Modeling Heterogeneous Network Interference

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***Abstract*—Cellular systems are becoming more heterogeneous with the introduction of low power nodes including femtocells, relays, and distributed antennas. Unfortunately, the resulting interference environment is also becoming more complex, making evaluation of different communication strategies for cellular systems more challenging in both analysis and simulation. This paper suggests a simplified interference model for heterogeneous network. Leveraging recent applications of stochastic geometry to analyze cellular systems, this paper propose to analyze performance in a fixed-size typical cell surrounded by an interference field consisted of superposition of marked Poisson point processes outside a guard region. The proposed model simplifies the simulation of cellular systems and may provide analytical insights for certain signaling strategies.**

I. Introduction

Cellular network deployment is approaching a phase transition. After this point, networks will take on a massively heterogeneous character. A wide variety of infrastructure will be deployed in these new networks including macro, pico, and femto base stations, as well as fixed relay stations and distributed antennas. A major challenge in deploying heterogeneous cellular networks is managing interference. The problem is compounded by the observation that the deployment of much of the small cell infrastructure will be demand based and more irregular than traditional infrastructure deployments. Further, as urban areas are built out, even macro and micro base station infrastructure is becoming less like points on a hexagonal lattice and more random [1]. Consequently, the aggregate interference environment is more challenging to model and evaluating the performance of communication techniques in the presence of heterogeneous interference is challenging.

A promising approach to model interference in modern cellular networks is to treat the base station locations as points distributed according to a spatial point process [1]–[5]. This approach leverages the mathematical framework known as stochastic geometry [6]–[9] and often uses the Poisson Point Process (PPP) for its tractability. PPPs have been used to model co-channel interference in ad hoc networks [7]–[10], and in generic settings as well [11]–[13]. One of the most promising approaches for system analysis is presented in [1], where key system performance metrics like coverage (or outage probability) and average rate are computed by averaging over all deployment scenarios. The approach in [1] is quite powerful, leading to closed form solutions for some special cases and also extensions to heterogeneous multi-tier networks [14]. From our perspective, the main drawback of the approach in [1] for application to existing systems is that performance is characterized for an entire system not for a given cell. As cellular networks are already built out, a cellular provider will likely want to know the performance they achieve in some given cells by adding additional infrastructure (and thus interference) in the rest of the network. It is also challenging to incorporate more complex kinds of heterogeneous infrastructure like fixed relays or distributed antennas into the signal and interference models.

In this paper we propose a simplified interference model for heterogeneous networks. The key idea is to consider a typical cell, where the interferers outside the fixed-size cell of interest are distributed according to a marked Poisson point process. By capturing the density of the interference field, which consists of superposition of marked PPPs outside a guard region, various complex scenarios can be evaluated in the typical cell. Under certain assumptions, heterogeneous interference can be modeled as a single marked PPP with a suitably chosen guard region and mark distribution. Our mathematical approach follows the approach of [1] and leverages basic results on PPPs [8], [9], [15]. We demonstrate how to employ the proposed typical cell analysis in a heterogeneous network consisting of mixtures of the different kinds of infrastructure. Compared with [1], our approach is more specific since we consider a fixed cell size and not a Voronoi tessellation. We also do not calculate system-wide performance measures, rather we focus on performance for a typical cell of interest. The advantage of our approach is that more complex types of communication and network topologies can be analyzed in a given target cell and key insights on the performance of advanced transmission techniques with heterogeneous interference using stochastic geometry can be obtained, at the expense of generality. Compared with the conventional approach to evaluating performance in cellular systems based on simulating multiple tiers of interfering cells, our approach does not require the explicit generation and simulation of interferer locations instead modeling the aggregate interference power from certain co-channel interference distributions.

II. Downlink Network Model

In the classic model for cellular systems in Fig. 1(a), base stations are located at the centers of hexagons in a hexagonal tessellation and interference is computed in a typical cell from multiple tiers of interferers. The hexagonal model requires simulation of multiple tiers of interferers and makes analysis difficult. In the stochastic geometry model for cellular systems illustrated in Fig. 1(b), base stations positions follow a PPP and cells are derived from the Voronoi tessellation. Cell sizes are random and performance metrics are computed in an aggregate sense accounting for all base station distributions.

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Our proposed model, which we call the *hybrid approach*, is illustrated in Fig. 1(c). We consider a *typical cell* of fixed size, nominally a circle with radius R_c. The base station locations outside of the typical cell are modeled according to a PPP and the interference becomes a shot-noise process [16]. Since the edge of the cell is fixed, and is not determined by interference levels, a guard region of radius R_g from the cell edge is imposed around the typical cell in which no other transmitters can occupy. Performance is evaluated inside the typical cell accounting for interference sources outside the guard region. In general R_g is a parameter of the model. In the simulations we show that the choice of $R_g = R_c/2$ provides good performance relative to a hexagonal tessellation.

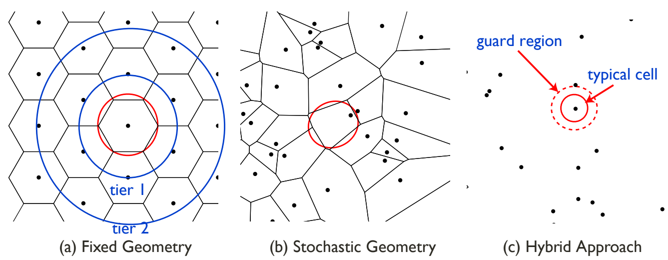

Fig. 1. Models for cellular communication. (a) The common fixed geometry model with hexagonal cells and multiple tiers of interference. (b) A stochastic geometric model where all base stations are distributed according to some 2D random process. (c) The proposed hybrid approach where there is a *typical cell* of a fixed size surround by base stations distributed according to some 2D random process, possibly with an exclusion region around the cell.

To make the calculations concrete, in this paper we focus on the downlink. We consider the received signal power at distance r from the transmitter presumably located at the cell center. Extensions to distributed antennas for example requires a more complex signal model. Let the received signal power be

$$S(r) \quad = \quad P(r)LH, \tag{1}$$

where $P(r) = P_s/\ell(r)$ is the distant-dependent average received signal power with P_s being the transmit power, L a random variable corresponding to the large-scale fading power usually log-normal, and H is a random variable corresponding to the small-scale fading power. We assume that a non-singular path-loss model is used with

$$\ell(r) \quad = \quad C \max(d_0, r)^n, \tag{2}$$

where $n > 2$ is the path-loss exponent, $d_0 > 0$ is the reference distance, and $C > 0$ is a constant. In addition to resulting in finite interference moments, (2) takes into account the realistic RF design constraints on the maximum received power and is often assumed in a standards based channel model like 3GPP LTE Advanced [17].

We consider the power due to small-scale fading as occurring after processing at the receiver, e.g. diversity combining. Furthermore, we assume that the transmit strategy is independent of the strategies in the interfering cells and no adaptive power control is employed. We model the small-scale fading power H as a Gamma distributed random variable with $\Gamma(k_h; \theta_h)$. The Gamma distribution is a general type of statistical distribution, which is related to the Nakagami distribution by a simple transformation and includes Rayleigh and Chi-Squared as special cases. It also has the flexibility to analyze several practical relevant scenarios assuming i.i.d. complex Gaussian signaling. For example, in conventional Rayleigh fading H is $\Gamma(1; 1)$ while Rayleigh fading with N_r receive antennas and maximum ratio combining H is $\Gamma(N_r; 1)$. With multiuser MIMO with N_t transmit antennas, $U \leq N_t$ active users, and zero-forcing precoding then H is $\Gamma(N_t - U + 1; 1/U)$ where the $1/U$ follows from splitting the power among different users.

III. Homogeneous Network Interference

Our primary interest in this paper is suggesting models for homogeneous and heterogeneous interference. First we consider the homogeneous case, where interference is just from a single kind of interferer, e.g. macrocells. We model the interferer locations according to a marked point process, where the marks correspond to the channel between the interferers and the target receiver. Specifically we consider a PPP Φ with density λ and marks modeled similarly to the signal power distribution. For interferer k, L_k follows a lognormal distribution and the small-scale fading distribution G_k is $\Gamma(k_g; \theta_g)$. The transmit power of interferers is denoted by P_g. The parameters of the interference mark distribution are usually different than the signal channel even when the same transmission scheme is employed in all cells. For example with single antenna transmission and conventional Rayleigh fading G_k is $\Gamma(1; 1)$ for any N_r at the receiver, while with multiuser MIMO zero-forcing beamforming with multiple transmit antennas serving $U \leq N_t$ users G_k is $\Gamma(U; 1)$.

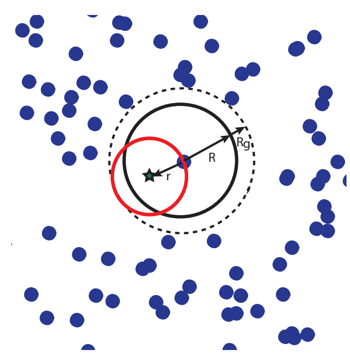

Fig. 2. Interference power calculation at a point r away from the transmitter. An upper bound is considered as the interference is not excluded uniformly around the receiver.

We aim at computing the interference power received at a distance $0 < r < R$ away from the center of the cell denoted by $I(r)$. Unfortunately, the exclusion distance to the nearest interferer is asymmetric, c.f. Fig. 2. Consider the interference contribution in a ball of radius $R_c + R_g - r$ around the received signal denoted $B(R_c + R_g - r)$. This results in an upper bound on the aggregate interference due to the fact that less area is excluded from the calculation. Then the receiver is at the center of the reduced interference region, and because it is a PPP can be assumed to be at the origin. Then we write the received interference power as

$$I(r) \quad = \sum_{k\in\Phi\backslash B(R_c+R_g-r)} \frac{G_k L_k P_g}{\ell(R_k)}. \quad (3)$$

The Laplace transform of $I(r)$ when the interference marks $G_k L_k$ follow an arbitrary but identical distribution for all k is calculated along the lines of [1] and is given by

$$\begin{aligned}
\mathcal{L}_{I(r)}(s) &= \mathbb{E}_{I(r)}[e^{-sI(r)}] \\
&= \mathbb{E}_{\Phi,G_k,L_k}\left(e^{-s\sum_{k\in\Phi\backslash B(R_c+R_g-r)}\frac{P_g G_k L_k}{\ell(R_k)}}\right) \\
&= \mathbb{E}_{\Phi,G_k,L_k}\left(\prod_{k\in\Phi\backslash B(R_c+R_g-r)} e^{-s\frac{P_g G_k L_k}{\ell(R_k)}}\right) \\
&\overset{(a)}{=} \mathbb{E}_{\Phi}\left(\prod_{k\in\Phi\backslash B(R_c+R_g-r)} \mathbb{E}_{G,L}\left(e^{-s\frac{P_g G L}{\ell(R_k)}}\right)\right) \\
&\overset{(b)}{=} e^{-2\pi\lambda\int_{R_c+R_g-r}^{\infty}\left(1-\mathbb{E}_{G,L}\left(e^{-sP_g G L C^{-1}v^{-n}}\right)\right)v dv} \\
&= e^{\lambda\pi(R_c+R_g-r)^2 - \frac{2\pi\lambda(sP_g)^{\frac{2}{n}}}{nC^{2/n}}F_{n,R_c+R_g-r,C}(s)}, \quad (4)
\end{aligned}$$

where (a) follows from the i.i.d. distribution of $G_k L_k$ and its further independence from the point process Φ, (b) follows assuming that $R_c + R_g - r > d_0$ and using the probability generating functional (PGFL) of the PPP [6], $Z = G_k L_k P_g$, and

$$F_{n,x,C}(s) = \mathbb{E}_z z^{\frac{2}{n}}\left[\Gamma(-\frac{2}{n}, sC^{-1}zx^{-n}) - \Gamma(-\frac{2}{n})\right]. \quad (5)$$

In some cases the expectation can be further evaluated, e.g. when $Z \sim \Gamma(k;\theta)$ then

$$\begin{aligned}
F_{n,x,C}(s) &= \frac{(sC^{-1}x^{-n})^{-\frac{2}{n}-k} n\theta^{-k}}{2+kn} \\
&\times \; {}_2F_1\left[k, k+\frac{2}{n}, 1+k+\frac{2}{n}, -\frac{1}{sC^{-1}x^{-n}\theta}\right] \\
&- \; \theta^{2/n}\mathcal{B}(k+\frac{2}{n}, -\frac{2}{n}), \quad (6)
\end{aligned}$$

where $\mathcal{B}(x, y)$ is the Beta Euler function and ${}_2F_1$ is the Gauss hypergeometric function.

IV. Heterogeneous Network Interference

In this section we extend the proposed interference model to the case of a heterogeneous network, where the macro-cellular network is underlaid other kinds of infrastructure including low-power nodes like small-cells, femtocells, fixed relays, and distributed antennas. The key idea is to model the interference from each type of infrastructure separately using a marked PPP described by the model in Section III, and then develop an equivalent marked PPP for the sum interference term in order to facilitate analysis. We consider two different interference scenarios as illustrated in Fig. 3. The scenario in Fig. 3(a) corresponds to multiple kinds of out-of-cell interference. For example, there might be interference from both high power base stations and low power distributed antenna system transmission points. The guard regions associated with different processes may be different. The scenario in Fig. 3(b) corresponds to cross-tier interference from a second tier of low-power nodes, e.g. small-cells or femtocells or uncoordinated transmission points. The main difference in this case is that there is interference within the cell so effectively the interference contribution is a constant. There may still be a guard radius in this case but it is likely to be small.

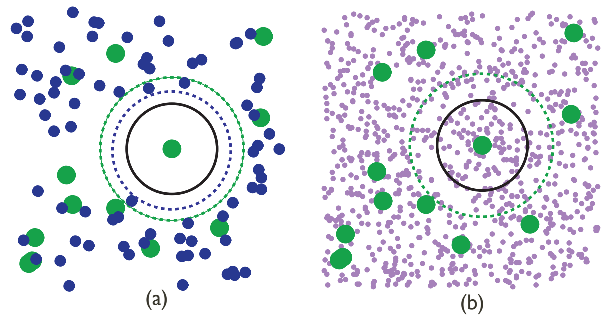

Fig. 3. Calculating interference in a heterogeneous network with multiple kinds of infrastructure in the cell. (a) Heterogeneous out-of-cell interference. (b) Homogeneous interference and cross-tier interference from low power nodes.

A. Heterogeneous Out-of-Cell Interference

Suppose that there are M different kinds of infrastructure deployed in a homogenous way through the network. A more complex model would take non-uniformity or clustering into account, but this is beyond the scope of this work. Each interference source can be modeled as a marked PPP with marks corresponding to the composite fading distribution distributed as $\Gamma(k_m;\theta_m)$ where $m = 1,\ldots,M$. The path-loss function is the same for each process. The transmitting power is given by P_m, the transmitter density by λ_m, and the guard radius by $R_g^{(m)}$. Consequently, each process is parameterized by $(k_m, \theta_m, P_m, \lambda_m, R_g^{(m)})$.

The difference in guard regions adds a complication to developing an equivalent marked PPP. Consequently we upperbound the interference level as described in the following lemma.

Lemma 1: Consider the sum of m PPPs Φ_m with parameters $(k_m, \theta_m, P_m, \lambda_m, R_g^{(m)})$. Then

$$\begin{aligned}
&\sum_{m=1}^{M}\sum_{k\in\Phi_m\backslash B(R_c+R_g^{(m)}-r)}\frac{G_k^{(m)}L_k^{(m)}P_m}{\ell(R_k^{(m)})} \quad (7) \\
&\overset{st}{\leq} \sum_{m=1}^{M}\underbrace{\sum_{k\in\Phi_m\backslash B(R_c+R_{\min}-r)}\frac{G_k^{(m)}L_k^{(m)}P_m}{\ell(R_k^{(m)})}}_{J^{(m)}(r)} := J(r) \quad (8)
\end{aligned}$$

where $R_{\min} = \min_{1\leq m\leq M} R_g^{(m)}$ and $X \overset{st}{\leq} Y$ denotes that X is stochastically dominated by Y.

Proof: Follows directly by noting that $\mathrm{vol}\left(B(R_c + R_g^{(m)} - r)\right) \geq \mathrm{vol}\left(B(R_c + R_{\min} - r)\right)$ and that $\Phi_m \setminus B(R_c + R_g^{(m)} - r) \subset \Phi_m \setminus B(R_c + R_{\min} - r)$. ■

The superposition of M different homogeneous PPPs can be treated as a single equivalent PPP by modifying appropriately the mark distribution.

Proposition 2: The cumulative interference received from all M interfering networks

$$\sum_{m=1}^{M} \sum_{k\in\Phi_m \setminus B(R_c+R_{\min}-r)} \frac{G_k^{(m)} L_k^{(m)} P_m}{\ell(R_k^{(m)})} = \sum_{k\in\Phi_e} \frac{X_k^{(e)}}{\ell(R_k^{(e)})},$$

where $\Phi_e = \left\{ \bigcup_{m=1}^{M} \Phi_m \setminus B(R_c + R_{\min} - r) \right\}$ and $X_k^{(e)}$ is a random variable that is equal to $G^{(m)} L^{(m)} P_m$ with probability $\lambda_m / \sum_{m=1}^{M} \lambda_m$ for $m = 1, \ldots, M$.

Proof: The equivalent PPP, which is the union of M PPPs, i.e. $\Phi_s = \left\{ \bigcup_{m=1}^{M} \Phi_m \setminus B(R_c + R_{\min} - r) \right\}$, is a PPP with intensity measure $\lambda_e = \sum_{m=1}^{M} \lambda_m$ since the superposition of M independent PPPs is also a PPP [8]. Consider now a randomly chosen point (typical point) of $x \in \Phi_s$, then the probability for the event that x belongs to $\Phi_m \setminus B(R_c + R_{\min} - r)$, i.e. $\mathbb{P}\left(x \in \Phi_m \setminus B(R_c + R_{\min} - r)\right) = \frac{\lambda_m}{\sum_{m=1}^{M} \lambda_m}$. Hence, the random variable $X_k^{(e)}$ (interference mark) of any point $x \in \Phi_s$ is equal to $G^{(m)} L^{(m)} P_m$ with probability $\mathbb{P}\left(x \in \Phi_m \setminus B(R_c + R_{\min} - r)\right) = \lambda_m / \sum_{n=1}^{M} \lambda_n$. ■

The marks of the cumulative heterogeneous interference has a finite mixture distribution $f_X(x)$ given by

$$f_X(x) = \sum_{n} \frac{\lambda_n}{\sum_{m=1}^{M} \lambda_m} f_Z^{(n)}(z), \tag{9}$$

where $f_Z^{(m)}(z)$ denotes the PDF of $Z^{(m)} = G^{(m)} L^{(m)} P_m$.

From the independence of the M PPPs and using similar arguments as for deriving (4), the Laplace transform of the heterogeneous out-of-cell interference is

$$\mathcal{L}_J(s) = \prod_{m=1}^{M} \mathcal{L}_{J^{(m)}(r)}(s), \tag{10}$$

with $\mathcal{L}_{J^{(m)}(r)}(s)$ the Laplace transform of $J^{(m)}(r)$ defined as in Lemma 1 with appropriate substitution for P_m, λ_m, and $R_{\min}$.

Expanding the integrals in (10) along the lines of the calculations that lead to (4), combining terms, and using the equivalent mark distribution from Proposition 2 leads to the following result.

Corollary 3: The Laplace transform of the cumulative interference from all M tiers is also given by

$$\mathcal{L}_J(s) = e^{\lambda_e \pi (R_c + R_{\min} - r)^2 - \frac{2\pi\lambda_e (sC^{-1})^{\frac{2}{n}}}{n} \mathbb{E}_x x^{\frac{2}{n}} \mathcal{A}(x)}, \tag{11}$$

where $\lambda_e = \sum_{m=1}^{M} \lambda_m$ and

$$\mathcal{A}(x) = \Gamma\left[-\frac{2}{n}, \frac{sx(R_c + R_{\min} - r)^{-n}}{C}\right] - \Gamma\left[-\frac{2}{n}\right]. \tag{12}$$

Using Corollary 3 we can replace calculations involving the heterogeneous interference term with an equivalent marked interference process. Consequently the homogeneous interference results can be applied but with a different mark distribution and a total density term.

B. Cross-Tier Interference

Certain kinds of infrastructure, like femtocells, are not associated with the macro base station. Rather they form another tier of nodes and create cross-tier interference. We propose to use the same stochastic geometry framework to model the interference with a main difference in how the notion of guard zone is defined. Let B_{cross} denote an exclusion region around the desired receiver with radius R_{cross}. This region might be quite small, just a few meters radius, and is designed to avoid the case where the low-power node is co-located with the target receiver. Let us suppose that the cross-tier interference is associated with a PPP given by Φ_{cross}, the transmitting power is given by P_{cross}, and the transmitter density by λ_{cross}. The received interference power is given by

$$I_{cross} = \sum_{k\in\Phi_{cross}\setminus B_{cross}} \frac{G_k L_k P_{cross}}{\ell(R_k)}, \tag{13}$$

which does not depend on r due to the shift invariance property of the PPP. Consequently cross-tier interference creates a constant interference that is independent of the mobile receiver location.

V. Multiuser MIMO with Heterogeneous Interference

To provide some example of the calculations in this paper, we characterize the performance of multiuser MIMO using the proposed interference model. We consider a typical cell where the transmitter has N_t transmit antennas and communicates with $U \leq N_t$ single-antenna receivers using zero-forcing (ZF) precoding with perfect channel state information. The typical cell performance is characterized as a function of the random quantity SINR given by

$$SINR_r = \frac{S(r)/\ell(r)}{I(r) + \sigma^2} \tag{14}$$

where $S(r)$ is the signal, $I(r)$ is the interference power, and σ^2 is the receiver noise variance. We assume that $S(r) = P_s S_r$ where $S_r \sim \Gamma(k_s, \theta_s)$ with $k_s = N_t - U + 1$ and $\theta_s = 1/U$ as the power is splitted equally among different users.

A. Success Probability

The success probability is defined as $\mathbb{P}(SINR_r > T)$ where T is a prescribed threshold. The success probability, which is one minus the outage probability, gives a measure of diversity performance and is related to the probability of coverage. Using the results from [18], the success probability at location r is given by (15) at the top of next page. Now assuming a kind of heterogeneous interference where the interferers in all M different kinds of infrastructure are also serving U single-antenna receivers using ZF precoding, the marks of the interference of each marked PPP follow a gamma distribution, i.e. $G^{(m)} \sim \Gamma(U, 1)$. The Laplace transform of $I(r)$ is then given by

$$\mathcal{L}_{I(r)}(s) = e^{\lambda_e \pi (R_c + R_{\min} - r)^2 - \frac{2\pi\lambda_e (sC^{-1})^{\frac{2}{n}}}{n} \mathbb{E}_x x^{\frac{2}{n}} \mathcal{A}(x)},$$

$$\mathbb{P}(SINR_r > T) = \sum_{t=0}^{N_t-U} \frac{1}{t!}\left(-\frac{\ell(r)TU}{P_s}\right)^t e^{-\frac{\ell(r)TU\sigma^2}{P_s}} \sum_{j=0}^{t}\binom{t}{j}\left(-\sigma^2\right)^{t-j} \frac{d^j\mathcal{L}_I(s)}{ds^j}\Big|_{s=\ell(r)TU/P_s} \quad (15)$$

where the distribution of X is given by (9). While this is special case, it should be clear that the proposed framework could be used to have random numbers of active users in each cell by modifying the mark distribution.

In the case of homogeneous interference, we again have that $G_k \sim \Gamma(U,1)$ and the Laplace transform of $I(r)$ is given by (4) with $F_{n,R_c+R_g-r,C}(s)$ given by (6) with $k = U$ and $\theta = 1$.

The expressions can be simplified when $U = N_t$ users are served, in which case the success probability becomes

$$\mathbb{P}(SINR_r > T) = e^{-\frac{\ell(r)TU\sigma^2}{P_s}}\mathcal{L}_I(\ell(r)TU/P_s). \quad (16)$$

B. Average Rate

The ergodic achievable rate assuming i.i.d. Gaussian signaling given by

$$\tau(r,R,\lambda,n) := \mathbb{E}\ln(1+SINR_r) \quad (17)$$

as a function of the location r. A useful form of the achievable rate, based on similar expressions for the sum rate in [1], can be obtained as follows

$$\begin{aligned}\tau(r,R,\lambda,n) &= \int_0^\infty \log(1+x)\mathrm{d}\mathbb{P}(SINR_r < x) \quad (18)\\ &= -\int_0^\infty \log(1+x)\mathrm{d}\left[\mathbb{P}(SINR_r > x)\right]\\ &= \int_0^\infty \frac{\mathbb{P}(SINR_r > x)}{1+x}\mathrm{d}x \quad (19)\end{aligned}$$

where the last step follows from integration by parts. Unfortunately the resulting average rate expressions do not provide simple or closed-form solutions without further assumptions. The integrals, however, can be evaluated numerically still avoiding the need for Monte Carlo simulation of the entire network.

VI. Simulations

In this section we consider a system model of the form in Fig. 2 with a single antenna base station and a Poisson field of base station interferers. We evaluate performance in a cell of radius $R_c = 300m$ with a guard region of $R_g = 150m$. The base station locations are modeled as a PPP with $\lambda = 2/\sqrt{27}R_c^2 = 4.3\ 10^{-6}$ transmitters per m^2 which corresponds to the same density as a hexagonal lattice with cells of radius R_c. The base station has $40W$ transmit power with a single antenna. Independent identically distributed complex Gaussian fading is assumed (corresponds to Rayleigh distributed channel amplitudes). The path-loss model of (2) is used with $n = 3.76$, $d_0 = 35m$, and $C = 33.88$. Log-normal shadowing is assumed with $6dB$ variance. Monte Carlo simulations are performed by simulating base station locations over a square of dimension $15R_c \times 15R_c$. The mutual information and the outage probability are estimated from their sample averages over 200 small scale fading realizations, and 500 different interferer positions. Performance is evaluated as a function of the distance from the center of the typical cell.

First we compare the performance between the proposed hybrid approach and a hexagonal layout. We consider three different guard regions for the hybrid approach: $R_g = 0$, $R_g = R_c$, and $R_g = R_c/2$. We compute the received interference power for a user at radius r from the cell center by considering that the interference lies outside a ball of radius R_c+R_g-r as illustrated in Fig. 2. We compare with a hexagonal layout of cells with either one tier of interferers (six total) or two tiers of interferers (eighteen total). We plot the ergodic rate in Fig. 4 and the probability of success in Fig. 5. Our approach with a guard region of $R_g = R_c/2$ matches the multi-tier hexagonal layout both in terms of ergodic rate and probability of success.

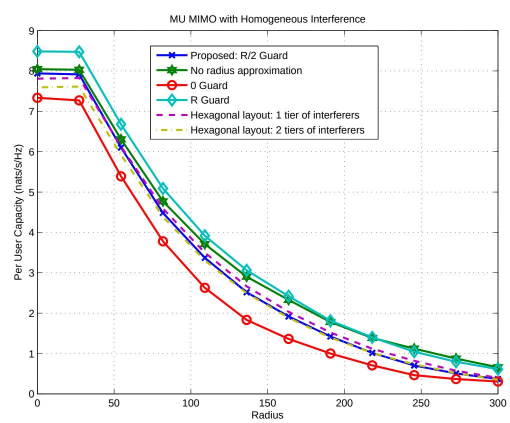

Fig. 4. Illustrates the ergodic rate as a function of radius from the cell center. The flat part at the beginning is due to the non-singular path-loss model that is constant until the reference distance of d_0. The case of no guard interval provides a lower bound on performance while a guard interval of R_c provides an upper bound on performance. The case with no radius approximation (not using the ball of radius $R_c + R_g - r$ assumption) and a guard of $R_c/2$ is also shown. Our proposed approach lower bounds the no radius approximation case, with a larger error at the cell edge. Without the radius approximation, a smaller guard region could give a better fit. The hexagonal layout with one and two tiers of interference almost exactly overlap the performance our our proposed approach with a guard of $R_c/2$. This justifies both the symmetric interference field approximation and the use of a guard interval $R_g = R_c/2$.

To illustrate the application of our model to heterogeneous networks, we consider MU MIMO at the base station with $N_t = 4$ and $U = 3$ active users with the same total power. We also consider a MU MIMO distributed antenna system (DAS) with four single antenna transmission points and the same total power. The DAS system is also modeled using a PPP but with density $4\lambda = 1.7\ 10^{-5}$. The distribution of the effective channels are derived from [19]. We also consider single cell femtocells with a single antenna, $0.1W$ transmit power, $20dB$ of indoor-outdoor penetration loss, with a density of $1.9\ 10^{-4}$ corresponding to about 45 femtocells on average

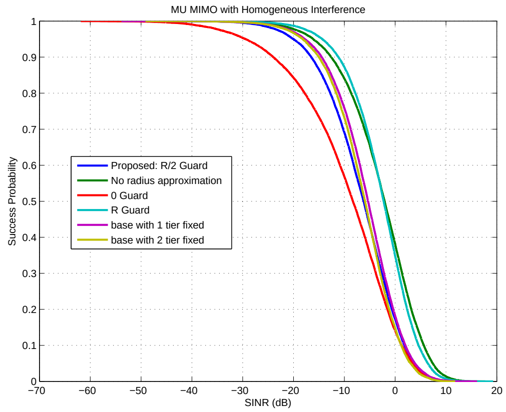

Fig. 5. Probability of success curves derived from the Kaplan-Meier estimate of the cumulative distribution function at a distance of $r = 300m$ for an SINR of $15dB$. Due to the number of points this curve is best viewed in color. The hexagonal layout with one and two tiers of interference almost exactly overlap the performance with a guard of $R_c/2$. This further justifies both the symmetric interference field approximation and the use of a guard interval $R_g = R_c/2$.

per base station. The results are illustrated in Fig. 6. The show how our model can be used to examine the performance of different kinds of infrastructure on MU MIMO and other signaling strategies.

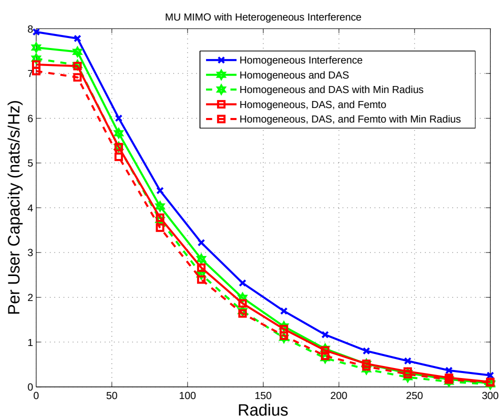

Fig. 6. The ergodic rate as a function of radius from the cell center for a single user in a MU MIMO system. The addition of distributed antennas in the interference environment reduces performance slightly at the cell center and more substantially at the cell edge. The cross-tier interference power is constant thus it is more significant at the cell center, where the co-channel interference is low. Because of the low transmit power and the large indoor-to-outdoor penetration loss, the femtocells have little impact on the macrocell user. This means that in this cell there are (on average) $45+3 = 48$ users being served in total, 3 from the macro cell and the duration from femtocells. The dashed lines show the performance with the minimum radius approximation. As expected, the predicted performance is reduced since the minimum radius considers higher levels of interference, with the benefit of simpler analytical expressions.

VII. Conclusions and Future Work

In this paper we proposed a hybrid model for determining the impact of interference in cellular systems. The key idea is to develop a model for the composite interference distribution outside a fixed typical cell, as a function of the user position in the typical cell. From a numerical perspective, our approach simplifies the simulation study of cellular systems by replacing the sum interference term with an equivalent interference random variable. Then functions like average rate and success probability can be computed through numerical integration rather than Monte Carlo simulation. Unfortunately the numerical integrals may still require a fair amount of computational power to compute. In future work, we are considering approximations of the interference distribution using Gamma and Inverse Gamma distributions motivated by [20], to facilitate simplified analysis and simulation in more complex settings.

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