteq U\).

Essentially, for a sub-modular function the incremental gain from adding an extra element in the set decreases with the size of the set. The main interest in sub-modular functions is from adding an extra element in the set decreases with the size of the set. The main interest in sub-modular functions is from the fact that entropy function is sub-modular. Let \(\Sigma := \left( I + \frac{P}{N_t} H_R L H_R^\dagger \right) \). Consider a set \(V = \{1, 2, \ldots, N_r\}\), and let \(A \subseteq V\) of cardinality \(|A| = L\). Let \(x_A\) be a zero-mean, multi-variate Gaussian random vector with covariance matrix \(\Sigma\), i.e. the probability density function of \(x_A\) is \(g_{x_A}(x) = \frac{1}{\sqrt{2\pi \det(\Sigma)}} \exp^{-\frac{1}{2} x^\dagger \Sigma^{-1} x} \). This construction is valid since \(\Sigma\) is a symmetric matrix. Also note that \(\Sigma\) is a positive definite matrix with \(\det(\Sigma) > 1\), and hence \(\log(\det(\Sigma)) > 0\). Then, from [20], the entropy of \(x_A\) is \(h(x_A) = \log \left( \sqrt{2\pi e} \right)^L \det(\Sigma) \). Hence

\[
h(x_A) = \log \left( \sqrt{2\pi e} \right)^L + \frac{1}{2} \log(\det(\Sigma)),
\]

\[
= \log \left( \sqrt{2\pi e} \right)^L + C_{RL}.
\]

Since entropy is a sub-modular function (Lemma 1), it follows that \(C_{RL}\) is sub-modular.

We present another proof of Theorem 4 to illustrate the connections between sub-modularity of \(C_{RL}\) and mutual information of some information theoretic channels.

**Proof:** In this proof we will directly show that for \(f(R_L) = C_{RL}\), \(R_L \subseteq U = \{1, 2, \ldots, N_r\}\), we denote the set of receive antennas, \(\{S \cup \{a\}\} - f(S) \geq f(T \cup \{a\}) - f(T)\), for all elements \(a \in U, a \notin T\) and all pairs of subsets \(S \subseteq T \subseteq U\).

For \(U = \{1, 2, \ldots, N_r\}\), and \(a \in U, a \notin T\), \(S \subseteq T \subseteq U\), let the channel coefficient matrix between the \(N_t\) transmit antennas and the set \(S\) of receiver antennas be \(H_S \in \mathbb{C}^{(S \times N)}\),

\[
C_{RL} = \log \det \left( I + \frac{P}{N_t} H_R L H_R^\dagger \right).
\]
between the $N_t$ transmit antennas and the set $T$ of receiver antennas be $\mathbf{H}_T \in \mathbb{C}^{[T] \times N_t} = [\mathbf{H}_{T \setminus S} \mathbf{H}_S]^T$, and the channel coefficient vector between the $N_t$ transmit antennas and the $i^{\text{th}}$ receive antenna be $\mathbf{h} \in \mathbb{C}^{1 \times N_t}$. Then for $f(\mathcal{R}_L) = C_{\mathcal{R}_L}$, $f(S \cup \{a\}) - f(S)$

$$= \log \det \left( I_{\left| S \right| + 1} + \frac{P}{N_t} \begin{bmatrix} \mathbf{H}_S & \mathbf{H}_S^\dagger \mathbf{h}\dagger \end{bmatrix} \right)$$

$$- \log \det \left( I_{\left| S \right|} + \frac{P}{N_t} \mathbf{H}_S \mathbf{h}\dagger \right),$$

$$= \log \det \left( I_{N_t} + \frac{P}{N_t} \begin{bmatrix} \mathbf{H}_S & \mathbf{H}_S^\dagger \mathbf{h}\dagger \end{bmatrix} \right)$$

$$- \log \det \left( I_{N_t} + \frac{P}{N_t} \mathbf{H}_S \mathbf{h}\dagger \mathbf{H}_S^\dagger \right),$$

where the second statement follows from the determinant equality $\det(I + AB) = \det(I + BA)$. Similarly, $f(T \cup \{a\}) - f(T)$

$$= \log \det \left( I_{N_t} + \frac{P}{N_t} \begin{bmatrix} \mathbf{H}_S & \mathbf{H}_S^\dagger \mathbf{h}\dagger \end{bmatrix} \right)$$

$$- \log \det \left( I_{N_t} + \frac{P}{N_t} \mathbf{H}_S \mathbf{h}\dagger \mathbf{H}_S^\dagger \right),$$

Thus, in light of Theorem 1, and the sub-modularity of the objective function in the receive antenna selection problem (Theorem 4), we propose the following greedy algorithm to maximize the mutual information while selecting the $L$ out of the $N_t$ receive antennas.

Greedy Algorithm (GA): At step $i$, $\mathcal{R}_L = \mathcal{R}_L \cup \{i^\star\}$, where

$$i^\star = \arg \max_{i \in \{1, 2, \ldots, N_r\}, i \notin \mathcal{R}_L} \log \det \left( I + \frac{P}{N_t} \mathbf{H}_S \mathbf{h}\dagger \mathbf{H}_S^\dagger \right),$$

and repeat for $i = i + 1$. Stop when $|\mathcal{R}_L| = L$.

Formally, we write our main result as follows.

**Theorem 5:** Let the output of the greedy algorithm (GA) be the set $S$. Then, if $S^\star$ is the set that maximizes the value of $f(\mathcal{R}_L) = C_{\mathcal{R}_L}$ over all $L$-element sets. Then $f(S) \geq (1 - \frac{1}{e}) f(S^\star)$.

**Proof:** Since $\det \left( I + \frac{P}{N_t} \mathbf{H}_S \mathbf{h}\dagger \mathbf{H}_S^\dagger \right) > 1$, $C_{\mathcal{R}_L} > 0$ and it follows that $f = C_{\mathcal{R}_L}$ is a function from $\{1, 2, \ldots, N_t\}$ to $\mathbb{R}^+$. Thus, the result follows from Theorem 1, using the monotonicity of the receive antenna selection problem from Theorem 3, and sub-modularity of the receive antenna selection problem from Theorem 4.

**Discussion:** In this section, we showed that in a point-to-point MIMO channel, the receive antenna selection problem is sub-modular, and a greedy optimization approach is guaranteed to be within $(1 - 1/e)$ fraction of the optimal solution. While the transmit/receive antenna selection problem has received tremendous amount of attention and is well studied in literature, however, to the best of our knowledge, no such theoretical guarantees have been proven before this work. Prior work on finding the optimal antenna subset primarily uses heuristic low-complexity approaches and does not provide with theoretical bounds. By making use of results available in the approximation algorithms literature, we have been able to justify the use of greedy approaches for receive antenna selection and derive a lower bound on their performance.

### VI. Relay Antenna Selection

In this section, we consider a multiple relay network, where multiple relays with total $N$ antennas (can be co-located or distributed) aid the communication between the source and the destination, equipped with single antenna each. We show...
that the relay selection problem in a multiple relay network is modular when each relay has full CSI for both of its channels and uses an amplify and forward strategy. Using the modularity of the relay selection problem, then we conclude that a greedy optimization approach leads to the optimal solution for the relay selection problem.

The relay selection problem is invariant to any duplexer assumption. We assume that relays work in half-duplex mode, and for simplicity of exposition assume the absence of a direct path between the source and the destination. Hence the transmission takes place in two phases, where in the first phase the source transmits to all relays, and then in the next phase the selected relays transmit to the destination. Without loss of generality, we assume that the source transmits with unit average power, and there is an unit average sum power constraint on the relays. Let the channel between the source and the $i^{th}$ relay be $f_i$, and the channel between the $j^{th}$ relay and destination be $g_j$. We assume that the source has no CSI, while the $i^{th}$ relay has CSI for both its channel coefficients $f_i$ and $g_i$, and the destination is assumed to know all the channel coefficients $f_i, g_i \forall i$.

Let $y_k$ be the signal received at relay $k$, where

$$y_k = f_kx + n_k,$$

where $x$ is the signal transmitted by the source with $E(x^2) = 1$, and $n_k$ is the AWGN with zero mean and unit variance. The relay $k$ then transmits $t_k = \frac{w_ky_k}{\gamma_k}$, where $\gamma_k = \sqrt{|f_k|^2 + 1}$ is the normalization factor to ensure that $E\left(\frac{w_k^2}{\gamma_k^2}\right)^2 = 1$, and $\sum_{k=1}^{N} w_k w_k^\dagger = 1$ to ensure a unit power constraint.

Under these assumptions, if $T_L \subseteq \{1, 2, \ldots, N\}$ is the subset of relays chosen for transmission, the received signal at the destination is

$$z = \sum_{i \in T_L} \frac{g_i w_i f_i}{\gamma_i} x + \sum_{i \in T_L} \frac{g_i w_i}{\gamma_i} n_i + v,$$

where $v$ is the AWGN with zero mean and unit variance received at the destination. Let $T_L = \{t_1, \ldots, t_L\}$. Hence the mutual information with the relay selected set $T_L$ is

$$C_{T_L}^{\text{relay}}(w) = \log(1 + \text{SNR}_{T_L}(w)), \quad (6)$$

with

$$\text{SNR}_{T_L}(w) := \frac{w^\dagger\Delta \Delta^\dagger w}{w^\dagger\Sigma \Sigma^\dagger w + I}w,$$

where $w = \begin{bmatrix} w_1 & \ldots & w_L \end{bmatrix}^T, \Delta = \begin{bmatrix} g_1 f_1 & \ldots & g_L f_L \end{bmatrix}^T, \Sigma = \begin{bmatrix} \frac{g_1^2}{\gamma_1} & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & \frac{g_L^2}{\gamma_L} \end{bmatrix}$, since $w^\dagger w = 1$.

From the relay selection point of view, the optimization problem is

$$\max_{T_L \subseteq \{1, 2, \ldots, N\}} \max_{w} C_{T_L}^{\text{relay}}(w),$$

or equivalently

$$\max_{T_L \subseteq \{1, 2, \ldots, N\}} \max_{w} \text{SNR}_{T_L}(w). \quad (7)$$

Next, we show that the relay selection problem (7) is modular and thereafter from Theorem 2, we conclude that the greedy solution to the relay antenna selection problem is optimal.

**Theorem 6:** The objective function of the optimal relay selection problem (7) is modular.

**Proof:** Consider $\text{SNR}_{T_L}(w) := \frac{w^\dagger \Delta \Delta^\dagger w}{w^\dagger \Sigma \Sigma^\dagger w + I}w$. Let $A = \Delta \Delta^\dagger$ and $B = \Sigma \Sigma^\dagger + I$. Then

$$\text{SNR}_{T_L}(w) = \frac{w^\dagger A w}{w^\dagger B w},$$

$$= \frac{w^\dagger B^{1/2}(\Sigma I)^{-1/2} A B^{-1/2}(\Sigma I)^{-1/2} B^{1/2} w}{w^\dagger B^{1/2}(\Sigma I)^{-1/2} w},$$

$$= \frac{y^\dagger (B^{1/2} I)^{-1/2} B^{-1/2} y}{y^\dagger y}, \quad y := (B^{1/2})^{-1/2} w,$$

$$= \frac{y^\dagger C y}{y^\dagger y}, \quad C := (B^{1/2} I)^{-1/2} B^{-1/2},$$

where $a$ is valid since $B$ is positive definite and symmetric. Thus, $\max_{w} \text{SNR}_{T_L}(w) = \max_{y} \frac{y^\dagger C y}{y^\dagger y} = \lambda_{\text{max}}(C)$ from the Rayleigh-Ritz theorem [22], and the optimal $y$ is the eigen-vector of $C$ corresponding to the largest eigen-value of $C$. Moreover, since $A$ is a rank-1 matrix, $C$ is also rank-1, with $\lambda_{\text{max}}(C) = tr(C)$, and the optimal $y$ is $B^{-1/2} \Delta$, and consequently the optimal $w = B^{-1/2} \Delta$. Since $tr(C) = \sum_{i \in T_L} \frac{|g_i|^2 |f_i|^2}{|f_i|^2 + |g_i|^2 + 1}$, we have that

$$\max_{w} \text{SNR}_{T_L}(w) = \sum_{i \in T_L} \frac{|g_i|^2 |f_i|^2}{|f_i|^2 + |g_i|^2 + 1}.$$

Thus, clearly the objective function is modular in the number of relay antennas.

The greedy algorithm to maximize the capacity while selecting $L$ relay antennas out of $N$ is as follows. Greedy Algorithm for Relay Selection (GARS): Initialize $n = 1$ and $T_L = \phi$. At step $n$, $T_L = T_L \cup \{i^*\}$, where

$$i^* = \arg\max_{i \in \{1, 2, \ldots, N\}, i \not\in T_L} \text{SNR}_{T_L \cup \{i\}},$$

and repeat for $n = n + 1$. Stop when $|T_L| = L$.

The main result of this Section is as follows.

**Theorem 7:** Let the output of the greedy algorithm (GARS) be the set $S$. Then, if $S^*$ is the set that maximizes the value of $f = C_{T_L}$ over all L-element sets. Then $f(S) = f(S^*)$.

**Proof:** Since the relay selection problem is modular (Theorem 6), the result follows from Theorem 2.

**Discussion:** In this section, we showed that in a multiple relay network with a single antenna equipped source-destination pair, the relay antenna selection problem for maximizing the mutual information is modular. Thus, a greedy optimization approach achieves the optimal solution. The modularity of the objective function in this case follows by using matrix theory results to show that each relay antenna contributes an additive
term to the objective function. Modular functions are special functions that do not exhibit the diminishing returns property, and where the incremental gain of adding a new element to an existing set is identical no matter how large the existing set is. Relay antenna selection has been extensively studied in the literature [10], [23]–[26], with various objective functions, but to the best of our knowledge this is the first work that derives theoretical guarantees on relay antenna selection algorithms, let alone the optimality of the greedy approach.

VII. SIMULATIONS

In this Section we present some numerical results to illustrate the results derived in this paper. In Fig. 3, we first consider the point-to-point MIMO case and plot the achievable rate (mutual information) versus the number of chosen receive antennas for both the greedy as well the optimal strategy (brute-force) for \(N_t = 4, N_r = 16\) with unit power transmission. In Fig. 4, we plot the performance of greedy algorithm for massive MIMO uplink with increasing number of receive antennas \(N_r\) with \(N_t = 4\) and number of selected receive antennas \(L = 4\). As can be seen from Fig. 3, the performance of the greedy algorithm is almost similar to the optimal strategy, and far better than the theoretically derived result (being \((1 - 1/e)\) fraction of the optimal). The point to note here is that the theoretical bound is somewhat pessimistic and corresponds to the worst-case scenario, however, in most cases the performance of greedy algorithms is significantly better than the promised worst-case bound. Similar simulations results have been obtained in [6] to show that greedy algorithms almost achieve the optimal performance in antenna selection for point-to-point MIMO setting. In Fig. 5, we consider the relay network and consider \(N = 16\) relay antennas to select from, and plot the achievable rate (mutual information) versus the number of chosen relay antennas for both the greedy as well the optimal strategy (brute-force). As established in this paper, the greedy algorithm achieves the optimal performance.

VIII. CONCLUSION

In this paper, we used the concept of sub-modular functions to obtain theoretical guarantees on the performance of greedy algorithms for receive antenna selection in point-to-point MIMO channels and relay selection in multiple relay network. For receive antenna selection in point-to-point MIMO channels, we showed that the performance of linear complexity greedy algorithms is only a constant fraction away from the optimal solution. For relay selection in multiple relay network, the greedy algorithm is shown to achieve the optimal solution due to the modularity of the objective function. The simulated performance of greedy algorithms has been well known in the literature, however, no known theoretical guarantees were available.

REFERENCES


