Outage Probability Analysis for Asynchronous Cognitive Radio Networks

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***Abstract*—In this paper, we consider a primary network coexisting with a secondary network in a spectrum sharing asynchronous cognitive radio (CR) network. We analyze the outage probability of the secondary network under the maximum power constraint and interference constraint. The validity of the analysis is verified by computer simulations.**

***Index Terms*—Cognitive radio network, spectrum sharing, dual-hop, outage probability.**

I. Introduction

To meet the demands for frequency spectrum in wireless services, a new approach called a cognitive radio (CR) network is introduced [1], [2]. In the CR network, the users in the secondary network, called the secondary users (SUs), are allowed to utilize the spectrum of the primary network opportunistically [3], [4].

In the spectrum sharing CR network, if the quality of service (QoS) of primary users (PUs) which utilize spectrum is guaranteed, the SUs are allowed to utilize the spectrum [5]. The signal from PUs becomes interference to the SUs and vice versa as both the PUs and SUs utilize the same spectrum.

Previous works on the spectrum sharing CR network did not consider the interference at the SUs caused by the PUs when multiple relays are available in the secondary network [6], [7]. In most of them, it is assumed the perfect synchronization between the primary network and secondary network. However, in practical environment, the primary network and secondary network are not perfectly synchronized.

This paper investigates the spectrum sharing CR network where the primary network and secondary network are not synchronized. The outage probability of the secondary network is analyzed.

The remainder of this paper is organized as follows. Section II describes the system model. In Section III, we investigate relay selection and the power constraint of secondary transmitters. In Section IV, the outage probability of the secondary network is derived. Numerical results are presented in Section V. Conclusion is drawn in Section VI.

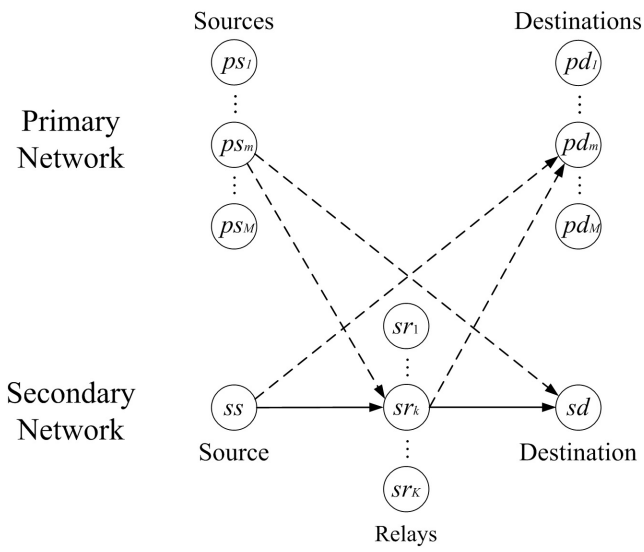

Fig. 1. System model of the cognitive radio network where the primary network and the secondary network co-exist. Solid lines and dashed lines represent the data transmission and the interference, respectively.

II. System model

Consider a primary network coexisting with a secondary network in a spectrum sharing cognitive radio network as shown in Fig. 1. The primary network consists of M source-destination pairs, ps_m-$pd_m, m = 1, \cdots, M$. The secondary network consists of a source ss, a destination sd, and K relays $sr_k, k = 1, \cdots, K$. Assume that a channel between terminals have an additive white Gaussian noise (AWGN) with unit variance, i.e., $N_0 = 1$.

Let $h_{i,j}$ denote the coefficient of the channel from the terminal i to the terminal j. Assume that $h_{i,j}$ is a zero-mean circularly symmetric complex Gaussian random variable and its variance is given by

$$\lambda_{i,j} = (d_{i,j})^{-\alpha} \tag{1}$$

where α is the path-loss exponent and $d_{i,j}$ is the distance

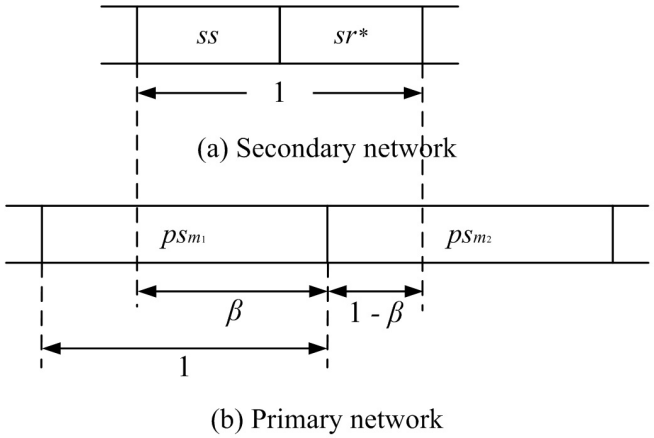

Fig. 2. Frames of the secondary network and primary network.

between the terminal i and terminal j. Assume that all channel coefficients are independent each other. Assume that the terminals in the primary network are placed contiguously and the terminals in the secondary network are also placed contiguously, so that $\lambda_{ss,sr_k} = \lambda_{ss,sr}$, $\lambda_{sr_k,sd} = \lambda_{sr,sd}$, $\lambda_{ps_m,sr_k} = \lambda_{ps,sr}$, $\lambda_{ps_m,sd} = \lambda_{ps,sd}$, $\lambda_{ss,pd_m} = \lambda_{ss,pd}$, and $\lambda_{sr_k,pd_m} = \lambda_{sr,pd}$ for $k = 1, \cdots, K$ and $m = 1, \cdots, M$. Assume that each terminal in the secondary network knows only the statistics of the channel from and to a terminal in the primary network and the instantaneous coefficient of the channel from and to a terminal in the secondary network.

Suppose that the frames of both networks have length of 1 and they are not synchronized. In the primary network, the source ps_m transmits its data signal x_{ps_m} with power P_{ps} for one frame. Assume that the primary network operates in the time division duplex (TDD). In the secondary network, ss transmits its data signal x_{ss} to sd in two phases of equal length for one frame. In the first phase, ss transmits x_{ss} with power P_{ss} to all relays. Suppose that a relay decodes x_{ss} successfully if the received signal-to-interference-plus-noise ratio (SINR) is not less than the threshold SINR which is given by

$$\gamma_{th} = 2^{2R} - 1 \tag{2}$$

where R is the target rate of the secondary network. Define the decoding set of the secondary network as

$$\mathcal{D} = \{sr_k : \gamma_{ss,sr_k} \geq \gamma_{th}\} \tag{3}$$

where $\gamma_{i,j}$ is the received SINR at the terminal j corresponding to the transmission from the terminal i. The relay is selected as follows:

$$sr^* = \underset{sr_k \in \mathcal{D}}{\arg\max}\, \gamma_{sr_k,sd}. \tag{4}$$

The selected relay sr^* forwards x_{ss} to sd with power P_{sr} in the second phase.

Fig. 2 shows the asynchronous frames of the secondary network and the primary network. Let β denote the length of the frame of ps_{m_1} overlapped with that of ss and sr^*, $\beta = (0, 1]$.

III. Power Constraint and Relay Selection

There are two constraints on the transmission power of the secondary network: the maximum transmission power constraint and the interference constraint to guarantee the quality of service (QoS) of the primary network [6]. For the terminal i, the maximum transmission power constraint is given by

$$P_i \leq P_i^{\max} \tag{5}$$

where $P_i^{\max}$ is its maximum transmission power. The interference constraint is given by [10], [11]

$$\Pr[P_i |h_{i,pd_m}|^2 \geq \bar{I}] \leq \epsilon \tag{6}$$

where $\bar{I}$ is the threshold of interference to the primary user and ϵ is the outage constraint of the primary user. For convenience, define a random variable

$$Y_{i,j} \triangleq P_i |h_{i,j}|^2. \tag{7}$$

Its probability density function (PDF) and cumulative distribution function (CDF) are given by [12]

$$f_{Y_{i,j}}(y) = \frac{1}{P_i \lambda_{i,j}} \exp\left(-\frac{y}{P_i \lambda_{i,j}}\right) \tag{8}$$

and

$$F_{Y_{i,j}}(y) = 1 - \exp\left(-\frac{y}{P_i \lambda_{i,j}}\right), \tag{9}$$

respectively. From (6) and (9), we obtain

$$P_i \leq \frac{-\bar{I}}{\lambda_{i,pd} \ln \epsilon}. \tag{10}$$

From (5) and (10), the transmission power of the terminal i in the secondary network is constrained as

$$P_i \leq \min\left\{P_i^{\max}, \frac{-\bar{I}}{\lambda_{i,pd} \ln \epsilon}\right\}. \tag{11}$$

The received SINR at sr_k corresponding to the transmission from ss is given by

$$\gamma_{ss,sr_k} = \begin{cases} \frac{|h_{ss,sr_k}|^2 P_{ss}}{2\beta |h_{ps_{m_1},sr_k}|^2 P_{ps} + (1-2\beta)|h_{ps_{m_2},sr_k}|^2 P_{ps} + 1}, & \beta < \frac{1}{2}, \\ \frac{|h_{ss,sr_k}|^2 P_{ss}}{|h_{ps_{m_1},sr_k}|^2 P_{ps} + 1}, & \text{o.w.} \end{cases} \tag{12}$$

If the relay sr_k is selected, the received SINR at sd corresponding to the transmission from sr_k is given by

$$\gamma_{sr_k,sd} = \begin{cases} \frac{|h_{sr_k,sd}|^2 P_{sr_k}}{|h_{ps_{m_2},sd}|^2 P_{ps} + 1}, & \beta < \frac{1}{2}, \\ \frac{|h_{sr_k,sd}|^2 P_{sr_k}}{(2\beta-1)|h_{ps_{m_1},sd}|^2 P_{ps} + 2(1-\beta)|h_{ps_{m_2},sd}|^2 P_{ps} + 1}, & \text{o.w.} \end{cases} \tag{13}$$

Substituting (12) and (13) into (4), the relay is selected as follows:

$$sr^* = \underset{sr_k \in \mathcal{D}}{\arg\max}\, Y_{sr_k,sd}. \tag{14}$$

$$P_{out}^{(3)} = \left\{1 - \eta_3 \exp\left(-\frac{\gamma_{th}}{P_{sr}\lambda_{sr,sd}}\right)\right\}^K$$
$$+K!\frac{2\beta-1}{4\beta-3}\exp\left(\frac{1}{(2\beta-1)P_{ps}\lambda_{ps,sd}}\right)\sum_{n=1}^{K}\frac{\eta_3^n(1-\eta_3)^{K-n}}{(K-n)!(n-1)!}B\left(\exp\left(-\frac{\gamma_{th}}{P_{sr}\lambda_{sr,sd}}\right);1+\frac{P_{sr}\lambda_{sr,sd}}{(2\beta-1)P_{ps}\lambda_{ps,sd}\gamma_{th}},n\right)$$
$$+K!\frac{2(1-\beta)}{4\beta-3}\exp\left(\frac{1}{2(1-\beta)P_{ps}\lambda_{ps,sd}}\right)\sum_{n=1}^{K}\frac{\eta_3^n(1-\eta_3)^{K-n}}{(K-n)!(n-1)!}B\left(\exp\left(-\frac{\gamma_{th}}{P_{sr}\lambda_{sr,sd}}\right);1+\frac{P_{sr}\lambda_{sr,sd}}{2(1-\beta)P_{ps}\lambda_{ps,sd}\gamma_{th}},n\right) \quad (20)$$

$$P_{out}^{(4)} = \left\{1 - \eta_3 \exp\left(-\frac{\gamma_{th}}{P_{sr}\lambda_{sr,sd}}\right)\right\}^K$$
$$+K!\exp\left(\frac{2}{P_{ps}\lambda_{ps,sd}}\right)\sum_{n=1}^{K}\frac{\eta_3^n(1-\eta_3)^{K-n}}{(K-n)!(n-1)!}B\left(\exp\left(-\frac{\gamma_{th}}{P_{sr}\lambda_{sr,sd}}\right);1+\frac{2P_{sr}\lambda_{sr,sd}}{P_{ps}\lambda_{ps,sd}\gamma_{th}},n\right)$$
$$+K!\frac{2\gamma_{th}}{P_{ps}\lambda_{ps,sd}P_{sr}\lambda_{sr,sd}}\exp\left(-\frac{\gamma_{th}}{P_{sr}\lambda_{sr,sd}}\right)\left(\frac{2}{P_{ps}\lambda_{ps,sd}}+\frac{\gamma_{th}}{P_{sr}\lambda_{sr,sd}}\right)^{-2}\sum_{n=1}^{K}\frac{\eta_3^n(1-\eta_3)^{K-n}}{(K-n)!(n-1)!}$$
$$\times{}_3F_2\left[\frac{2P_{sr}\lambda_{sr,sd}}{P_{ps}\lambda_{ps,sd}\gamma_{th}}+1,\frac{2P_{sr}\lambda_{sr,sd}}{P_{ps}\lambda_{ps,sd}\gamma_{th}}+1,1-n;\ \frac{2P_{sr}\lambda_{sr,sd}}{P_{ps}\lambda_{ps,sd}\gamma_{th}}+2,\frac{2P_{sr}\lambda_{sr,sd}}{P_{ps}\lambda_{ps,sd}\gamma_{th}}+2;\exp\left(-\frac{\gamma_{th}}{P_{sr}\lambda_{sr,sd}}\right)\right] \quad (22)$$

IV. Outage Probability

An outage occurs when the received SINR is less than the threshold SINR γ_{th}. The outage probability is given by

$$\begin{aligned} P_{out} &= \Pr\left[\gamma_{sr^*,sd} < \gamma_{th}\right] \\ &= \Pr\left[\mathcal{D} = \varnothing\right] \\ &\quad + \sum_{\mathcal{D}\neq\varnothing}\Pr\left[\mathcal{D}\right]\Pr\left[\gamma_{sr^*,sd} < \gamma_{th}|\mathcal{D}\right]. \end{aligned} \quad (15)$$

Theorem 1: For $0 < \beta < 1/4$ or $1/4 < \beta < 1/2$, the outage probability is given by

$$P_{out}^{(1)} = \left\{1 - \eta_1 \exp\left(-\frac{\gamma_{th}}{P_{sr}\lambda_{sr,sd}}\right)\right\}^K$$
$$+ K!\exp\left(\frac{1}{P_{ps}\lambda_{ps,sd}}\right)\sum_{n=1}^{K}\frac{\eta_1^n(1-\eta_1)^{K-n}}{(K-n)!(n-1)!}$$
$$\times B\left(\exp\left(-\frac{\gamma_{th}}{P_{sr}\lambda_{sr,sd}}\right);1+\frac{P_{sr}\lambda_{sr,sd}}{P_{ps}\lambda_{ps,sd}\gamma_{th}},n\right) \quad (16)$$

where $B\left(w;p,q\right) = \int_0^w t^{p-1}(1-t)^{q-1}dt$ is the incomplete Beta function [13] and

$$\eta_1 = \frac{1}{(4\beta-1)P_{ps}\lambda_{ps,sr}}\exp\left(-\frac{\gamma_{th}}{P_{ss}\lambda_{ss,sr}}\right)$$
$$\times\left\{\left(\frac{\gamma_{th}}{P_{ss}\lambda_{ss,sr}}+\frac{1}{2\beta P_{ps}\lambda_{ps,sr}}\right)^{-1}\right.$$
$$\left.-\left(\frac{\gamma_{th}}{P_{ss}\lambda_{ss,sr}}+\frac{1}{(1-2\beta)P_{ps}\lambda_{ps,sr}}\right)^{-1}\right\}. \quad (17)$$

Proof: See Appendix A. ■

Theorem 2: For $\beta = 1/4$, the outage probability is given by

$$P_{out}^{(2)} = \left\{1 - \eta_2 \exp\left(-\frac{\gamma_{th}}{P_{sr}\lambda_{sr,sd}}\right)\right\}^K$$
$$+ K!\exp\left(\frac{1}{P_{ps}\lambda_{ps,sd}}\right)\sum_{n=1}^{K}\frac{\eta_2^n(1-\eta_2)^{K-n}}{(K-n)!(n-1)!}$$
$$\times B\left(\exp\left(-\frac{\gamma_{th}}{P_{sr}\lambda_{sr,sd}}\right);1+\frac{P_{sr}\lambda_{sr,sd}}{P_{ps}\lambda_{ps,sd}\gamma_{th}},n\right) \quad (18)$$

where

$$\eta_2 = \frac{4}{(P_{ps}\lambda_{ps,sr})^2}\exp\left(-\frac{\gamma_{th}}{P_{ss}\lambda_{ss,sr}}\right)$$
$$\times\left(\frac{\gamma_{th}}{P_{ss}\lambda_{ss,sr}}+\frac{2}{P_{ps}\lambda_{ps,sr}}\right)^{-2}. \quad (19)$$

Proof: See Appendix B. ■

Theorem 3: For $1/2 < \beta < 3/4$ or $3/4 < \beta < 1$, the outage probability is given in (20) at the top of the page, where

$$\eta_3 = \exp\left(-\frac{\gamma_{th}}{P_{ss}\lambda_{ss,sr}}\right)\left(1+\frac{P_{ps}\lambda_{ps,sr}}{P_{ss}\lambda_{ss,sr}}\gamma_{th}\right)^{-1}. \quad (21)$$

Proof: See Appendix C. ■

Theorem 4: For $\beta = 3/4$, the outage probability is given in (22) at the second top of the page, where ${}_pF_q\left(a_1,a_2,\cdots,a_p;b_1,b_2,\cdots,b_q;z\right)$ is the Hypergeometric Function [13].

Proof: See Appendix D. ■

Theorem 5: For $\beta = 1/2$ or $\beta = 1$, the outage probability

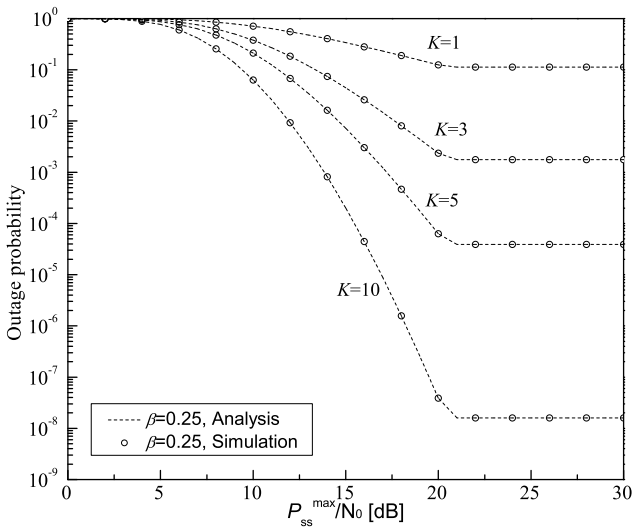

Fig. 3. Outage probability of the secondary network for various number of relays.

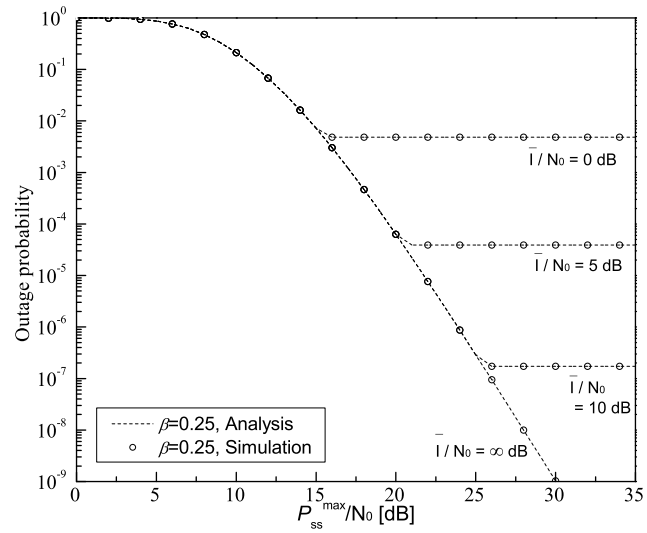

Fig. 4. Outage probability of the secondary network for various interference thresholds.

is given by

$$P_{out}^{(5)} = \left\{1 - \eta_3 \exp\left(-\frac{\gamma_{th}}{P_{sr}\lambda_{sr,sd}}\right)\right\}^K + K!\exp\left(\frac{1}{P_{ps}\lambda_{ps,sd}}\right)\sum_{n=1}^{K}\frac{\eta_3^n(1-\eta_3)^{K-n}}{(K-n)!(n-1)!} \times B\left(\exp\left(-\frac{\gamma_{th}}{P_{sr}\lambda_{sr,sd}}\right); 1 + \frac{P_{sr}\lambda_{sr,sd}}{P_{ps}\lambda_{ps,sd}\gamma_{th}}, n\right). \tag{23}$$

Proof: The probability of decoding set is given in (35). The outage probability for given decoding set is given in (28). By substituting (28) and (35) into (15), (23) is obtained. ■

V. Numerical Results

Consider a cognitive radio network consisting of two primary source-destination pairs, a secondary source-destination pair, and various number of secondary relays. Suppose that $R = 1$, $\alpha = 4$, and $d_{ss,sr} = d_{sr,sd} = 1$. Also, suppose that $d = d_{ps,sd} = d_{ss,pd} = d_{sr,pd}$ and $P_{ss}^{\max} = P_{sr}^{\max}$.

Fig. 3 shows the outage probability of the secondary network for $\bar{I}/N_0 = 5$dB, $\epsilon = 0.1$, P_{ps}/N_0=20dB, $d = 5$, and various number of relays. It is shown that the analytical results perfectly match the simulation results. It is also shown that the slope of the outage probability curve increases and the floor level decreases as the number of relays increases.

Fig. 4 shows the outage probability of the secondary network for $K = 5$, $\epsilon = 0.1$, P_{ps}/N_0=20dB, $d = 5$, and various interference thresholds. It is shown that the slope of the outage probability curves are unchanged for various interference thresholds before the floors begin. It is also shown that the floor level decreases as the interference threshold increases. Also, the outage probability curve does not have floor for $\bar{I}/N_0 = \infty$.

VI. Conclusion

In this paper, we consider a asynchronous cognitive radio network. We analyze the outage probability of the secondary network under the maximum power constraint and interference constraint. Analysis is verified by computer simulations. It is shown that the interference to the primary network causes a floor on outage probability curve. Also, we investigate the effect of the number of relays and the interference threshold on the outage probability.

Appendix A
Proof of Theorem 1

For convenience, define a random variable

$$Z_1 \triangleq 2\beta|h_{ps_{m_1},sr_k}|^2 P_{ps} + (1-2\beta)|h_{ps_{m_2},sr_k}|^2 P_{ps}. \tag{24}$$

Its PDF is given by [9]

$$f_{Z_1}(z) = \frac{1}{(4\beta-1)P_{ps}\lambda_{ps,sr}}\left\{\exp\left(-\frac{z}{2\beta P_{ps}\lambda_{ps,sr}}\right) - \exp\left(-\frac{z}{(1-2\beta)P_{ps}\lambda_{ps,sr}}\right)\right\}. \tag{25}$$

The probability that sr_k belongs to $\mathcal{D}$ is given by

$$\begin{aligned}\Pr\left[sr_k \in \mathcal{D}\right] &= \Pr\left[\frac{Y_{ss,sr_k}}{Z_1+1} \geq \gamma_{th}\right] \\ &= \int_0^\infty \left\{1 - F_{Y_{ss,sr_k}}\left(\gamma_{th}(\gamma+1)\right)\right\} f_{Z_1}(\gamma)d\gamma \\ &= \frac{1}{(4\beta-1)P_{ps}\lambda_{ps,sr}}\exp\left(-\frac{\gamma_{th}}{P_{ss}\lambda_{ss,sr}}\right) \\ &\quad \times \left\{\left(\frac{\gamma_{th}}{P_{ss}\lambda_{ss,sr}} + \frac{1}{2\beta P_{ps}\lambda_{ps,sr}}\right)^{-1} \right. \\ &\quad \left. -\left(\frac{\gamma_{th}}{P_{ss}\lambda_{ss,sr}} + \frac{1}{(1-2\beta)P_{ps}\lambda_{ps,sr}}\right)^{-1}\right\} \\ &= \eta_1.\end{aligned} \tag{26}$$

$$\begin{aligned}
\Pr[\gamma_{sr^*,sd} < \gamma_{th}|\mathcal{D}] &= \sum_{sr_k\in\mathcal{D}} \Pr[sr_k = sr^*|\mathcal{D}] \Pr[\gamma_{sr_k,sd} < \gamma_{th}|\mathcal{D}] \\
&= \sum_{sr_k\in\mathcal{D}} \int_0^{\gamma_{th}} \prod_{sr_l\in\mathcal{D}-\{sr_k\}} F_{Y_{sr_l,sd}}(\gamma) f_{Y_{sr_k,sd}}(\gamma) d\gamma \\
&\quad + \sum_{sr_k\in\mathcal{D}} \int_{\gamma_{th}}^{\infty} \left\{1 - F_{Y_{ps_{m_2},sd}}\left(\frac{\gamma}{\gamma_{th}} - 1\right)\right\} \prod_{sr_l\in\mathcal{D}-\{sr_k\}} F_{Y_{sr_l,sd}}(\gamma) f_{Y_{sr_k,sd}}(\gamma) d\gamma \\
&= \left\{1 - \exp\left(-\frac{\gamma_{th}}{P_{sr}\lambda_{sr,sd}}\right)\right\}^{|\mathcal{D}|} + \frac{\gamma_{th}}{P_{sr}\lambda_{sr,sd}} \exp\left(-\frac{\gamma_{th}}{P_{sr}\lambda_{sr,sd}}\right) \\
&\quad \times \sum_{sr_k\in\mathcal{D}} \int_0^{\infty} \exp\left(-\left(\frac{1}{P_{ps}\lambda_{ps,sd}} + \frac{\gamma_{th}}{P_{sr}\lambda_{sr,sd}}\right)\gamma\right) \left\{1 - \exp\left(-\frac{\gamma_{th}}{P_{sr}\lambda_{sr,sd}}\right) \exp\left(-\frac{\gamma_{th}\gamma}{P_{sr}\lambda_{sr,sd}}\right)\right\}^{|\mathcal{D}|-1} d\gamma \\
&= \left\{1 - \exp\left(-\frac{\gamma_{th}}{P_{sr}\lambda_{sr,sd}}\right)\right\}^{|\mathcal{D}|} + |\mathcal{D}| \exp\left(\frac{1}{P_{ps}\lambda_{ps,sd}}\right) B\left(\exp\left(-\frac{\gamma_{th}}{P_{sr}\lambda_{sr,sd}}\right); 1 + \frac{P_{sr}\lambda_{sr,sd}}{P_{ps}\lambda_{ps,sd}\gamma_{th}}, |\mathcal{D}|\right) \qquad (28)
\end{aligned}$$

Then, the probability of decoding set is given by

$$\begin{aligned}
\Pr[\mathcal{D}] &= \prod_{sr_k\in\mathcal{D}} \Pr[sr_k \in \mathcal{D}] \prod_{sr_k\notin\mathcal{D}} \Pr[sr_k \notin \mathcal{D}] \\
&= \eta_1^{|\mathcal{D}|}(1-\eta_1)^{K-|\mathcal{D}|}. \qquad (27)
\end{aligned}$$

The outage probability for given decoding set is given in (28) at the top of the page. The last equality in (28) comes from

$$\begin{aligned}
&\int_0^{\infty} \exp(-\mu w) \left\{1 - \tau \exp\left(-\frac{w}{\rho}\right)\right\}^{\nu-1} dw \\
&= \tau^{-\rho\mu} \rho B(\tau; \rho\mu, \nu). \qquad (29)
\end{aligned}$$

Substituting (27) and (28) into (15), we obtain (16).

Appendix B
Proof of Theorem 2

For $\beta = 1/4$, the random variable Z_1 in (24) becomes

$$Z_1 = \frac{1}{2}|h_{ps_{m_1},sr_k}|^2 P_{ps} + \frac{1}{2}|h_{ps_{m_2},sr_k}|^2 P_{ps}. \qquad (30)$$

Its PDF is given by [9]

$$f_{Z_1}(z) = \frac{4z}{(P_{ps}\lambda_{ps,sr})^2} \exp\left(-\frac{2z}{P_{ps}\lambda_{ps,sr}}\right). \qquad (31)$$

The probability that sr_k belongs to $\mathcal{D}$ is given by

$$\begin{aligned}
\Pr[sr_k \in \mathcal{D}] &= \Pr\left[\frac{Y_{ss,sr_k}}{Z_1 + 1} \geq \gamma_{th}\right] \\
&= \frac{4}{(P_{ps}\lambda_{ps,sr})^2} \exp\left(-\frac{\gamma_{th}}{P_{ss}\lambda_{ss,sr}}\right) \\
&\quad \times \left(\frac{\gamma_{th}}{P_{ss}\lambda_{ss,sr}} + \frac{2}{P_{ps}\lambda_{ps,sr}}\right)^{-2} \\
&= \eta_2. \qquad (32)
\end{aligned}$$

Then, the probability of decoding set is given by

$$\begin{aligned}
\Pr[\mathcal{D}] &= \prod_{sr_k\in\mathcal{D}} \Pr[sr_k \in \mathcal{D}] \prod_{sr_k\notin\mathcal{D}} \Pr[sr_k \notin \mathcal{D}] \\
&= \eta_2^{|\mathcal{D}|}(1-\eta_2)^{K-|\mathcal{D}|}. \qquad (33)
\end{aligned}$$

Substituting (28), (32), and (33) into (15), we obtain (18).

Appendix C
Proof of Theorem 3

The probability that sr_k belongs to $\mathcal{D}$ is given by

$$\begin{aligned}
\Pr[sr_k \in \mathcal{D}] &= \Pr\left[\frac{Y_{ss,sr_k}}{Y_{ps_{m_1},sr_k} + 1} \geq \gamma_{th}\right] \\
&= \int_0^{\infty} \left\{1 - F_{Y_{ss,sr_k}}(\gamma_{th} \times (\gamma + 1))\right\} f_{Y_{ps_{m_1},sr_k}}(\gamma) d\gamma \\
&= \exp\left(-\frac{\gamma_{th}}{P_{ss}\lambda_{ss,sr}}\right) \left(1 + \frac{P_{ps}\lambda_{ps,sr}}{P_{ss}\lambda_{ss,sr}}\gamma_{th}\right)^{-1} \\
&= \eta_3. \qquad (34)
\end{aligned}$$

Then, the probability of decoding set is given by

$$\begin{aligned}
\Pr[\mathcal{D}] &= \prod_{sr_k\in\mathcal{D}} \Pr[sr_k \in \mathcal{D}] \prod_{sr_k\notin\mathcal{D}} \Pr[sr_k \notin \mathcal{D}] \\
&= \eta_3^{|\mathcal{D}|}(1-\eta_3)^{K-|\mathcal{D}|}. \qquad (35)
\end{aligned}$$

For convenience, define a random variable

$$Z_2 \triangleq (2\beta - 1)|h_{ps_{m_1},sd}|^2 P_{ps} + 2(1-\beta)|h_{ps_{m_2},sd}|^2 P_{ps}. \qquad (36)$$

Its CDF is given by [9]

$$\begin{aligned}
F_{Z_2}(z) &= 1 - \frac{2\beta - 1}{4\beta - 3} \exp\left(-\frac{z}{(2\beta-1)P_{ps}\lambda_{ps,sd}}\right) \\
&\quad + \frac{2(1-\beta)}{4\beta - 3} \exp\left(-\frac{z}{2(1-\beta)P_{ps}\lambda_{ps,sd}}\right). \qquad (37)
\end{aligned}$$

The outage probability for given decoding set is given in (38) at the top of the next page. The last equality in (38) comes from (29).

Substituting (35) and (38) into (15), we obtain (20).

$$\Pr\left[\gamma_{sr^*,sd}<\gamma_{th}|\mathcal{D}\right]=\sum_{sr_k\in\mathcal{D}}\Pr\left[sr_k=sr^*|\mathcal{D}\right]\Pr\left[\gamma_{sr_k,sd}<\gamma_{th}|\mathcal{D}\right]$$
$$=\sum_{sr_k\in\mathcal{D}}\int_0^{\gamma_{th}}\prod_{sr_l\in\mathcal{D}-\{sr_k\}}F_{Y_{sr_l,sd}}(\gamma)f_{Y_{sr_k,sd}}(\gamma)d\gamma+\sum_{sr_k\in\mathcal{D}}\int_{\gamma_{th}}^{\infty}\left\{1-F_{Z_2}\left(\frac{\gamma}{\gamma_{th}}-1\right)\right\}\prod_{sr_l\in\mathcal{D}-\{sr_k\}}F_{Y_{sr_l,sd}}(\gamma)f_{Y_{sr_k,sd}}(\gamma)d\gamma$$
$$=\left\{F_{Y_{sr,sd}}(\gamma_{th})\right\}^{|\mathcal{D}|}+\sum_{sr_k\in\mathcal{D}}\int_{\gamma_{th}}^{\infty}\frac{2\beta-1}{4\beta-3}\exp\left(-\frac{\gamma/\gamma_{th}-1}{(2\beta-1)P_{ps}\lambda_{ps,sd}}\right)\left\{1-\exp\left(-\frac{\gamma}{P_{sr}\lambda_{sr,sd}}\right)\right\}^{|\mathcal{D}|-1}$$
$$\times\frac{1}{P_{sr}\lambda_{sr,sd}}\exp\left(-\frac{\gamma}{P_{sr}\lambda_{sr,sd}}\right)d\gamma-\sum_{sr_k\in\mathcal{D}}\int_{\gamma_{th}}^{\infty}\frac{2(1-\beta)}{4\beta-3}\exp\left(-\frac{\gamma/\gamma_{th}-1}{2(1-\beta)P_{ps}\lambda_{ps,sd}}\right)$$
$$\times\left\{1-\exp\left(-\frac{\gamma}{P_{sr}\lambda_{sr,sd}}\right)\right\}^{|\mathcal{D}|-1}\frac{1}{P_{sr}\lambda_{sr,sd}}\exp\left(-\frac{\gamma}{P_{sr}\lambda_{sr,sd}}\right)d\gamma$$
$$=\left\{1-\exp\left(-\frac{\gamma_{th}}{P_{sr}\lambda_{sr,sd}}\right)\right\}^{|\mathcal{D}|}+|\mathcal{D}|\frac{2\beta-1}{4\beta-3}\exp\left(\frac{1}{(2\beta-1)P_{ps}\lambda_{ps,sd}}\right)B\left(\exp\left(-\frac{\gamma_{th}}{P_{sr}\lambda_{sr,sd}}\right);1+\frac{P_{sr}\lambda_{sr,sd}}{(2\beta-1)P_{ps}\lambda_{ps,sd}\gamma_{th}},|\mathcal{D}|\right)$$
$$-|\mathcal{D}|\frac{2(1-\beta)}{4\beta-3}\exp\left(\frac{1}{2(1-\beta)P_{ps}\lambda_{ps,sd}}\right)B\left(\exp\left(-\frac{\gamma_{th}}{P_{sr}\lambda_{sr,sd}}\right);1+\frac{P_{sr}\lambda_{sr,sd}}{2(1-\beta)P_{ps}\lambda_{ps,sd}\gamma_{th}},|\mathcal{D}|\right)\tag{38}$$

$$\Pr\left[\gamma_{sr^*,sd}<\gamma_{th}|\mathcal{D}\right]=\sum_{sr_k\in\mathcal{D}}\Pr\left[sr_k=sr^*|\mathcal{D}\right]\Pr\left[\gamma_{sr_k,sd}<\gamma_{th}|\mathcal{D}\right]$$
$$=\sum_{sr_k\in\mathcal{D}}\int_0^{\gamma_{th}}\prod_{sr_l\in\mathcal{D}-\{sr_k\}}F_{Y_{sr_l,sd}}(\gamma)f_{Y_{sr_k,sd}}(\gamma)d\gamma$$
$$+\sum_{sr_k\in\mathcal{D}}\int_{\gamma_{th}}^{\infty}\left\{1-F_{Z_2}\left(\frac{\gamma}{\gamma_{th}}-1\right)\right\}\prod_{sr_l\in\mathcal{D}-\{sr_k\}}F_{Y_{sr_l,sd}}(\gamma)f_{Y_{sr_k,sd}}(\gamma)d\gamma$$
$$=\left\{1-\exp\left(-\frac{\gamma_{th}}{P_{sr}\lambda_{sr,sd}}\right)\right\}^{|\mathcal{D}|}$$
$$+\sum_{sr_k\in\mathcal{D}}\int_{\gamma_{th}}^{\infty}\exp\left(-\frac{2\gamma/\gamma_{th}-2}{P_{ps}\lambda_{ps,sd}}\right)\left\{1-\exp\left(-\frac{\gamma}{P_{sr}\lambda_{sr,sd}}\right)\right\}^{|\mathcal{D}|-1}\frac{1}{P_{sr}\lambda_{sr,sd}}\exp\left(-\frac{\gamma}{P_{sr}\lambda_{sr,sd}}\right)d\gamma$$
$$+\sum_{sr_k\in\mathcal{D}}\int_{\gamma_{th}}^{\infty}\frac{2\gamma/\gamma_{th}-2}{P_{ps}\lambda_{ps,sd}}\exp\left(-\frac{2\gamma/\gamma_{th}-2}{P_{ps}\lambda_{ps,sd}}\right)\left\{1-\exp\left(-\frac{\gamma}{P_{sr}\lambda_{sr,sd}}\right)\right\}^{|\mathcal{D}|-1}\frac{1}{P_{sr}\lambda_{sr,sd}}\exp\left(-\frac{\gamma}{P_{sr}\lambda_{sr,sd}}\right)d\gamma$$
$$=\left\{1-\exp\left(-\frac{\gamma_{th}}{P_{sr}\lambda_{sr,sd}}\right)\right\}^{|\mathcal{D}|}+\zeta_1+\zeta_2\tag{41}$$

Appendix D
Proof of Theorem 4

The probability of decoding set is given in (35). For $\beta=3/4$, the random variable Z_2 in (36) becomes

$$Z_2=\frac{1}{2}|h_{ps_{m_1},sd}|^2P_{ps}+\frac{1}{2}|h_{ps_{m_2},sd}|^2P_{ps}.\tag{39}$$

Its CDF is given by [9]

$$F_{Z_2}(z)=1-\exp\left(-\frac{2z}{P_{ps}\lambda_{ps,sd}}\right)-\frac{2z}{P_{ps}\lambda_{ps,sd}}\exp\left(-\frac{2z}{P_{ps}\lambda_{ps,sd}}\right).\tag{40}$$

The outage probability for given decoding set is given in (41) at the second top of the page. From (29), we have

$$\zeta_1=|\mathcal{D}|\exp\left(\frac{2}{P_{ps}\lambda_{ps,sd}}\right)\times B\left(\exp\left(-\frac{\gamma_{th}}{P_{sr}\lambda_{sr,sd}}\right);1+\frac{2P_{sr}\lambda_{sr,sd}}{P_{ps}\lambda_{ps,sd}\gamma_{th}},|\mathcal{D}|\right).\tag{42}$$

From the formula

$$\int_0^{\infty}w\left\{1-\tau\exp\left(-\frac{w}{\rho}\right)\right\}^{\nu-1}\exp\left(-\mu w\right)dw=\frac{{}_3F_2\left(\rho\mu,\rho\mu,1-\nu;1+\rho\mu,1+\rho\mu;\tau\right)}{\mu^2},\tag{43}$$

we have (44) at the top of the next page. Substituting (35), (42), and (44) into (15), we obtain (22).

$$\zeta_2 = \frac{2\left|\mathcal{D}\right|\gamma_{th}}{P_{ps}\lambda_{ps,sd}P_{sr}\lambda_{sr,sd}}\exp\left(-\frac{\gamma_{th}}{P_{sr}\lambda_{sr,sd}}\right)\left(\frac{2}{P_{ps}\lambda_{ps,sd}}+\frac{\gamma_{th}}{P_{sr}\lambda_{sr,sd}}\right)^{-2}$$
$$\times 3F_2\left[\frac{2P_{sr}\lambda_{sr,sd}}{P_{ps}\lambda_{ps,sd}\gamma_{th}}+1, \frac{2P_{sr}\lambda_{sr,sd}}{P_{ps}\lambda_{ps,sd}\gamma_{th}}+1, 1-\left|\mathcal{D}\right|; \frac{2P_{sr}\lambda_{sr,sd}}{P_{ps}\lambda_{ps,sd}\gamma_{th}}+2, \frac{2P_{sr}\lambda_{sr,sd}}{P_{ps}\lambda_{ps,sd}\gamma_{th}}+2; \exp\left(-\frac{\gamma_{th}}{P_{sr}\lambda_{sr,sd}}\right)\right] \quad (44)$$

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