Controlled Sampling using an Energy Harvesting Sensor

Alireza Seyedi
Department of Electrical Engineering and Computer Science
University of Central Florida

***Abstract*—The problem of sampling from a remote sensor, powered by energy harvesting, is considered. The problem is formulated as a partially observable Markov decision process (POMDP), since the controller only has partial knowledge of the energy reserve at the sensor. Three policies are proposed and their performances are evaluated and compared to that of a clairvoyant policy.**

I. Introduction

The problem of sensing using a remote sensor is of interest in a wide array of sensing applications. This problem becomes significantly more interesting when the sensor node is powered by energy harvesting.

The idea of energy harvesting is that the sensor node is equipped with a harvester device which collects energy from an ambient source such as light, wind, vibrations, or electromagnetic waves. The harvested energy is stored in an energy storage device (battery or super-capacitor), and is used for sensing, signal processing and communication when needed (Figure 1). The intermittent and stochastic nature of the majority of ambient energy sources adds a new dimension to the problem: the level of available energy is no longer monotonically decreasing. Consequently, when energy harvesting is used, interesting stochastic decision and control problems arise.

It is, therefore, natural to take a controlled sensing approach when sampling using an energy harvesting sensor. In general, the control of a sensor may consist of occurrence and timing of the sensing events as well as adjustment of different sensor parameters or modalities. Conventionally, in controlled sensing, the information acquired from the sensor(s) consists of the measurements taken by the sensor. When energy harvesting is involved, however, one can consider the harvested energy as a second phenomenon affecting the sensor node. Thus, the information obtained from the sensor can include the state of energy harvesting as well as the amount of energy stored at the sensor, in addition to the conventional information (Figure 2).

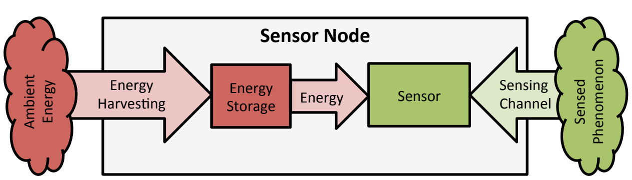

Fig. 1. Energy harvesting sensor node.

In this paper we consider the problem of controlled sampling using an energy harvesting sensor. That is, we assume that the controller neglects the values of the samples, and only tries to maximize the number of samples. We formulate the problem as a partially observable Markov decision process (POMDP) [1], and provide a number of heuristic sub-optimal policies. We evaluate the performance of these policies and compare them to that of a clairvoyant policy.

II. Related Work

There has been a large body of literature developing on energy harvesting sensor nodes, particularly during the last year. However, the vast majority of these works focus on the communications aspect and do not consider sensing or sampling, much less controlled sensing or sampling. These literature have covered many aspects of communications with energy harvesting including source and channel coding [2], power control [3], power allocation [4], scheduling [5], routing [6], multiple access [7] and throughput maximization or delay constraints [8]. Information theoretic aspects are studied in [9] and [10]. Energy harvesting has been considered in the context of broadcast [11], cellular networks [12], cognitive communications [13], and cooperative and relayed communications [14]. There have also been a few works considering non-idealities such as the imperfections of energy harvesting and storage components [15] as well as correlation between energy sources [16].

Few works consider the sensing problem in conjunction with energy harvesting. In [17]–[19] remote estimation systems with energy harvesting nodes are considered. They focus on how the transmit powers must be allocated. Thus, although they do consider a sensing objective, it is the communication that is the subject of control. To the best of our knowledge, works on controlled sensing with energy harvesting are limited

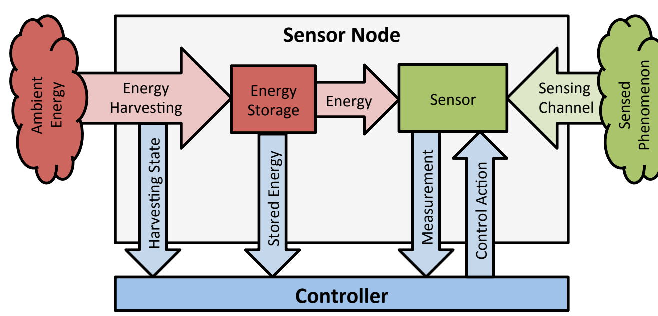

Fig. 2. Controlled sensing with an energy harvesting sensor.

to [20] and [21]. The problems of estimation and quickest change detection with energy harvesting nodes are considered in [21] and [20], respectively.

III. Problem Definition

Consider an energy harvesting sensor being remotely controlled by a controller.

Energy harvesting: Let $h_k \in \mathbb{Z}_+$ be the number of units of energy harvested at time k. We assume that h_k is iid and has a probability mass function (pmf)

$$q_i = Pr(h_k = i). \tag{1}$$

Clearly, $q_i = 0$ if $i < 0$, as the amount of harvested energy is non-negative.

Energy storage: The sensor stores the harvested energy in a storage device with capacity of B energy units. Thus, the amount of stored energy at each time is given by

$$b_{k+1} \quad = \quad \min\{\max\{b_k - c_k, 0\} + h_k, B\} \tag{2}$$

where c_k is the energy consumed at time k. We note that when the storage is full ($b_k = B$), any harvested energy is lost, and when sufficient energy is not available ($b_k < c_k$), the task requiring consumption c_k will fail, yet the storage will be emptied. Furthermore, we note that imperfections of the storage device, namely inefficiency of charge/discharge can be absorbed in the variables in (2).

Sampling operation: To request a sample from the sensor, the controller transmits a "sample request packet". We denote the decision of the controller to do so with $u_k = \{\text{R}, \text{D}\}$, where R stands for "Request" and D stands for "Do not request". Receiving a sample request packet results in consumption of c units of energy at the sensor. Upon receiving this request, the sensor will take a sample from a given physical phenomenon and transmits it back to the controller in a "response" packet, if it has sufficient energy. We assume that this process, i.e. sampling and response, costs $C - c$ units of energy. In other words, requesting a sample by the controller may result in successfully receiving a sample at the energy cost of C units to the sensor, or it may be unsuccessful and cost the sensor c energy units. In summary

$$c_k \quad = \quad \begin{cases} 0 & u_k = \text{D} \\ c & u_k = \text{R}, b_k < C \\ C & u_k = \text{R}, b_k \geq C \end{cases} . \tag{3}$$

Also we denote the reward at time k by r_k, where

$$r_k \quad = \quad \begin{cases} 0 & u_k = \text{D} \\ 0 & u_k = \text{R}, b_k < C \\ 1 & u_k = \text{R}, b_k \geq C \end{cases} , \tag{4}$$

where 0 indicates that a sample was not received (not requested or failed), and 1 indicates that a sample was received.

Objective: The objective of the controller is to maximize the number of samples taken. That is, we would like to maximize the total reward function,

$$R \quad = \quad \sum_{k=0}^{H} r_k, \tag{5}$$

where $H < \infty$ is the horizon.

Knowledge of controller: When the sensor responds, it will also report its current level of energy storage (before new harvested energy is added),

$$\bar{b}_k = \max\{b_k - c_k, 0\}. \tag{6}$$

Thus, in the case of sampling success. However, since the amount of harvested energy is random, the controller does not know b_{k+1} with certainty. Moreover, when sampling fails, the controller does not receive a response. In this case it can only infer that $0 \leq b_k \leq C - c$. Thus, the controller only has partial knowledge of the energy level at the sensor. Thus, we can only model the problem as a partially observable Markov decision process (POMDP) [1].

IV. State Space and Sufficient Statistics

A. State Space

In general, given a POMDP, all past information, may be required for an optimal policy. Let us define the information that the controller learns at each time as

$$y_k = \begin{cases} \text{N} & u_k = \text{D} \\ \text{F} & u_k = \text{R}, b_k < C \\ \bar{b}_k & u_k = \text{R}, b_k \geq C \end{cases} , \tag{7}$$

where $\bar{b}_k \in \{0, ..., B - C\}$, is the amount of remaining energy reported by the sensor in the response packet. Using this definition we can describe the total information at the controller at time k as

$$\mathcal{I}_k \quad = \quad \{y_m\}_{m=0}^{k}. \tag{8}$$

We note that the control actions u_k are implied from y_k, and thus are omitted. Clearly, in general, $\mathcal{I}_k$, takes values in a space which grows exponentially, as $(B - C + 3)^k$. Thus, including the actual energy state $b_k \in \{0, ..., B\}$, the size of the state space at time k is $(B + 1)(B - C + 3)^k$. This means that for any reasonable horizon a dynamic programming approach is computationally prohibitive.

B. Sufficient Statistics

However, it is well known that the posterior probability mass function (pmf) of the unknown variable is a sufficient statistic [22]. That is, the controller can keep track of

$$\mathbf{p}_k \quad = \quad [\ p_{k,0} \quad ... \quad p_{k,B}\ ]^T, \tag{9}$$

where $p_{k,i} = Pr(b_k = i|\mathcal{I}_k)$, instead of all the past information, $\mathcal{I}_k$. The posterior pmf, $\mathbf{p}_k$ takes values in the space $[0, 1]^{B+1}$. Thus, although the state space is infinite, the number of states is fixed, which makes the problem easier to handle.

The evolution of the posterior pmf, $\mathbf{p}_k$, can be described as follows. If the control action is $u_k = \text{D}$, then the posterior pmf must be updated to reflect the addition of the newly harvested energy. That is

$$\mathbf{p}_{k+1} \quad = \quad \mathbf{Q}\mathbf{p}_k, \tag{10}$$

where

$$\mathbf{Q} = \begin{bmatrix} q_0 & 0 & \cdots & 0 & 0 \\ q_1 & q_0 & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ q_{B-1} & q_{B-2} & \cdots & q_1 & 0 \\ q_B & q_B + q_{B-1} & \cdots & \sum_{i=0}^{B-2} q_{B-i} & 1 \end{bmatrix} \tag{11}$$

where q_i are given in (1). We note that the multiplication by $\mathbf{Q}$ convolves the pmf of the harvested energy with the posterior pmf, while also limiting it to B from above.

Alternatively, if $u_k = \text{R}$, but the sampling fails (i.e. $y_k = \text{F}$), then the controller can only infer that before the reception of the request packet, there was not sufficient energy, i.e. $b_k < C$. Since the request packet itself consumes c, thus, the remaining energy $\bar{b}_k$ must be in the range $\{0, ..., C-c-1\}$. This means that

$$\mathbf{p}_{k+1} = A_k \mathbf{QTp}_k, \quad (12)$$

where

$$\mathbf{T} = \left[\begin{array}{c|c|c} \begin{matrix} \mathbf{1}_{1\times c} \\ \mathbf{0}_{(C-c-1)\times c} \end{matrix} & \mathbf{I}_{(C-c)\times(C-c)} & \mathbf{0}_{(C-c)\times(B-C+1)} \\ \hline \multicolumn{3}{c}{\mathbf{0}_{(B-C+c+1)\times(B+1)}} \end{array} \right]$$

truncates the pmf to the allowable range and

$$A_k(\mathbf{p}_k) = \frac{1}{\sum_{i=0}^{C} p_{k,i}} \quad (13)$$

is a normalization factor to ensure that $\mathbf{p}_{k+1}$ is a proper pmf.

Finally, if $u_k = \text{R}$, and the sampling succeeds, the controller will receive $y_k = \bar{b}_k$, which means

$$\mathbf{p}_{k+1} = \mathbf{Q}\mathbf{e}_{\bar{b}_k}, \quad (14)$$

where $\mathbf{e}_i$ is the ith standard basis vector.

V. Policy Design

Although the optimal policy proves to be elusive, one may design heuristic sub-optimal, yet simple, policies based on the understanding of the information carried in the posterior pmf. We consider four different policies as described in the following:

A. Kalman Estimator (KE) Policy

Given the posterior pmf, the Kalman estimate of the unknown variable b_k is found by

$$\begin{aligned} \hat{b}_k &= E[b_k|\mathbf{p}_k] \\ &= \sum_{i=0}^{B} i p_{k,i} \end{aligned} \quad (15)$$

This policy, compares this estimate of the remaining energy level with a predetermined threshold and requests a sample, if $\hat{b}_k$ exceeds that threshold, i.e.

$$u_k = \begin{cases} \text{R} & \hat{b}_k \geq B\tau \\ \text{D} & \hat{b}_k < B\tau \end{cases}. \quad (16)$$

B. Probability of Sufficient Energy (PSE) Policy

Another intuitive approach is to decide based on the conditional probability that the sensor has sufficient energy. That is

$$u_k = \begin{cases} \text{R} & P_{\text{SE}} \geq \tau \\ \text{D} & P_{\text{SE}} < \tau \end{cases}. \quad (17)$$

where

$$P_{\text{SE}} = \sum_{i=C}^{B} p_{k,i}, \quad (18)$$

is the posterior probability that $b_k \geq C$.

C. Deterministic Approximation (DA) Policy

A much simpler approach is to neglect the stochastic nature of energy harvesting and assume that at each time slot, the system harvests energy $\bar{h}$, where $\bar{h} = E[h_k] = \sum_{i=0}^{B} i q_i$. This means that we can estimate b_k by $\check{b}_k$, where

$$\check{b}_{k+1} = \begin{cases} \min\{\check{b}_k + \bar{h}, B\} & u_k = \text{D} \\ \max\{\check{b}_k - c, 0\} & u_k = \text{R}, b_k < C \\ \bar{b}_k & u_k = \text{D}, b_k \geq C \end{cases}. \quad (19)$$

The decision is then made based on

$$u_k = \begin{cases} \text{R} & \check{b}_k \geq B\tau \\ \text{D} & \check{b}_k < B\tau \end{cases}. \quad (20)$$

D. Clairvoyant Policy

Finally, since an optimal policy is not known, we consider a clairvoyant policy as benchmark, where the exact value of b_k is known to the controller, and the decision is made based on

$$u_k = \begin{cases} \text{R} & \hat{b}_k \geq C \\ \text{D} & \hat{b}_k < C \end{cases}. \quad (21)$$

Clearly, the performance of such a policy will serve as an upper bound on that of the optimal policy.

VI. Numerical Results

We use Monte Carlo simulations to evaluate the performance of the policies described above. For our simulations, we assume that the capacity of the energy storage device is $B = 12$ units of energy. Furthermore, we assume that receiving the sample request packet costs $c = 2$ units of energy, and measurement of the sample and sending the response packet costs $C = 6$ units of energy. To demonstrate the effect of randomness on the performance of policies, we consider two different distributions for the energy harvesting. In the fist case, we assume that the pmf of the harvested energy (in energy units) is

$$q_i = Pr(h_k = i) = \begin{cases} \frac{1}{2} & i = 0 \\ \frac{1}{2} & i = 1 \\ 0 & \text{otherwise} \end{cases} \quad (22)$$

and in the second case, we assume that

$$q_i = Pr(h_k = i) = \begin{cases} \frac{6}{7} & i = 0 \\ \frac{1}{42} & 1 \leq i \leq 6 \\ 0 & \text{otherwise} \end{cases}. \quad (23)$$

We note that in both cases, an average of $\bar{h} = \frac{1}{2}$ units of energy is harvested in every time unit, which ensures a proper comparison between the two cases. However, the variance in the first case is $\sigma_h^2 = \frac{1}{2}$, where as in the second case, it is $\sigma_h^2 = \frac{13}{6} = 2.1667$.

Given these parameters, we simulate the system for a given threshold τ and horizon H. Figure 3, for instance, depicts the normalized average number of samples as a function of the threshold τ for the above policies assuming a horizon of $H = 50$. We assume that the energy storage device is empty at the beginning ($b_0 = 0$). We average the number of samples taken over $10,000$ repetitions. For easier interpretation of the results, we normalize the number of samples by $B\bar{h}/C$, which

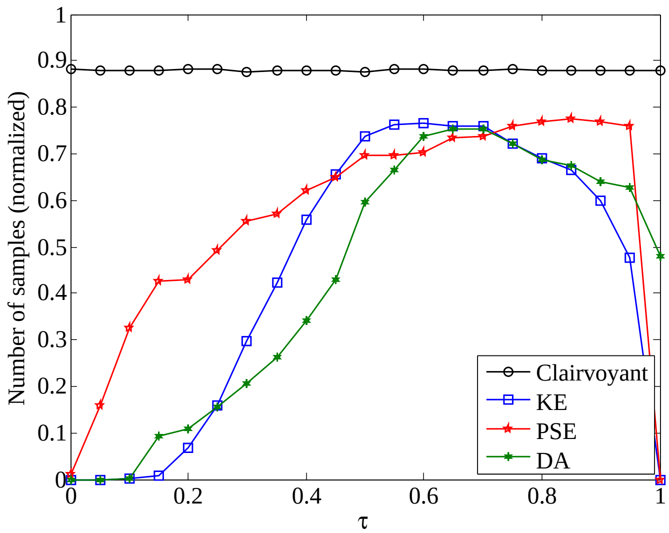

Fig. 3. Normalized number of samples versus threshold τ for horizon $H = 50$, assuming harvesting distribution (22).

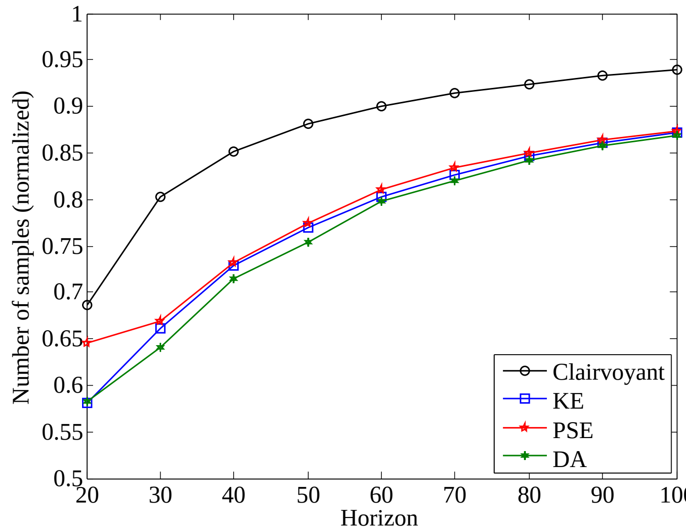

Fig. 4. Normalized number of samples versus horizon, H, assuming harvesting distribution (22).

is the number of samples taken with deterministic harvesting ($\bar{h}$ units per time step), full information, and long horizons.

From results such as those presented in Figure 3 we will choose the optimal values of threshold τ for each policy and horizon. We note that we have used a resolution of 0.05 for τ. Figure 4 depicts the obtained results for the harvesting distribution given in (22). As expected, we observe that the clairvoyant policy performs quite well. In fact, since the controller has perfect information, no energy is lost due to bad decisions. Similarly, no energy is lost due to the fullness of the energy storage device. Thus, the only reason that the normalized number of samples os less than one is that some energy is remained in the storage device, when the horizon is reached. As we can see, this gap narrows as horizon increases. The KE, PSE and DA policies perform quite similarly, with a slight advantage for the PSE policy. All of these policies perform at $80\% - 95\%$ of that of the GA policy.

Figure 5 presents similar results, but assuming (23) as the distribution of the harvested energy. Compared to Figure 4 We can see that as expected, as the variance of the harvested energy increases, the performance of KE, PSE and DA policies fall. We also note that as expected, the difference in performance of these policies is more pronounced. In particular, the DA policy is now performs noticeably worse than KE and PSE policies. This is of course expected as the level of randomness has risen.

Overall, we observe that the heuristic policies perform more-or-less similarly. From this, we conjecture that the performance of an optimal policy will be closer to the performance of these policies, rather than close to the upper bounding clairvoyant policy.

VII. Conclusion and Discussion

This paper, for the first time, considers the problem of controlled sampling from an energy harvesting sensor. The problem is formulated as a partially observed Markov decision

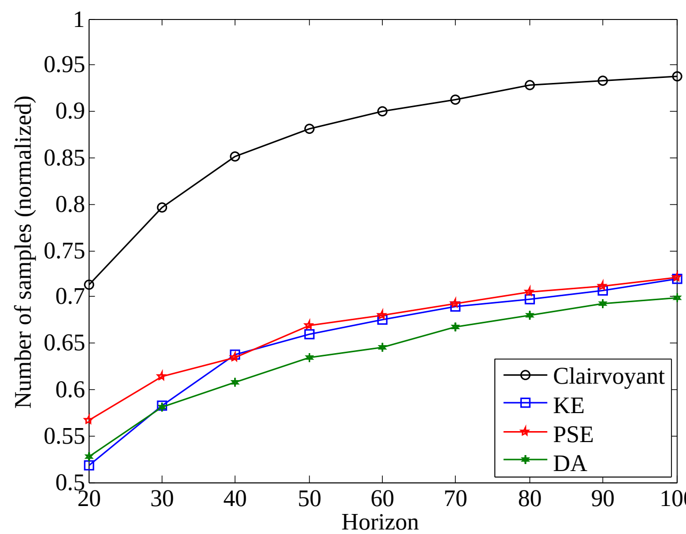

Fig. 5. Normalized number of samples versus horizon, H, assuming harvesting distribution (23).

process (POMDP). A number of sub-optimal policies are proposed and their performances are studied though simulations, and compared to that of a clairvoyant policy. We conjecture that the performance of the optimal policy is closer to those of the proposed policies, than to that of the clairvoyant policy, which provides an upper bound.

The extensions of this problem, in terms of extending the objective function in order to regulate the sample rate, as well as to the case of multiple sensors are quite interesting and are the subject of our ongoing work.

References

[1] M. L. Puterman, *Markov Decision Processes: Discrete Stochastic Dynamic Programming*. John Wiley & Sons, 2009.

[2] P. Castiglione, O. Simeone, E. Erkip, and T. Zemen, "Energy Management Policies for Energy-Neutral Source-Channel Coding," *IEEE Transactions on Communications*, vol. 60, no. 9, pp. 2668–2678, Sep. 2012.

[3] A. Seyedi and B. Sikdar, "Energy Efficient Transmission Strategies for Body Sensor Networks with Energy Harvesting," *IEEE Transactions on Communications*, vol. 58, no. 7, pp. 2116–2126, Jul. 2010.

[4] K. Tutuncuoglu and A. Yener, "Sum-rate optimal power policies for energy harvesting transmitters in an interference channel," *Journal of Communications and Networks*, vol. 14, no. 2, pp. 151–161, Apr. 2012.

[5] J. Yang and S. Ulukus, "Optimal Packet Scheduling in an Energy Harvesting Communication System," *IEEE Transactions on Communications*, vol. 60, no. 1, pp. 220–230, Jan. 2012.

[6] S. Sarkar, M. H. R. Khouzani, and K. Kar, "Optimal Routing and Scheduling in Multihop Wireless Renewable Energy Networks," *IEEE Transactions on Automatic Control*, vol. 58, no. 7, pp. 1792–1798, Jul. 2013.

[7] F. Iannello, O. Simeone, and U. Spagnolini, "Medium Access Control Protocols for Wireless Sensor Networks with Energy Harvesting," *IEEE Transactions on Communications*, vol. 60, no. 5, pp. 1381–1389, May 2012.

[8] V. Sharma, U. Mukherji, V. Joseph, and S. Gupta, "Optimal energy management policies for energy harvesting sensor nodes," *IEEE Transactions on Wireless Communications*, vol. 9, no. 4, pp. 1326–1336, Apr. 2010.

[9] O. Ozel and S. Ulukus, "Achieving AWGN Capacity Under Stochastic Energy Harvesting," *IEEE Transactions on Information Theory*, vol. 58, no. 10, pp. 6471–6483, Oct. 2012.

[10] K. Huang, "Spatial Throughput of Mobile Ad Hoc Networks Powered by Energy Harvesting," *IEEE Transactions on Information Theory*, vol. 59, no. 11, pp. 7597–7612, Nov. 2013.

[11] J. Yang, O. Ozel, and S. Ulukus, "Broadcasting with an Energy Harvesting Rechargeable Transmitter," *IEEE Transactions on Wireless Communications*, vol. 11, no. 2, pp. 571–583, Feb. 2012.

[12] D. W. K. Ng, E. S. Lo, and R. Schober, "Energy-Efficient Resource Allocation in OFDMA Systems with Hybrid Energy Harvesting Base Station," *IEEE Transactions on Wireless Communications*, vol. 12, no. 7, pp. 3412–3427, Jul. 2013.

[13] N. Pappas, A. Ephremides, and A. Traganitis, "Optimal utilization of a cognitive shared channel with a rechargeable primary source node," *Journal of Communications and Networks*, vol. 14, no. 2, pp. 162–168, Apr. 2012.

[14] Chuan Huang, Rui Zhang, and Shuguang Cui, "Throughput Maximization for the Gaussian Relay Channel with Energy Harvesting Constraints," *IEEE Journal on Selected Areas in Communications*, vol. 31, no. 8, pp. 1469–1479, Aug. 2013.

[15] B. Devillers and D. Gunduz, "A general framework for the optimization of energy harvesting communication systems with battery imperfections," *Journal of Communications and Networks*, vol. 14, no. 2, pp. 130–139, Apr. 2012.

[16] N. Michelusi, K. Stamatiou, and M. Zorzi, "Transmission Policies for Energy Harvesting Sensors with Time-Correlated Energy Supply," *IEEE Transactions on Communications*, vol. 61, no. 7, pp. 2988–3001, Jul. 2013.

[17] Y. Zhao, B. Chen, and R. Zhang, "Optimal power allocation for an energy harvesting estimation system," in *2013 IEEE International Conference on Acoustics, Speech and Signal Processing*. IEEE, May 2013, pp. 4549–4553.

[18] C. Huang, Y. Zhou, T. Jiang, P. Zhang, and S. Cui, "Power allocation for joint estimation with energy harvesting constraints," in *2013 IEEE International Conference on Acoustics, Speech and Signal Processing*. IEEE, May 2013, pp. 4804–4808.

[19] A. Limmanee, S. Dey, and A. Ahlen, "Distortion minimization via multiple sensors under energy harvesting constraints," in *2013 IEEE 14th Workshop on Signal Processing Advances in Wireless Communications (SPAWC)*. IEEE, Jun. 2013, pp. 225–229.

[20] J. Geng and L. Lai, "Non-Bayesian Quickest Change Detection With Stochastic Sample Right Constraints," *IEEE Transactions on Signal Processing*, vol. 61, no. 20, pp. 5090–5102, Oct. 2013.

[21] A. Nayyar, T. Basar, D. Teneketzis, and V. V. Veeravalli, "Optimal Strategies for Communication and Remote Estimation With an Energy Harvesting Sensor," *IEEE Transactions on Automatic Control*, vol. 58, no. 9, pp. 2246–2260, Sep. 2013.

[22] C. Striebel, "Sufficient statistics in the optimum control of stochastic systems," *Journal of Mathematical Analysis and Applications*, 1965.