The Generalized Degrees of Freedom of the Interference Relay Channel with Strong Interference

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Abstract—The interference relay channel (IRC) under strong interference is considered. A high-signal-to-noise ratio (SNR) generalized degrees of freedom (GDoF) characterization of the capacity is obtained. To this end, a new GDoF upper bound is derived based on a genie-aided approach. The achievability of the GDoF is based on cooperative interference neutralization. It turns out that the relay increases the GDoF even if the relay-destination link is weak. Moreover, in contrast to the standard interference channel, the GDoF is not a monotonically increasing function of the interference strength in the strong interference regime.

I. INTRODUCTION

Information theoretic results indicate that relays increase the achievable rate of a point-to-point system [1]. Even wireless networks, where interference caused by concurrent transmissions is the main challenging problem, benefit from the deployment of relays which provide multiplicative gains in terms of achievable rates. A multiplicative gain can be shown by comparing the generalized degrees of freedom (GDoF) of a network with and without a relay. The GDoF is an information theoretic measure which was introduced in the context of the basic interference channel by Etkin et al. in [2] and is a useful approximation for the capacity of a network in the high signal-to-noise ratio (SNR) regime. The benefit of a relay in the IC was also shown in [3] by studying the GDoF of the so-called interference relay channel (IRC), an elemental network which consists of two transmitters (TX), two receivers (RX) and a relay (see Fig. 1). The authors of [3] considered the case in which the source-relay link is weaker than the interference link. Complementary to [3], the goal of this work is to study the impact of a relay on the GDoF when the source-relay link is stronger than the interference link under the condition that the interference itself is strong. Thus, associated with the result in [3], the characterization of the GDoF for the strong interference regime is completed. By comparing the GDoF of the IRC with that of the IC, we observe an increase in the GDoF even if the relay-destination link is weak. Even more surprising, the analysis shows that in the strong interference regime the GDoF can decrease as a function of the interference strength, which is a behavior not observed in the IC. The results are interesting, given the previous results in [4] indicated that the degrees of freedom (DoF) of the IRC, a special and important case of the GDoF, is not increased at all by the use of relays (DoF = 1).

For the achievability, we use a transmission strategy which is a combination of decode-forward [1], compute-forward [5], and a strategy named “cooperative interference neutralization” (CN) which is a modified version of the strategy in [6]. While in the setup considered in [6], the destinations receive interference only from the relays, in our fully connected IRC, the destinations receive interference from both the relay and the undesired transmitter. Our CN strategy is designed to deal with both interferers. Since our IRC is fully connected, we utilize block-Markov coding [7]. The relay is causal, and therefore, it is only able to neutralize the interference from the previously decoded blocks. This constitutes yet another major difference with [6]. Moreover, [6] only considered the deterministic channel. In this work, we design the CN scheme for the Gaussian channel by using nested lattice codes [5]. These codes are used in order to enable the relay to decode the sum of codewords [5] which is then scaled and transmitted in such a way that reduces interference at both receivers.

A new upper bound on the sum capacity is derived based on a genie aided complementing existing upper bounds from [3] to fully characterize the GDoF.

The rest of the paper is organized as follows. In Section II, we introduce the notations and the Gaussian IRC. The main result of the paper is summarized in Section III. Then, in Section IV, the new upper bound is proved. In Section V, the proposed transmission scheme is motivated by considering the linear-high SNR deterministic channel model, followed by details on the relaying strategy “CN” and the achievability scheme for the Gaussian case. In Section VI, we discuss the reason of decreasing behavior of the GDoF versus interference strength by studying the transmission scheme in details. Finally, we conclude in Section VII.

II. MODEL DEFINITION

Let us first define the notations which are used in this paper. We denote a length-\(n\) sequence \((x_1, \ldots, x_n)\) by \(x^n\).

The functions \(C(x)\) and \(C^+(x)\) are defined as

\[
C(x) = \frac{1}{2}\log(1 + x), \quad C^+(x) = (C(x))^+, \quad (1)
\]
where \((x)^+ = \max\{0, x\}\). A Gaussian distribution with mean \(\mu\) and variance \(\sigma^2\) is denoted as \(\mathcal{N}(\mu, \sigma^2)\).

### A. System Model

The information theoretic model of the IRC is shown in Fig. 1. Transmitter \(i\) (TX\(_i\)), \(i \in \{1, 2\}\), has a message \(m_i\) which is a random variable uniformly distributed over the set \(\mathcal{M}_i \equiv \{1, \ldots, 2^{nR_i}\}\) for its respective receiver (RX\(_i\)). The message is encoded into a codeword \(x^n_i = f_i(m_i)\), where \(x_{ik}, k = 1, \ldots, n\), is a realization of a real valued random variable \(X_{ij}\). The transmitters must satisfy a power constraint given by

\[
\frac{1}{n} \sum_{j=1}^{n} \mathbb{E}[X_{ij}^2] \leq P. \tag{2}
\]

In time instant \(k\), the relay receives

\[
y_{rk} = h_d x_{1k} + h_c x_{2k} + z_{rk}, \tag{3}
\]

where \(h_d\) denotes the real valued channel gain of the source-relay channel. Moreover, \(z_{rk}\) represents the additive Gaussian noise at the relay with zero mean and unit variance (\(Z_c \sim \mathcal{N}(0, 1)\)). The relay is causal, which means that the transmitted symbol \(x_{rk}\) at time instant \(k\) is a function of the received signals at the relay in the previous time instants, i.e. \(x_{rk} \equiv f_r(y_{rk}^{k-1})\). The average transmit power of the relay cannot exceed \(P\). The received signals at the destinations are given by

\[
y_{jk} = h_d x_{jk} + h_c x_{1k} + h_r x_{rk} + z_{jk}, \quad j \neq l \tag{4}
\]

where \(j, l \in \{1, 2\}\), and \(h_d, h_c, h_s,\) and \(h_r\) represent the real valued channel gains of the desired, interference, source-relay, and relay-destination channels, respectively. The additive noise at the receivers is \(Z_j \sim \mathcal{N}(0, 1)\). The probability of error, achievable rates \(R_1, R_2\), capacity region \(C\) are defined in the standard Shannon sense [8]. The sum capacity is the maximum achievable sum-rate which is given by

\[
C_\Sigma = \max_{(R_1, R_2) \in C} R_\Sigma, \tag{5}
\]

where \(R_\Sigma = R_1 + R_2\). Clearly, the sum capacity of the channel depends on the channel gains.

Since the focus of the paper is on the GDoF of the IRC, we need to define the following parameters. Let \(\alpha, \beta,\) and \(\gamma\) be defined as

\[
\alpha = \frac{\log(P h_2^c)}{\log(P h_2^d)}, \quad \beta = \frac{\log(P h_2^c)}{\log(P h_2^d)}, \quad \gamma = \frac{\log(P h_2^c)(\max(\alpha, \beta) + (\gamma - \alpha)^+) + \gamma + \alpha}{\log(P h_2^d)}. \tag{6}
\]

Fig. 1: System model for the symmetric Gaussian IRC.

Fig. 2: A comparison of the GDoF of the IRC (where \(\beta = 2\) and \(\gamma = 3\)) and IC. The GDoF of the IRC is the minimum of the illustrated upper bounds.

Then, the GDoF of the IRC, \(d(\alpha, \beta, \gamma)\) is defined as

\[
d(\alpha, \beta, \gamma) = \lim_{P h_2^d \to \infty} \frac{C_\Sigma(\alpha, \beta, \gamma)}{\frac{1}{2} \log(P h_2^d)}. \tag{7}
\]

This paper studies the IRC with strong interference \(h_2^c > h_2^d\). According to (6), the strong interference regime corresponds to \(\alpha > 1\). The next section summarizes the main result of the paper.

### III. SUMMARY OF THE MAIN RESULT

In this work, we derive a new upper bound for the GDoF of the IRC which is given in Lemma 1.

**Lemma 1.** The GDoF of the IRC is upper bounded by

\[
d \leq \alpha + \beta. \tag{8}
\]

The proof of the new upper bound is given in Section IV. In addition to the new GDoF upper bound, we use some known upper bounds for the IRC which are derived in [3]. These upper bounds are restated in Lemma 2.

**Lemma 2.** The GDoF of the IRC is upper bounded by

\[
d \leq \min \{2 \max\{1, \beta\}, 2 \max\{1, \gamma\}, \max\{\alpha, \beta\} + (\gamma - \alpha)^+, \gamma + \alpha\}. \tag{9}
\]

Then, these upper bounds are compared with the achievable sum-rate given in Lemma 3, whose proof is deferred to Section V.

**Lemma 3.** Let \(R_{cn}^{(w)}\), \(R_{cf}^{(l)}\), \(R_{cm}\), and \(R_{df}\) be the rates associated with the sub-messages referred to as the with cooperative interference neutralization message, the \(l\)th compute-forward message, the common message, and the decode-forward message, respectively. A sum-rate \(R_\Sigma\) is achievable with

\[
R_\Sigma = 2 \left( \sum_{w=1}^{W} R_{cn}^{(w)} + \sum_{l=1}^{L} R_{cf}^{(l)} + R_{cm} + R_{df} \right). \tag{10}
\]
if the constraints (59)-(63), and (52)-(58) are satisfied under power constraints (51) and (65).

Using the parameters in (6) in addition to the definition of the GDoF, we convert the sum-rate in Lemma 3 into the achievable GDoF of the IRC. Finally, by comparing this achievable GDoF expression, with the upper bounds given in Lemma 1 and Lemma 2, we get the GDoF in Theorem 1. Notice that the GDoF of the IRC with $1 \leq \alpha$ and $\gamma \leq \alpha$ is characterized completely in [3]. The result for the remaining part of the strong interference regime is presented in the following Theorem.

**Theorem 1.** The GDoF of the IRC with $1 < \alpha < \gamma$ is given by

\[
d = \min\{2 \max\{1, \beta\}, \max\{\alpha, \beta\} + \gamma - \alpha, \gamma + \alpha, \alpha + \beta\}
\] (11)

In order to see the impact of the relay, we compare the derived GDoF of the IRC with the GDoF of the IC in the strong interference regime given in [2]

\[
d_{IC} = \min\{\alpha, 2\}. \quad (12)
\]

In Fig. 2, the new and the known GDoF upper bounds for the IRC and the GDoF of the IC are illustrated. As it is shown in this figure, the new upper bound is more binding than the old one for some values of $\alpha$. Therefore, the new upper bound is required in addition to the known upper bounds for characterizing the GDoF of the IRC. The minimum of the upper bounds gives us the GDoF of the IRC. Moreover, comparing the GDoF expression in Theorem 1 with (12), we conclude that the GDoF performance of the IRC is better than the IC. This increase is also obtained even if the relay-destination link is weak ($\beta < 1$) (cf. (11)).

The other important observation is the decreasing behavior of the GDoF versus $\alpha, \beta$ is $< \gamma$.

Moreover, comparing the GDoF expression in Theorem 1 with (12), we get the GDoF in Theorem 1.

Now, consider every term in (17) separately. The first term in (17) can be rewritten as

\[
I(m_1, m_2; S^n) \leq I(m_1, m_2, X^n_r; S^n) \leq nC(P h^n_{1c}). \quad (18)
\]

The second term in (17) is given by

\[
I(m_2, m_1; Y^n_1|S^n) \leq I(m_2, m_1, X^n_r; Y^n_1|S^n) \leq h(Y^n_1|S^n) - h(Y^n_1|S^n, m_1, m_2, X^n_r) \leq h(Y^n_1 - S^n - h(Y^n_1 - h_r X^n_r + Z^n, m_1, m_2, X^n_r)) \leq h(h_d X^n_2 + h_c X^n_2 + Z^n_2 - Z^n h_r X^n_r + Z^n) - h(Z^n_2) \leq nC \left(1 + P \left(\frac{h_d^2 + h_c^2}{h_c^2}\right)\right), \quad (19)
\]

where in (a), we dropped the conditioning in the first term because it does not increase the entropy. Moreover, in the second term in (a), we dropped the conditions because they are all independent from $Z^n_1$. Finally, the third term is rewritten as

\[
I(m_2; Y^n_2|S^n, m_1, Y^n_1) \leq h(Y^n_2 - h_d X^n_2 + Z^n - Z^n h_d X^n_2 + Z^n - Z^n h_c X^n_2 + Z^n_2) \leq h\left(h_d X^n_2 + Z^n - Z^n h_d X^n_2 + Z^n - Z^n h_c X^n_2 + Z^n_2\right) \leq h\left(h_d X^n_2 + Z^n_2 - Z^n h_d X^n_2 + Z^n_2 - Z^n h_c X^n_2 + Z^n_2 - Z^n\right) - h(Z^n_2) \leq h\left(h_d X^n_2 + Z^n_2 - Z^n h_d X^n_2 + Z^n_2 - Z^n h_c X^n_2 + Z^n_2 - Z^n h_c X^n_2 + Z^n_2 - Z^n\right) \leq nC \left(1 + P \left(\frac{h_d^2 + h_c^2}{h_c^2}\right)\right), \quad (20)
\]

Since conditioning does not increase entropy, we drop some conditions in the first term of (a) and (b). Moreover, we remove the conditions in the second term of (a) because they are independent from $Z^n_2$.

**IV. NEW UPPER BOUND (PROOF OF LEMMA 1)**

In this section, we prove the upper bound given in Lemma 1. To do this, we give $S^n = h_r X^n + Z^n$ as side information to both receivers, where $Z^n$ is i.i.d. $\mathcal{N}(0, 1)$, independent of all other random variables. Moreover, we give $Y^n_1$ and $m_1$ to receiver 2. Then, using Fano’s inequality, the chain rule, and the independence of $m_1$ and $m_2$, we write

\[
n(R_1 + R_2 - \epsilon_n) \leq I(m_1; Y^n_1, S^n) + I(m_2; Y^n_2, S^n, Y^n_1, m_1) \leq I(m_1; S^n) + I(m_1; Y^n_1|S^n) + I(m_2; m_1) + I(m_2; S^n|m_1) + I(m_2; Y^n_1|m_1) + I(m_2; Y^n_2|S^n, m_1, Y^n_1) \leq I(m_1, m_2; S^n) + I(m_1, m_2; Y^n_1|S^n) + I(m_2; Y^n_2|S^n, m_1, Y^n_1). \quad (21)
\]

(21) (22) (23) (24) (25) (26) (27)
In the next step, RX$_2$ decodes the first three bits of $y_2(b)$. While $x_{2,df}(b)$ is desired for RX$_2$, the other ones are required in the next decoding step for interference cancellation. The receiver decodes $x_{1,cf}(b)$ and adds it to $x_{1,cf}^{(1)}(b) + x_{2,cf}^{(1)}(b)$ to obtain the desired bit $x_{2,cf}^{(1)}(b)$. Next, the receiver removes the interference of $x_{2,cf}^{(1)}(b)$ and $x_{1,df}(b)$ from $y_2(b)$ and decodes $x_{2,cn}(b)$ which is also desired. Finally, the contribution of $x_{1,cf}(b)$ is removed from the last bit of vector $y_2(b)$ and $x_{2,cn}(b)$ is decoded. Due to the symmetry, RX$_1$ does the same decoding process. Notice that the receivers decode the CN bits successively bit by bit. This will lead to the idea of rate splitting of the CN message in the Gaussian case considered in the next subsection.

B. Cooperative interference neutralization:

Cooperative interference neutralization (CN) is a relay strategy which was introduced recently in [11], [12] and [13].

In this strategy, the transmitters and the relay transmit in such a way that the interference from the undesired transmitter is neutralized at the receiver.

We introduce rate splitting to the original CN strategy [11]. For the sake of simplicity, we discuss a CN strategy with only two splits. Consider a block of transmission $b$, where $b \in \{0, \ldots, B\}$ for some $B \in \mathbb{N}$, TX$_1$ wants to send the messages $m_1(1), \ldots, m_1(B)$ in $B \in \mathbb{N}$ blocks of transmission to RX$_1$. First, TX$_1$ splits its message $m_1(b)$ into two parts, i.e. $m_1^{(1)}(b)$ and $m_1^{(2)}(b)$, and then encodes them using nested lattice codes. TX$_1$ and TX$_2$ use the same nested-lattice codebook $(\Lambda_{f,cn}^{(w)}, \Lambda_{c,cn}^{(w)})$ with rate $R_{cn}$ and power $P_{cn}$, where $\Lambda_{c,cn}^{(w)}$ denotes the coarse lattice, $\Lambda_{f,cn}^{(w)}$ denotes the fine lattice, and $w$ is the split index ($w \in \{1, 2\}$). For more details about nested lattice-codes, the reader is referred to [5], [14] and [15]. The transmitters encode their messages into length-$n$ codewords $x_{c,cn}^{(w)}(b)$ from the nested lattice code $(\Lambda_{f,cn}^{(w)}, \Lambda_{c,cn}^{(w)})$. Then, they construct the following signals

$$x_{i,cn}^{(w),n}(b) = (x_{i,cn}^{(w)}(b) - d_{i,cn}^{(w)}) \mod \Lambda_{c,cn}^{(w)},$$

where $d_{i,cn}^{(w)}$ is $n$-dimensional random dither vector. Since the length of all sequences in the paper is $n$, we drop the superscript $n$ in the rest of the paper since it is clear from the context. The transmitted signal by TX$_1$ is given by

$$x_1(b) = \sum_{w=1}^{2} x_{1,cn}^{(w)}(b) + \sqrt{\frac{P_{cn}}{R_{cn}}} x_{1,cn}^{(w)}(b+1),$$

where $b = 1, \ldots, B-1$, and $P_{cn}$ denote the power of the future signal of the $w$th split, respectively. Notice that we need to consider an initialization block ($b = 0$) in which the transmitter sends only the future information. Moreover, in the last block $b = B$, the users send only their current information. The other user constructs the transmit signals in the same way. The relay is interested only in the modulo-sum of the future CN codewords, which is

$$(\Lambda_{c,cn}^{(w)}(b+1) + \Lambda_{c,cn}^{(w)}(b+1)) \mod \Lambda_{c,cn}^{(w)}$$
in block $b$. Let us assume that the decoding process at the relay was successful in block $b - 1$. Therefore, the modulogram of the current codewords is known at the relay at the end of block $b - 1$. The relay constructs $h_s (x_{1,cn} (w) + x_{1,cn} (b))$ from $(\lambda_1^{(w)} (b) + \lambda_2^{(w)} (b)) \mod \Lambda_{c,cn}$ as shown in [16]. Then, the relay removes it from the received signal in block $b$. Next, the relay decodes the modulo-sum of the future codewords corresponding to $w = 1$ and then for $w = 2$ successively as follows. First, sum of the signals corresponding to $w = 1$ is decoded while treating the signals $w = 2$ as noise. Then, the relay removes the interference caused by $w = 1$. Next the relay decodes the sum of the signals $w = 2$. Using the result of [17], we conclude that the relay can decode the sum of the future CN codewords successively, if the rate satisfies

\[
R_{cn}^{(w)} \leq C^+ \left( \sum_{i=w+1}^{2} \frac{P_{cn} h_{c}^2}{2 P_{cn} h_{c}^2 + 1} + \frac{1}{2} \right). \tag{39}
\]

The decoded mod-$\Lambda_{c,cn}$ sum has power $P_{cn}^{(w)}$ as the original nested-lattice code. In every block $b = 1, \ldots, B$, the relay sends

\[
x_r (b) = \sum_{w=1}^{2} h_c \left( \lambda_1^{(w)} (b) + \lambda_2^{(w)} (b) \mod \Lambda_{c,cn} \right). \tag{40}
\]

RX$_1$ wants to decode $\lambda_1^{(w)} (b)$ by performing backward decoding. Assume now that the future desired CN signal is decoded successfully and is known at the destination. Thus, RX$_1$ removes it from the received signal, and then divides the remaining signal by $h_c$ and adds the dither $d_2^{(w)}$. Then, it calculates the quantization error with respect to $\Lambda_{c,cn}$. Similar to the decoding at the relay, the destination decodes the codeword corresponding to the first split, and then after removing its interference, it decodes the codeword of the second split. The decoding of $\lambda_1^{(1)} (b)$ is as follows

\[
\begin{align*}
\frac{y_1^{(1)} + d_2}{h_c} & \mod \Lambda_{c,cn} \tag{41} \\
= & \left[ x_{2,cn}^{(1)} (b) + x_{1,cn}^{(1)} (b) + y_1^{(1)} (b) + d_2^{(1)} \right] \mod \Lambda_{c,cn} \\
& = \left( \lambda_2^{(1)} (b) - d_2^{(1)} \right) \mod \Lambda_{c,cn} \tag{42} \\
& = \left( \lambda_1^{(1)} (b) + \lambda_2^{(1)} (b) \right) \mod \Lambda_{c,cn} \tag{43} \\
& + y_1^{(1)} (b) + d_2^{(1)} \mod \Lambda_{c,cn} \tag{44} \\
& = -\lambda_1^{(1)} + y_1^{(1)} (b) \mod \Lambda_{c,cn} \tag{45}
\end{align*}
\]

where $y_1^{(1)} (b)$ is the remaining part of the received signal given in (46) at the top of the next page. In this way, RX$_1$ can decode $\lambda_1^{(1)} (b)$ successfully if the rate constraint in (47) is satisfied with $w = 1$.

\[
P_{cn}^{(w)} \leq C^+ \left( \sum_{i=w+1}^{2} \frac{P_{cn} h_{c}^2}{2 P_{cn} h_{c}^2 + 1} + \frac{1}{2} \right). \tag{47}
\]

After decoding $\lambda_1^{(1)} (b)$, the signal $y_1^{(1)} (b)$ can be reconstructed as follows

\[
\begin{align*}
\left[ -\lambda_1^{(1)} (b) + y_1^{(1)} (b) \right] \mod \Lambda_{c,cn} + \lambda_1^{(1)} (b) & \mod \Lambda_{c,cn} \\
= & y_1^{(1)} (b) \mod \Lambda_{c,cn} = y_1^{(1)} (b), \tag{48}
\end{align*}
\]

where the last equality holds with high probability for some power allocations $P_{cn}^{(i)} \geq P_{cn}^{(2)}$ [18]. By using $y_1^{(1)} (b)$, RX$_1$ decodes the second CN split with the rate constraint in (47) where $w = 2$. Then, RX$_1$ proceeds backwards till block 1.

Fig. 3: An example for linear deterministic IRC with $n_d = 2$, $n_c = 3$, $n_r = 6$, and $n_s = 5$. The scheme is shown for time slot $b$. Only RX$_2$ is shown for clarity.
the modulo-sum of the CF and CN codewords corresponding to jth split, respectively. The rate constraints for successful decoding at the relay are given by

\[ R_{cm} \leq C \left( \frac{h^2 P_{cm}}{2h^2 \left( P_{df} + P_{cf} + P_{df} + P_{df} \right) + 1} \right) \]

\[ 2R_{cm} \leq C \left( \frac{2h^2 P_{cf}}{2h^2 \left( P_{cf} + P_{cnF} + P_{cf} \right) + 1} \right) \]

These constraints on the power constraints for successful decoding are to ensure that the relay is able to decode the messages correctly.

\[ R^{(l)}_{cf} \leq C^+ \left( \frac{h^2 P^{(l)}_{cf}}{2h^2 \left( \sum_{i=l+1}^{N} P^{(i)}_{cf} + P_{cnF} + P_{df} \right) + 1} - \frac{1}{2} \right) \]

\[ R_{df} \leq C \left( \frac{h^2 P_{df}}{2h^2 P_{cf} + 1} \right), \quad 2R_{df} \leq C \left( \frac{2h^2 P_{df}}{2h^2 P_{cf} + 1} \right) \]

\[ R^{(w)}_{cf} \leq C^+ \left( \frac{h^2 P^{(w)}_{cf}}{2h^2 \sum_{i=w+1}^{W} P^{(w)}_{cf} + 1} - \frac{1}{2} \right) \]

The relay encodes the DF messages and all modulo-sum of the CF into length-n codewords \( x_{r,df} \) and \( x_{r,cf} \) using a Gaussian random codebook with powers \( P_{r,df} \) and rates \( R_r \). Moreover, \( x_{r,cf} \) is constructed as in (40). Due to the causality, the relay sends the DF, CF, and CN signals in the next transmission block as follows

\[ x_r(b) = x_{r,df}(b) + x_{r,cf}(b) - \frac{h_e}{h_r} \sum_{w=1}^{W} x_{r,cf}(w) \]

where \( b = 1, \ldots, B - 1 \). Moreover, the relay needs to satisfy the following power constraint

\[ P_{r,cf} + P_{r,df} + \frac{h^2}{h_r} \sum_{w=1}^{W} P^{(w)}_{cf} \leq P \]

\[ P_{r,cf} + P_{r,df} + \frac{h^2}{h_r} \sum_{w=1}^{W} P^{(w)}_{cf} \leq P \]

G. Decoding

First, \( RX_1 \) starts decoding at the end of the last block \( B \). It decodes the messages in the following order

\[ m^{(2)}_{1,cm}, m^{(2)}_{2,cm}, \ldots, m^{(1)}_{r,cf}, m^{(2)}_{r,df}, m^{(2)}_{2,cf}, \ldots, m^{(L)}_{1,cm} \]

Notice that, if \( h_e > h_d \), \( RX_1 \) receives the CF signal from \( TX_2 \) on a higher power level than \( x_{r,cf} \). Therefore, \( RX_1 \) needs to decode the CF message of \( TX_2 \) i.e. \( m^{(1)}_{2,cf} \) before that of the relay \( m_{r,cf} \). In the opposite case, if \( h_e < h_d \), the optimal decoding order is vice versa. Therefore, the second to \( L \)th split of CF messages are all decoded after \( m_{r,cf} \). Similar to CN, we need \( L - 1 \) splits for CF messages to perform the successive decoding. The rate constraints for successive decoding at the destination are given in (52)-(58).
\[ R_{cn} \leq C \left( \frac{h_d^2 P_{cm}}{(h_d^2 + h_c^2) [P_{sf} + P_{cn}] + h_d^2 P_{cnF} + h_c^2 [P_{r,cf} + P_{r,cn} + P_{r,df}] + 1} \right) \]

\[ 2R_{cn} \leq C \left( \frac{h_d^2 P_{cm}}{(h_d^2 + h_c^2) [P_{sf} + P_{cn}] + h_d^2 P_{cnF} + h_c^2 [P_{r,cf} + P_{r,cn} + P_{r,df}] + 1} \right) \]

\[ 2R_{df} \leq C \left( \frac{h_d^2 P_{df}}{(h_d^2 + h_c^2) [P_{sf} + P_{cn}] + h_d^2 P_{cnF} + h_c^2 [P_{r,cf} + P_{r,cn} + P_{r,df}] + 1} \right) \]

\[ R_{cf}^{(1)} \leq C \left( \frac{h_d^2 P_{cf}^{(1)}}{h_d^2 [P_{sf} + P_{cn}] + h_c^2 \left( P_{cnF} + P_{cn} + \sum_{i=2}^{L} P_{cf}^{(i)} \right) + h_c^2 [P_{r,cf} + P_{r,cn}] + 1} \right) \]

\[ R_{r,cf} \leq C \left( \frac{h_d^2 P_{r,cf}}{(h_d^2 + h_c^2) \sum_{i=2}^{L} P_{cf}^{(i)} + P_{cn} + h_d^2 P_{cnF} + h_c^2 [P_{r,cn} + 1} \right) \]

\[ R_{cf}^{(i)} \leq C \left( \frac{h_d^2 P_{cf}^{(i)}}{(h_d^2 + h_c^2) P_{cn} + h_d^2 \left( P_{cnF} + \sum_{i=1}^{L} P_{cf}^{(i)} \right) + h_d^2 \sum_{i=1}^{L} P_{cf}^{(i)} + h_c^2 P_{r,cf} + 1} \right) \]

\[ R_{cn}^{(w)} \leq C \left( \frac{h_d^2 P_{cn}^{(w)}}{h_d^2 \sum_{i=w}^{W} P_{cn}^{(w)} + h_c^2 \left( P_{cnF} + \sum_{i=w+1}^{W} P_{cn}^{(w)} \right) + h_c^2 \sum_{i=w+1}^{W} P_{cn}^{(w)} + 1} \right) \]

VI. DISCUSSION

In this section, we highlight the reason of the decrease of the GDoF versus interference strength in some cases (see Fig. 2). To this end, we study the optimal transmission schemes for different interference strength with \( 1 < \alpha < \beta \) and with \( \beta < \gamma \) and \( \beta < 2\alpha \).

First, consider the case that the capacity of the TX-relay channel is higher than twice that of the capacity of the interference channel (\( \alpha < \gamma / 2 \)). In this case, the transmission scheme is a combination of the CN and the DF strategies. From the transmission scheme, we know that the sum of current CN signals is available at the relay. Therefore, the relay is able to remove this sum before decoding the DF codeword. The relay encodes the DF codeword into \( x_{r,df} \) and the sum of the CN codewords into \( x_{r,cn} \). The received signal at RX1 which is a superposition of the signals from TX1, TX2, and the relay, is shown in Fig. 4(a). Note that the illustrations in Fig. 4 can be understood in a similar manner as in the linear deterministic model. A detailed description of such signal illustrations can be found in [11]. Since \( x_{r,df} \) is received at the destination on a higher power level than the interference signal, it is decoded first. By using backward decoding, the RX reconstructs \( x_{2,df} \) from \( x_{r,df} \) and cancels its interference. As it can be seen in Fig. 4(a), the GDoF assigned to the DF signal cannot exceed \( \beta - \alpha \). Moreover, it is shown in Fig. 4(a) that the relay CN signal \( (x_{2,cn}) \) is received on the same power level as the undesired CN signal \( (x_{2,cn}) \). Therefore, \( x_{2,cn} \) is neutralized by the superposition with \( x_{r,cn} \) and RX is able to decode its desired CN signal completely. Since in the CN strategy, we neutralize the interference signal, the GDoF of the CN signal cannot be higher than \( \alpha \) (See Fig. 4(a)).

As it is shown, the relay uses its resources for neutralizing the interference (CN) and sending extra signals (DF). Roughly speaking, while a strong relay-RX channel (\( \beta \)) is required for forwarding extra signals, a strong TX-relay channel (\( \gamma \)) is needed to provide the future signals to the relay. In this region (\( \alpha < \gamma / 2 \)), the capacity of the TX-relay channel is high enough for sending all current and future signals to the relay, which can then perform as a cognitive relay. Now, suppose that the strength of the interference channel increases. Then, the TX’s will use their strong channel to relay to provide more future signal (by exploiting the empty power levels under \( x_{1,cnF} \) and \( x_{2,cnF} \) in Fig. 4(a)). Therefore, the relay becomes more capable to neutralize the interference. While the relay will assign more power levels to neutralize the interference, the remaining power levels for extra signals (DF) will be reduced. Therefore, the GDoF of the CN signal increases while that of the DF signal decreases. Since the CN signal is desired at both users while the DF signal is only desired at RX2, the overall GDoF increases versus \( \alpha \). The increase of the GDoF stops, when \( \alpha = \gamma / 2 \).

At this point, the capacity of the TX-relay channel is exactly twice that of the interference channel. This is shown in Fig. 4(b). Now, let the interference strength increase further. Obviously, the TX’s will not be able to forward more future signal to the relay. Therefore, the relay cannot neutralize the interference completely. In order to avoid reception of the future signal \( (x_{2,cnF}) \) over the noise level (the 0 level in Fig. 4(c)) and to align the CN signals of the relay with that of the undesired transmitter, we decrease the GDoF of the CN signal. Note that reducing the GDoF of the CN can cause that the GDoF of the DF signal exceeds the GDoF of the CN signal. In this case, TX2 needs to assign some power levels over \( x_{2,cn} \) to the DF signal which is not desired at RX1. To avoid this, we need to decrease the GDoF of the DF signal as it is shown in Fig. 4(c). By reducing the GDoF of the CN and DF signals, some empty power levels appear, which
are used for adding CF signals ($x_{1,cf}$, $x_{2,cf}$, and $x_{rc,cf}$ in Fig. 4(c)). While the increase of the GDoF of the CN signals compensate the decrease of that of the CF signals, reducing the GDoF of the DF signal causes a decrease in the overall GDoF versus $\alpha$ when $\gamma/2 < \alpha < \beta$.

In summery, this analysis shows that the relay uses its resources to remove the interference by neutralization and cancellation. Moreover, the remaining resources are utilized for forwarding extra signals. When the interference gets stronger, the relay reduces the GDoF of the extra signals in order to be able to remove the interference completely. This explains the non-increasing behavior of the GDoF versus interference strength in this region.

VII. CONCLUSION

We characterized the GDoF of the IRC in the strong interference regime. To this end, we proposed a new upper bound for the GDoF of the IRC which is required in addition to some old upper bounds. Moreover, we suggested a transmission scheme which achieves the upper bound. This scheme is a combination of compute-forward, decode-forward, and cooperative interference neutralization. The achievability scheme is shown for a toy example based on the linear-deterministic model. The new relaying strategy “cooperative interference neutralization” is extended for the Gaussian channel by using nested lattice codes.

REFERENCES


Fig. 4: The received signal at RX$_1$ is illustrated for three different cases when $\beta < \gamma$. The interference gets stronger from the case (a) to (c). While in (a), the transmission scheme uses the interference to enhance the GDoF, in (c), the scheme cannot derive benefit from the increase of the interference to enhance the GDoF.