Burst or random error correction based on Fire and BCH codes

Wei Zhou, Shu Lin, Khaled Abdel-Ghaffar
Department of Electrical and Computer Engineering
University of California, Davis, CA 95616, USA
Email: {weizhou, shulin, ghaffar}@ucdavis.edu

***Abstract*—A class of codes obtained by combining Fire codes, which are burst correcting codes, with BCH codes, which are random error correcting codes, is proposed. The proposed codes are subcodes of both Fire codes and BCH codes. Lower bounds on the burst error-correcting capabilities of the proposed codes are derived. The codes can be used over a compound channel that causes burst errors or random errors.**

I. Introduction

In 1959, Fire [5] presented a construction of cyclic codes generated by a product of a binomial and a primitive polynomial. The binomial has the form $x^c + 1$ for some positive odd integer c. The primitive polynomial generates a cyclic Hamming code that can correct a single error. The length of the Fire code is the least common multiple (LCM) of c and the length of the Hamming code. If the degree of the primitive polynomial satisfies certain conditions, then the Fire code can correct a single burst of length up to $(c+1)/2$. In 1968, Hsu, Kasami, and Chien [9] presented an interesting modification of Fire codes. They replaced the primitive polynomial generating the Hamming code with a polynomial generating a code that can correct more than one random error. Their motivation was to develop codes that can work over the compound burst/random error channel, i.e., a channel that causes either a burst of errors or random errors to the transmitted codeword and neither the encoder nor the decoder knows, a priori, the type of errors caused by the channel. The codes Hsu, Kasami, and Chien presented are not cyclic but rather shortened cyclic codes of length equal to the length of the random error correcting code used in the construction.

Let $\mathbf{v} = (v_0, v_1, \ldots, v_{n-1})$ be a binary sequence of length n transmitted over a channel and let $\mathbf{r} = (r_1, r_2, \ldots, r_{n-1})$ be the received sequence. Then $\mathbf{e} = \mathbf{r} - \mathbf{v} = (e_0, e_1, \ldots, e_{n-1})$ is the error sequence caused by the channel. If the indices of the 1's in $\mathbf{e}$ are confined within b consecutive integers, $i, i+1, \ldots, i+b-1$, then we say that the channel caused a burst of length up to b. If e_i and e_{i+b-1} are both equal to 1, then we say that the burst has starting position i and length b. If the indices of the 1's in $\mathbf{e}$, reduced modulo n to be in the set $\{0, 1, \ldots, n-1\}$, are confined within b consecutive integers, then $\mathbf{e}$ is a cyclic burst. Thus, $\mathbf{e} = (11010000000010)$ forms a cyclic burst of length six starting at position 12. Such a burst of errors, affecting the beginning and the end of a transmitted sequence, is called an end-around burst. A code C is said to have burst error-correcting capability of b if it can correct all bursts, not necessarily cyclic, of length up to b.

A channel may also cause random errors rather than a burst of errors. If the number of 1's in $\mathbf{e}$ is t, i.e., $\mathbf{e}$ has weight t, then we say that the channel caused t random errors. A code C is said to have random error-correcting capability of t if it can correct all random errors of weight t or less.

Throughout this paper, we will only consider binary codes although generalizations to codes over larger finite fields are possible. For convenience, we represent a sequence $\mathbf{e} = (e_0, e_1, \ldots, e_{n-1})$ by a polynomial $\mathbf{e}(x) = \sum_{i=0}^{n-1} e_i x^i$. For brevity, we identify the sequence $\mathbf{e}$ with its polynomial $e(x)$ and say that $e(x)$ has weight t, denoted by $w(e(x)) = t$, if the sequence $\mathbf{e}$ itself is of weight t, i.e., $\mathbf{e}(x)$ has exactly t nonzero terms. We also say that $e(x)$ is a burst of length b if the sequence $\mathbf{e}$ is a burst of length b. In this case, we have $e(x) = x^i(e_i + e_{i+1}x + \cdots + e_{i+b-1}x^{b-1})$, i.e., a monomial multiplied by a polynomial of degree $b-1$. We say that $e(x)$ is a cyclic burst if $\mathbf{e}$ is a cyclic burst. In this case, $e(x)$ is congruent modulo $x^n + 1$ to a product of a monomial multiplied by a polynomial of degree $b-1$. If $\mathbf{e}$ is a codeword, then we say that $e(x)$ is a code polynomial.

In this paper, we follow the work in [9] but restrict ourselves to cyclic codes. An advantage of cyclic codes over non-cyclic codes, including shortened cyclic codes, is the ability of cyclic codes to correct end-around bursts. Suppose that $\ldots, \mathbf{v}_{i-1}, \mathbf{v}_i, \mathbf{v}_{i+1}, \ldots$ is a sequence of transmitted codewords. If the channel causes a burst of errors affecting both $\mathbf{v}_{i-1}$ and $\mathbf{v}_i$ and another burst of errors affecting both $\mathbf{v}_i$ and $\mathbf{v}_{i+1}$, then the two bursts affecting $\mathbf{v}_i$ can be considered as a single end-around burst which may be correctable by a code capable of correcting such bursts. By modifying the construction in [9], we construct a class of cyclic codes that generalize Fire codes. However, we show that a code in this class of length larger than that of the random error-correcting code has poor random error-correcting capability regardless of that of the random error-correcting code. Hence, for the compound burst/random error channel, we consider cyclic codes in the proposed class of length equal to that of the random error-correcting code.

The paper is organized as follows. In Section II, we propose a class of cyclic codes that generalize Fire codes. The use of the proposed codes over the compound channel is considered in Section III. The paper is concluded in Section IV.

II. Generalization of Fire Codes

First, we present the construction of the well-known Fire codes [5].

Construction 1. *(Fire Codes) Let $g_0(x)$ be an irreducible polynomial of degree m_0 and period n_0 that does not divide x^c+1, where c is a positive odd integer. Then, $g(x)=(x^c+1)g_0(x)$ generates a Fire code, which is cyclic, of length $n=\mathrm{LCM}\{c,n_0\}$.*

Fire codes are capable of correcting bursts of errors as shown next.

Theorem 1. *The Fire code has burst error-correcting capability of* $\min\{m_0,(c+1)/2\}$.

In particular, if $c=2b-1$ and $m_0\geq b$, then the code can correct every single burst of length equal to or less than b.

The polynomial $g_0(x)$ in the construction of Fire codes is an irreducible polynomial of period n_0 and, hence, generates a code of length n_0 that can correct a single random error. By replacing $g_0(x)$ with the generator polynomial of any random error-correcting code, Hsu, Kasami, and Chien [9] presented the following construction of codes, which we call *shortened cyclic HKC codes*.

Construction 2. *(Shortened Cyclic HKC Codes) Let $g_0(x)$ be the generator polynomial of a cyclic code C_0 of length n and minimum distance $\geq 2t_0+1$. Let m_0 be the sum of the degrees of those irreducible factors of $g_0(x)$ whose periods are equal to n. Then, the code, C, obtained by shortening the cyclic code generated by $(x^c+1)g_0(x)$ to length n is a shortened cyclic HKC code.*

The following result shows that shortened cyclic HKC codes have burst error-correcting capability.

Theorem 2. *The shortened cyclic HKC code has burst error-correcting capability of* $\min\{m_0,(2c+3t_0-2)/4,c-1\}$.*

We notice that the class of shortened cyclic HKC codes is an extension of the class of Fire codes. However, by setting $t_0=1$, it is clear that Fire codes is not a special case of shortened cyclic HKC codes. First, Fire codes, which are cyclic, have lengths equal to the least common multiple of c and the period of $g_0(x)$ while the lengths of the shortened cyclic HKC codes, which are not cyclic if c is not a factor of n, are equal to the period of $g_0(x)$. Second, with the choice $c=2b-1$ and $m_0\geq b$, the burst error-correcting capability of the shortened cyclic HKC codes as stated in Theorem 2 is $b-1$ while that of Fire codes is b as stated in Theorem 1. However, by a simple modification of the construction of shortened cyclic HKC codes, we can obtain the following construction of cyclic codes, which we call *cyclic HKC codes*, that include Fire codes as a special case.

Construction 3. *(Cyclic HKC Codes) Let $g_0(x)$ be the generator polynomial of a cyclic code C_0 of length n_0 and minimum distance $\geq 2t_0+1$. Let m_0 be the sum of the degrees of those irreducible factors of $g_0(x)$ whose periods are equal to n_0. Then, the code, C, generated by $g(x)=\mathrm{LCM}\{x^c+1,g_0(x)\}$ is a cyclic HKC code of length $n=\mathrm{LCM}\{c,n_0\}$.*

*The statement of the theorem in [9] does not list $c-1$ in the argument of the minimum function although this is explicit in the proof.

Similar to their closely related shortened cyclic HKC codes, the cyclic HKC codes have also burst error-correcting capability as stated next. The proof of the theorem, which very much follows that of Theorem 2 in [9], is placed in the appendix.

Theorem 3. *The cyclic HKC code has burst error-correcting capability of*

$$\begin{array}{ll} \min\{m_0,(c+t_0)/2,c-1\} & \text{for } 1\leq t_0\leq 2,\\ \min\{m_0,(2c+3t_0-2)/4,c-1\} & \text{for } t_0\geq 3. \end{array}$$

Notice that setting $t_0=1$ in the construction of cyclic HKC and Theorem 3 gives the construction of Fire codes and Theorem 1, respectively.

The following result shows that if $n>n_0$, then no matter how large m_0 and t_0 are, the burst error-correcting capability of the cyclic HKC code C in Construction 3 is less than c and its random error-correcting capability is one.

Theorem 4. *If $n>n_0$, then the cyclic HKC code C has burst error-correcting capability less than c and random error-correcting capability of one.*

Proof: Let

$$v(x)=\sum_{i=0}^{c-1}x^i+x^{n_0}\sum_{i=0}^{c-1}x^i \pmod{x^n+1}.$$

Then, $v(x)$ is a nonzero code polynomial, which is the sum of two bursts of length c. We have

$$\sum_{i=0}^{c-1}x^i+x^{n_0}\sum_{i=0}^{c-1}x^i=\frac{(x^{n_0}+1)(x^c+1)}{x+1},$$

which is divisible by both x^c+1 and $g_0(x)$, and, hence, by $\mathrm{LCM}\{(x^c+1),g_0(x)\}$. Thus, $v(x)$ is a codeword in C. Since $v(x)$ is a sum of two bursts of length at most c, the burst error-correcting capability of C is less than c. We also notice that $(x+1)v(x)=1+x^c+x^{n_0}+x^{n_0+c}$ is a codeword in C of weight four. Hence, C has random error-correcting capability of one. ∎

Because of Theorem 4, we only consider in the following cyclic HKC codes of length $n=n_0$, which is the case if and only if n_0 is a multiple of c. In this case, the burst error-correcting capability of the code need not be less than c as dictated by Theorem 4. Actually, for shortened codes, the burst error-correcting capability can exceed c and, consequently, the guaranteed burst error-correcting capability specified in Theorem 2. Notice that Theorems 2 and 3 indicate that increasing the minimum distance of the code C_0 may increase the burst error-correcting capability of C. However, this is not the best way to achieve a larger burst error-correcting capability as it is more efficient, in terms of code rate, to keep the minimum distance of C_0 equal to three and increase the value of c. Therefore, what is interesting in the shortened cyclic HKC codes and cyclic HKC codes is not the use of the these codes over a channel causing bursts but rather over compound channels that either cause bursts or random errors as explained in the next section.

Before proceeding to the next section, let us mention the following two well-known bounds on the burst error-correcting capabilities of codes. Let C be a linear code of length n and dimension $n-r$ which can correct bursts of length b or less. Then, b is upper bounded by the Reiger bound [15] as $b \le \lfloor r/2 \rfloor$. If the code is cyclic, then b is also upper bounded by the Abramson bound [2], [12] $2^r \ge n2^{b-1}+1$, i.e., $b \le r - \lceil \log_2(n+1) \rceil + 1$. Codes satisfying the Abramson bound with equality were studied in [1].

Let C_0 be a binary BCH code of length n_0 and distance $\ge 2t_0+1$. Let m be the multiplicative order of 2 modulo n_0, i.e., m is the smallest positive integer for which 2^m-1 is divisible by n_0. Then, C_0 has dimension at least $n_0 - mt_0$ [11]. We conclude that the burst error-correcting capability, b, of the cyclic HKC code C of length $n = n_0$, which then has dimension at least $n-c-mt_0$, is upper bounded by the Reiger bound as $b \le \lfloor (c+mt_0)/2 \rfloor$ and by the Abramson bound as $b \le c + mt_0 - \lceil \log_2(n+1) \rceil + 1$, which reduces to $b \le c + m(t_0-1)+1$ in case $n+1$ is a power of two.

III. Coding for the Compound Burst/Random Error Channel

A compound channel consists of a set of channels with the same input and output alphabets. All transmissions use one of the channels in the set. Both encoder and decoder know the set of channels but do not know which channel is used for transmission. Compound channels and their capacities were introduced and studied in [3], [4], [16]. Explicit constructions of codes were proposed in [9], [6], [7], [8] in the case in which the compound channel consists of two channels: one causing bursts of errors and the other causing random errors. The codes constructed in these papers are shortened cyclic codes.

Consider a compound channel consisting of a burst error channel and a random error channel. For this compound channel, we are interested in a code, C, of length n and dimension $n-r$ that can correct a single burst of length up to b_{comp} or up to t_{comp} random errors. Such a code is said to be $(b_{\text{comp}}, t_{\text{comp}})$-compound error-correcting. We assume that $b_{\text{comp}} > t_{\text{comp}}$ otherwise the compound channel can be reduced to a single channel causing up to t_{comp} random errors.

Necessary and sufficient conditions for the linear code C to be $(b_{\text{comp}}, t_{\text{comp}})$-compound error-correcting are presented in [9]. These conditions can be stated as follows:

1) No nonzero codeword in C is a sum of two bursts of length up to b_{comp};
2) no nonzero codeword in C has weight less than $2t_{\text{comp}}+1$; and
3) no nonzero codeword in C is a sum of a burst of length up to b_{comp} and a vector of weight up to t_{comp}.

The first condition is equivalent to saying that the code can correct a burst of length up to b_{comp} if the channel causes a burst of that length or less. The second condition is equivalent to saying that the code can correct up to t_{comp} random errors if the channel causes up to that many random errors. The third condition is equivalent to saying that no two distinct codewords, one suffers from a burst of length up to b_{comp}, and the other suffers from up to t_{comp} random errors, can lead to the same word. Posner [14] proposed a simple test to check whether or not condition 3) is met in case $t_{\text{comp}} = 1$.

A necessary and sufficient condition for 1), 2), and 3) to hold is that all distinct bursts of weight up to b_{comp} and vectors of weight up to t_{comp} are in different cosets. Based on this, Hsu, Kasami, and Chien derived the following bound, which combines the Abramson and the Hamming bounds [11]:

$$2^r \ge n2^{b_{\text{comp}}-1} + \sum_{i=0}^{t_{\text{comp}}} \binom{n}{i} - n \sum_{i=1}^{t_{\text{comp}}} \binom{b_{\text{comp}}-1}{i-1}.$$

Clearly, if C is b-burst error-correcting, t-random error-correcting, and $(b_{\text{comp}}, t_{\text{comp}})$-compound error-correcting, and b and t are the largest numbers for which this is true, then $b_{\text{comp}} \le b$ and $t_{\text{comp}} \le t$. Typically, we are interested in the maximum possible b_{comp} for which the code C is (b_{comp}, t)-compound error-correcting. This is obtained by checking the largest value not greater than b for which condition 3) holds.

Following [9], a cyclic or a shortened cyclic $(b_{\text{comp}}, t_{\text{comp}})$-compound error-correcting code can be decoded as shown in Fig. 1. The receiver, which does not know whether the channel causing bursts or the channel causing random errors was used, decodes the received polynomial $r(x)$ using two decoders: one to correct a burst of length up to b, which can be done using burst trapping, and the other for correcting up to t random errors. From condition 1) above, it follows that the first decoder produces an output, which is the transmitted codeword, if the channel used for transmission causes a burst of length up to b. From condition 2), it follows that the second decoder produces an output, which is the transmitted codeword, if the channel used for transmission causes up to t random errors. The third condition guarantees that in either case only one of the two decoders succeeds in producing an output or that both produce identical outputs. This precludes the case in which the two decoders produce different codewords as outputs.

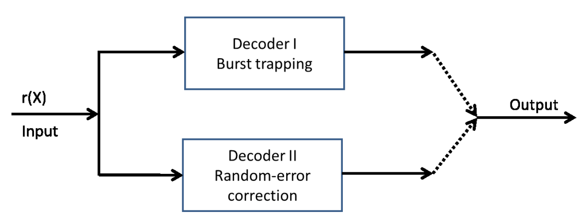

Fig. 1. Decoder for the compound channel

Hsu, Kasami, and Chien in [9] presented and proved the following theorem for the shortened cyclic HKC codes as given in Construction 2.

Theorem 5. *Let C be a shortened cyclic HKC code. If C is a b-burst error correcting code and t-random error correcting,* *then it is a* (b_{comp}, t)*-compound error correcting code where* $b_{\text{comp}} = \min\{b, c\}$.

Based on Theorem 5, Hsu, Kasami, and Chien constructed a class of shortened cyclic HKC codes, denoted by Class K, for the compound channel where C_0 is a BCH code. The codes constructed are not cyclic but rather shortened cyclic codes. By checking the proof of Theorem 5, which appears as Theorem 1 in [9], it follows that the theorem holds also for the cyclic HKC codes presented in Construction 3.

In Table I, we provide some cyclic HKC codes for the compound channel. Each code, C, is given by Construction 3, where c is a factor of $n_0 = n$ and the code C_0 is a BCH code. In particular, C is a subcode of a Fire code and a BCH code of equal length n. Each row in the table corresponds to a code C. The first column gives the length n and the dimension k of C. The second column gives the value of c and the third column gives roots of the generator polynomial $g_0(x)$ in terms of powers of an element β of order n. Since the roots are consecutive powers of β starting with β, the code C_0 generated by $g_0(x)$ is a narrow-sense BCH code [11]. The polynomial $g_0(x)$ is the product of the minimal polynomials over GF(2) of these roots and is given in octal notation in the fourth column. For example, 3525 represents the polynomial $x^{10} + x^9 + x^8 + x^6 + x^4 + x^2 + 1$. The fifth column gives the burst error-correcting capability, b, of the code C and the sixth column gives the BCH lower bound [11] on the random error-correcting capability, t, of the code C, i.e., the maximum number of consecutive powers of β that are roots of the generator LCM$\{x^c+1, g_0(x)\}$ of C is $2t$ or $2t+1$. Although t is a lower bound on the number of random errors correctable by the code, decoding algorithms, such as the Berlekamp-Massey algorithm and Euclid's algorithm, can efficiently decode up to the BCH bound [10]. The seventh column gives the (b_{comp}, t) compound error-correcting capability of the code C.

IV. Conclusion

In this paper, we considered cyclic codes for the compound burst/random error channel. The codes are obtained by combining Fire and BCH codes. We presented results on the burst error-correcting capabilities of some of the constructed codes. It is worth mentioning that the actual maximum burst error-correcting capabilities of the codes exceed those given in Theorems 3 and 5. Therefore, the burst error-correcting capabilities of the cyclic HKC codes need to be studied further.

Appendix

Proof of Theorem 3. Since $g_0(x)$ divides $x^{n_0}-1$ and both $x^{n_0}-1$ and x^c-1 are factors of x^n-1, it follows that $g(x)$ divides x^n-1 and C is a code of length n. To prove the burst error-correcting capability of C we assume that the code has a nonzero codeword which is the sum of two bursts of length at most $b < c$. Since the code is cyclic, we can assume that the starting position of one of the bursts is 0, i.e., $e(x)+e'(x)$ is a code polynomial where $e(x) \equiv \sum_{i=0}^{b-1} e_i x^i \pmod{x^n-1}$ and $e'(x) \equiv \sum_{i=i_0}^{i_0+b-1} e'_i x^i \pmod{x^n-1}$. Hence,

$$e(x) \equiv e'(x) \pmod{g(x)}, \tag{1}$$

which implies that

$$e(x) \equiv e'(x) \pmod{x^c+1} \tag{2}$$

and

$$e(x) \equiv e'(x) \pmod{g_0(x)}. \tag{3}$$

From (2), we have

$$e'(x) \equiv \sum_{i=0}^{b-1} e_i x^{i+cl_i} \pmod{x^n+1}, \tag{4}$$

where

$$i_0 \le i + cl_i < i_0 + b \tag{5}$$

for $0 \le i < b$. As $b < c$, then $l_i = l_0$ or $l_0 - 1$. If $l_i = l_0$ for all i, $0 \le i < b$, then $e'(x) = x^{cl_0} e(x) \pmod{x^n+1}$. Since $e(x)+e'(x)$ is a nonzero code polynomial, cl_0 is not divisible by n and, therefore, is not divisible by n_0 either. Hence, no irreducible factor of $g_0(x)$ of period n_0 divides $x^{cl_0}+1$. From (3),

$$(x^{cl_0}+1)e(x) \equiv 0 \pmod{g_0(x)}.$$

The product of all irreducible factors of $g_0(x)$ of period n_0 divides $e(x)$. Since this product is of degree m_0 while $e(x)$ is of degree less than b, it follows that $b \ge m_0 + 1$. Therefore, $l_1, \ldots, l_{b-1}$ are not all equal to l_0 as at least one of them equals $l_0 - 1$. Let j_0 be the largest index j for which $e_j = 1$ and $l_j = l_0$ and j_1 be the smallest index j for which $e_j = 1$ and $l_j = l_0 - 1$. From (5), we have $j_0 + cl_0 < i_0 + b$ and $i_0 \le j_1 + c(l_0 - 1)$. From these two inequalities, we get

$$j_1 - j_0 \ge c - b + 1. \tag{6}$$

Therefore, $e_j = 0$ for $j_0 < j < j_1$ and the weight of $e(x)$ is at most $b - (j_1 - j_0 - 1) \le 2b - c$. From (4), the weight of $e'(x)$, which is the same as that of $e(x)$, is also at most $2b-c$. As $e(x) + e'(x) \pmod{x^n+1}$ is a code polynomial in the code generated by $g(x) = \text{LCM}\{x^c+1, g_0(x)\}$ and, hence, of even weight not less than $2t_0+1$, we have

$$2(2b-c) \ge w(e(x)) + w(e'(x)) \ge w(e(x)+e'(x)) \ge 2t_0+2,$$

which gives

$$b \ge \frac{c+t_0+1}{2}. \tag{7}$$

This proves the theorem in case $t_0 = 1$ or 2. For $t_0 \ge 3$, the proof continues as given for Theorem 2 in [9] where t is used to denote t_0.

Acknowledgment

This work was supported in part by the NSF under Grant CCF-1015548 and in part by gift grants from Northrop Grumman Corporation, LSI Corporation and SanDisk.

TABLE I
PARAMETERS OF CYCLIC HKC CODES FOR THE COMPOUND CHANNEL.

(n,k)	c	roots of $g_0(x)$	$g_0(x)$	b	t	(b_{comp},t)
$(15,8)$	3	β	23	3	1	$(3,1)$
$(15,6)$	5	β	23	4	2	$(4,2)$
$(15,4)$	3	β,β^3	721	5	4	$(5,3)$
$(39,24)$	3	β	13617	6	1	$(6,1)$
$(39,12)$	3	β,β^3	153651205	13	3	$(13,3)$
$(45,30)$	3	β	10011	6	1	$(6,1)$
$(45,28)$	5	β	10011	7	2	$(7,2)$
$(45,26)$	3	β,β^3	230213	8	2	$(8,2)$
$(45,24)$	5	β,β^3	230213	8	2	$(8,2)$
$(45,20)$	3	β,β^3,β^5	21113023	12	3	$(12,3)$
$(45,18)$	5	β,β^3,β^5	21113023	13	3	$(13,3)$
$(51,40)$	3	β	763	4	1	$(4,1)$
$(51,32)$	3	β,β^3	266251	7	2	$(7,2)$
$(51,24)$	3	β,β^3,β^5	134531443	13	4	$(11,4)$
$(63,54)$	3	β	103	3	1	$(3,1)$
$(63,50)$	7	β	103	5	1	$(5,1)$
$(63,48)$	9	β	103	6	1	$(6,1)$
$(63,48)$	3	β,β^3	12471	5	2	$(5,2)$
$(63,44)$	7	β,β^3	12471	6	2	$(6,2)$
$(63,42)$	9	β,β^3	12471	9	2	$(9,2)$
$(63,42)$	3	β,β^3,β^5	1701317	7	3	$(7,3)$
$(63,38)$	7	β,β^3,β^5	1701317	10	3	$(9,3)$
$(63,36)$	3	$\beta,\beta^3,\beta^5,\beta^7$	166623567	12	4	$(9,4)$
$(63,33)$	3	$\beta,\beta^3,\beta^5,\beta^7,\beta^9$	1033500423	13	5	$(10,5)$
$(63,32)$	7	$\beta,\beta^3,\beta^5,\beta^7$	166623567	13	5	$(12,5)$
$(255,244)$	3	β	435	3	1	$(3,1)$
$(255,242)$	5	β	435	3	1	$(3,1)$
$(255,236)$	3	β,β^3	267543	7	2	$(3,2)$
$(255,234)$	5	β,β^3	267543	7	2	$(7,2)$
$(255,228)$	3	β,β^3,β^5	156720665	11	3	$(8,3)$
$(255,226)$	5	β,β^3,β^5	156720665	13	3	$(7,3)$
$(255,220)$	3	$\beta,\beta^3,\beta^5,\beta^7$	75626641375	14	4	$(7,4)$

REFERENCES

[1] K. A. S. Abdel-Ghaffar, R. J. McEliece, A. Odlyzko, and H. C. A. van Tilborg, "On the existence of optimum cyclic burst-correcting codes," *IEEE Trans. Inf. Theory*, vol. IT-32, no. 5, pp. 768–775, Nov. 1986.

[2] N. M. Abramson, "Error-correcting codes from linear sequential circuits," presented at the fourth Sypm. Inform. Theory, London, England, Aug. 29–Sep. 2, 1960.

[3] D. Blackwell, L. Breiman, and A. J. Thomasian, "The capacity of a class of channels," *Ann. Math. Statist.*, vol. 30, pp. 1229–1241, Dec. 1959.

[4] R. L. Dobrushin, "Optimum information transmission through a channel with unknown parameters," *Radio Eng. Electron.*, vol. 4, no. 12, pp. 1–8, 1959.

[5] P. Fire, E, P. (1959). "A class of multiple-error-correcting binary codes for non-independent errors," Sylvania Reconnaissance Systems Laboratory, Mountain View, CA, Rept. RSL-E-2, 1959.

[6] H. T. Hsu, "A class of binary shortened cyclic codes for a compound channel," *Inform. Contr.*, vol. 18, pp. 126–139, 1971.

[7] H. T. Hsu, "On the decoding of a class of shortened cyclic codes for a compound channel," *Inform. Contr.*, vol. 22, pp. 79–88, 1973.

[8] H. T. Hsu, "A class of shortened cyclic codes for a compound channel," *IEEE Trans. Inf. Theory*, vol. 20, no. 1, Jan. 1974.

[9] H. T. Hsu, T. Kasami, and R. T. Chien, "Error-correcting codes for a compound channel," *IEEE Trans. Inf. Theory*, vol. IT-14, no. 1, Jan. 1968.

[10] S. Lin and D. J. Costello, Jr., *Error Correcting Coding: Fundamentals and Applications*, 2nd ed. Upper Saddle River, NJ: Prentice-Hall 2004.

[11] F. J. MacWilliams and N. J. A. Sloane, *The Theory of Error-Correcting Codes*. Amsterdam, The Netherlands: North-Holland, 1977.

[12] R. J. McEliece, *The Theory of Information and Coding*, 2nd Edition. Cambridge, UK: Cambridge University Press, 2002.

[13] W. W. Peterson and E. J.Weldon, Jr., *Error-Correcting Codes*, 2nd Edition, Cambridge, MA, MIT Press, 1972.

[14] E. C. Posner, "Simultaneous error-correction and burst-error detection using binary linear cyclic codes," *J. Soc. Indust. Appl. Math.*, vol. 13, no. 4, pp. 1087–1095, Dec. 1965.

[15] S. H. Reiger, "Codes for the correction of clustered errors," *IRE Trans. Inf. Theory*, vol. IT-6, no. 1, pp. 16–21, Mar. 1960.

[16] J. Wolfowitz, "Simultaneous channels," *Arch. Rat. Mech. Anal.*, vol. 4, pp. 371–386, 1960.