Regenerative Hierarchical Codebooks for Limited Channel Feedback in MIMO Systems

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***Abstract*—It has been well known that channel-adaptive systems (closed-loop) have better performance than the systems without channel knowledge (open-loop). For closed-loop systems, the receiver need to feed back its own channel information to the transmitter through the reverse channel. As the codebook size gets larger, the system performance improves, but the computational (search) complexity grows exponentially with respect to the size in conventional codebook structure. In this letter, we propose 2 codebook design schemes with hierarchical structure to overcome the complexity problem, which maintains near-optimal performance.**

***Index Terms*—limited feedback, MIMO, hierarchical codebook**

I. Introduction

One of the popular techniques for channel feedback is vector quantization methods. Grassmannian codebooks were proposed in [1], [2]. It is generated so that minimum distance between codewords in the codebook is maximized. It has been well known that the codebook is optimal for independent identically distributed (i.i.d.) channel. The above conventional codebooks have a complexity problem because the computational complexity to find the best codeword grows exponentially with respect to the codebook size.

To reduce the complexity, we propose two hierarchical codebook design schemes for two channel environments: i.i.d. channel and time correlated channel. Hierarchical codebooks for limited channel feedback were proposed in [3], [4]. The hierarchical codebook in [3], however, is designed with a DFT-based codebook and its goal is not to reduce complexity. In fact, DFT codebooks do not perform well as the codebook size increases because of the power constraint on the elements in the codebook matrix [5]. On the other hand, the hierarchical codebook in [4] can only be used in multiuser MIMO systems. The proposed schemes are based on the centroid condition by the LBG algorithm [6], [7].

II. Hierarchical Codebook Design

In this section, we propose two hierarchical codebook design schemes to reduce search complexity for an i.i.d. channel and a time correlated channel. The proposed schemes rely on the centroid concept which was defined in the Linde, Buzo, and Gray (LBG) vector quantization algorithm [6], where mean-squared-error (MSE) was used as the distance metric. In communication systems, the chordal distance may be more meaningful [8]. The chordal distance between two vectors w_1 and w_2 is given by [1]

$$d(w_1, w_2) = \sin(\theta_{1,2}) = \sqrt{1 - |w_1^H w_2|^2} \tag{1}$$

where $\theta_{1,2}$ is the angel between w_1 and w_2.

In [7], a modified LBG vector quantization algorithm which uses the chordal distance [9] as a distance measure in centroid condition was proposed. The centroid with chordal distance is given by [7]

$$\mathbf{C}_n = \mathbf{U_R I}_{M\times N} \tag{2}$$

where $\mathbf{I}_{M\times N}$ is an $M \times N$ sub-matrix of an identity matrix, and $\mathbf{U_R}$ is the unitary eigenvector matrix in the eigen decomposition of the sample covariance matrix $\mathbf{R}$. The centroid condition minimizes the average distance in R_n which is determined by the nearest neighbor condition and updates the codebook.

A. Codebook Design for i.i.d. channel

We consider a single-user system in this letter. When we assume the receiver estimates its own channel perfectly for block fading channel, the system model is given by

$$\mathbf{y} = \sqrt{P}\mathbf{H}^H\mathbf{V}\mathbf{s} + \mathbf{n} \tag{3}$$

where P is the transmit power, $\mathbf{H}$ is the $M\times N$ channel matrix whose channel entries are i.i.d. complex Gaussian with zero mean and unit variance, $\mathbf{V}$ is the $M \times N$ precoding matrix, $\mathbf{s}$ is the transmit symbol, and $\mathbf{n}$ is the $N \times 1$ complex white Gaussian noise vector with unit variance.

The first hierarchical codebook design scheme for an i.i.d. channel uses the centroid condition of the LBG algorithm. Basically, the proposed algorithm uses a Grassmannian codebook since the codewords in a Grassmannian codebook are the farthest from each other than any other codebooks. The hierarchical codebook design for an i.i.d. channel takes the following steps.

1) Define Grassmannian codebook, $\mathcal{C}_1 = \{v_1, v_2, \ldots, v_{2^{B_1}}\}$, where v_i is the ith vector or matrix codeword and 2^{B_1} is the size of $\mathcal{C}_1$.
2) Compute all chordal distances between v_i and v_j for all i and j $(1 \le i \le 2^{B_1}, 1 \le j \le 2^{B_1}, j \ne i)$.
3) Sort the chordal distance with ascending order for each v_i, and find 2^{B_2} codewords which have minimum

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chordal distance for each v_i. Let the 2^{B_2} codewords for each v_i be the set of $\{n_{i,k}\}$ $(1 \leq k \leq 2^{B_2})$.
4) Find the centroid between v_i and $n_{i,k}$ for each i and k, and the centroid is denoted by $a_{i,k}$.
5) Find the centroid between v_i and $a_{i,k}$ again for each i and k.
6) The centroids in 5) produce the new children codebooks.

One of characteristics of the proposed codebook design scheme is to regenerate new child codebooks with the existing codebook. Thus, the hierarchical codebook consists of two stages: the parent codebook and the children codebook stage.

In the system with the proposed hierarchical codebook, the receiver estimates its own channel at first, and finds the best codeword with chordal distance in the first stage codebook, $\mathcal{C}_1$. Let us denote the best chosen codeword by v_{i^*}. The receiver then selects the best codeword in the second stage codebook, $\mathcal{S}_{i^*}$, where we denote the child codebook for v_i by $\mathcal{S}_i = \{c_{i,1}, c_{i,2}, \ldots, c_{i,2^{B_2}}\}$. If we denote the best codeword in $\mathcal{S}_{i^*}$ by c_{i^*,j^*}, the finally selected codeword is c_{i^*,j^*}, and the indices of i^* and j^* are fed back to the transmitter. An important characteristic of the proposed hierarchical codebook is to achieve complexity reduction with performance similar to the performance of conventional codebooks. The hierarchical codebook needs $2^{B_1} + 2^{B_2}$ comparisons with the estimated channel while conventional codebooks needs $2^{B_1+B_2}$ comparisons.

By using quantization cell approximation, the CDF of quantization error at the first stage is given by [11]

$$F_{d_{c_1}^2}(x) = \begin{cases} 2^{B_1} A x^{N(M-N)}, & 0 \leq x \leq \delta \\ 1, & \delta \leq x \end{cases} \quad (4)$$

where A is the same as $c_{n,p,q,\beta}$ in [11], d_{c_1} is the chordal distance between the channel and the nearest codeword of the 1st stage, and $\delta = \left(\frac{1}{2^{B_1} A}\right)^{1/(N(M-N))}$. The CDF of quantization error at the 2nd stage is given by

$$\begin{aligned} F_{d_{c_2}^2}(x) &= Pr\{d_{c_2}^2 \leq x | d_{c_1}^2 \leq \delta\} Pr\{d_{c_1}^2 \leq \delta\} \\ &+ Pr\{d_{c_2}^2 \leq x | d_{c_1}^2 \geq \delta\} Pr\{d_{c_1}^2 \geq \delta\}. \end{aligned} \quad (5)$$

Assuming $Pr\{d_{c_1}^2 \geq \delta\} \approx 0$, we have

$$\begin{aligned} F_{d_{c_2}^2}(x) &= Pr(d_{c_2}^2 \leq x | d_{c_1}^2 \leq \delta) \\ &= \begin{cases} \frac{2^{B_2} x^{N(M-N)}}{\delta^{N(M-N)}}, & 0 \leq x \leq \eta \\ 1, & \eta \leq x \end{cases} \end{aligned} \quad (6)$$

where d_{c_2} is the chordal distance between the channel and the nearest codeword of the 2nd stage, and $\eta = \delta(\frac{1}{2^{B_2}})^{1/N(M-N)}$. Thus, the expectation of the quantization error of the hierarchical codebook is given by

$$\begin{aligned} E[D] &= \int_0^\eta x \cdot f_{d_{c_2}^2}(x) dx \\ &= \frac{N(M-N)}{N(M-N)+1} \frac{1}{A^{1/(N(M-N))}} 2^{-\frac{(B_1+B_2)}{N(M-N)}}. \end{aligned} \quad (7)$$

From (7), $E[D]$ decreases exponentially with respect to $B_1 + B_2$. Since the search complexity is $\mathcal{O}(2^{B_1} + 2^{B_2})$, it is minimized when $B_1 = B_2$.

B. Codebook Design for Time Correlated Channel

A time correlated channel model is given by

$$\mathbf{H}(t) = \rho \cdot \mathbf{H}(t-1) + \sqrt{1-\rho^2} G(M, N) \quad (8)$$

where ρ is time correlated factor and $G(M, N)$ is an $M \times N$ random channel matrix. ρ is determined by the Jake's model as follows.

$$R(l) = J_0(2 \cdot \pi \cdot f_D \cdot T_s \cdot l) \quad (9)$$

where $J_0(\cdot)$ is the zeroth-order Bessel function of the first kind, f_D is the Doppler shift, and T_s is the sampling time.

The channel variation between the previous time slot and the present time slot is small in a time-correlated channel. Therefore codebook size does not have to be large since the present channel may be predicted with some degree from the channel of the previous time slot. We propose a hierarchical codebook design scheme by using the characteristics of the time correlated channel. The summary of the codebook design scheme is as follows.

1) Define Grassmannian codebook, $\mathcal{C}_1 = \{v_1, v_2, \ldots, v_{2^{B_1}}\}$, where v_i is the ith vector or matrix codeword, and 2^{B_1} is the size of $\mathcal{C}_1$.
2) Compute the chordal distances between v_i and v_j for all i and j $(1 \leq i \leq 2^{B_1}, 1 \leq j \leq 2^{B_1}, j \neq i)$.
3) Sort the chordal distance with ascending order for each v_i, and select the first 2^{B_2-1} codewords for each v_i. Denote the 2^{B_2-1} codewords for each v_i by the set $\{n_{i,k}\}$ $(1 \leq k \leq 2^{B_2-1})$.
4) Find the centroid between v_i and $n_{i,k}$ for each i and k, and it is denoted by $a_{i,k}$.
5) Find the centroid between v_i and $a_{i,k}$, and it is denoted by $b_{i,k,1}$.
6) Find the centroid between $n_{i,k}$ and $a_{i,k}$, and it is denoted by $b_{i,k,2}$.
7) The centroids in 5) and 6) produce the children codebooks.

The design process of the hierarchical codebook in a time correlated channel is similar to the design process in an i.i.d. channel. However, the codebook design scheme in a time correlated channel builds two centroids ($b_{i,k,1}$ and $b_{i,k,2}$) for v_i and $n_{i,k}$, and that is why it needs to find 2^{B_2-1} neighbor codewords. Here, $b_{i,k,1}$ represents the region of v_i, $b_{i,k,2}$ represents the centroid for the region of $n_{i,k}$, and both $b_{i,k,1}$ and $b_{i,k,2}$ build the children codebook for v_i.

In terms of codebook structure, the codebook for a time-correlated channel is composed of two stage codebooks as in the previously proposed hierarchical codebook schemes, and 2^{B_2-1} codewords ($b_{i,k,1}$) belong to the region of v_i, and the other 2^{B_2-1} codewords ($b_{i,k,2}$) belong to the region of $n_{i,k}$ $(1 \leq k \leq 2^{B_2-1})$.

In practical systems, the best codeword is selected in $\mathcal{C}_1$ in the first time slot of each refresh period, and it is denoted by v_{i^*}. From the second time slot, the best codeword is selected not in $\mathcal{C}_1$, but in the children codebook for v_{i^*} which has 2^{B_2} codewords. The next children codebook is determined by

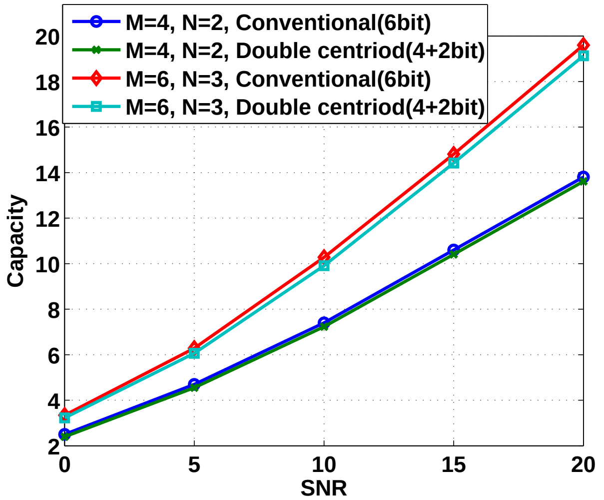

Fig. 1. Capacity comparison with respect to SNR between a conventional codebook and the hierarchical codebook with double centroid in an i.i.d. channel.

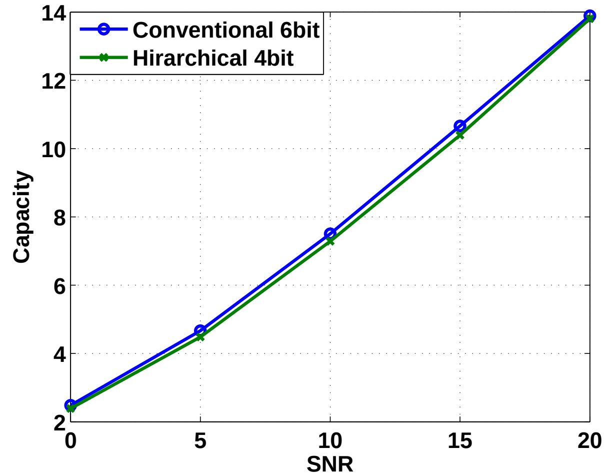

Fig. 2. Capacity comparison with respect to SNR between a conventional codebook and the hierarchical codebook scheme in a time correlated channel when $M = 4, N = 2, B = 6, DB = 4$.

the codeword (in $\mathcal{C}_1$) closest to the selected codeword in the previous time slot. For example, we assume $B_1 = 3, B_2 = 2$, and suppose that the two codewords which are the closest to v_1 are v_5 and v_8. If $b_{1,8,2}$ in the child codebook for v_1 is selected at the $t-1$ time slot, the best codeword at the t time slot is determined in the child codebook for v_8 since the channel is expected to be in the region of v_8. If $b_{1,8,1}$ in the child codebook for v_1 is selected at the $t-1$ time slot, the best codeword at the t time slot is determined in the child codebook for v_1 since the channel is expected to be in the region of v_1.

III. Simulation Results

In this section, we compare the capacity of the proposed hierarchical codebook schemes with that of conventional codebooks. The proposed hierarchical codebooks are designed with the Grassmannian codebook in [10], [8].

Fig.1 shows the capacity comparison between a conventional codebook and the hierarchical codebook with the double centroid scheme for an i.i.d. channel. It is observed that the proposed codebook achieves performance similar to the conventional codebook with about 70 % reduction in computational complexity. Fig.2 shows the capacity comparison between a conventional codebook and the hierarchical codebook scheme in a time-correlated channel when $M = 4, N = 2, B = 6, DB = 4$ where DB is the number of feedback bits in a time correlated channel. In this simulation, ρ is 0.9172 assuming the Doppler speed is 8 Km/h. We also assume the sampling time is 5 ms and the refresh period is 100 ms. It is observed that the capacity of the proposed 4-bit codebook in a time-correlated channel is close to that of the 6-bit conventional codebook.

IV. Conclusions

In this letter, we propose two novel hierarchical codebook schemes in an i.i.d. channel and a time-correlated channel to reduce computational (search) complexity with minimal performance loss. A common methodology of the proposed schemes is to build the second stage codebook based on the region of each codeword in the first stage codebook. Simulation results show the proposed hierarchical codebook schemes have significant complexity reduction compared to the conventional codebook schemes with negligible capacity degradation.

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