Quantum teleportation over hyper entangled states

Ankur Raina and Shayan Garani Srinivasa
Dept. of Electronic Systems Engineering
Indian Institute of Science, Bangalore, India
Email: {ankur, shayan.gs}@dese.iisc.ernet.in

Abstract—Teleportation is a well known protocol to transfer one qubit between users sharing a Bell pair. By treating teleportation as a transfer of qubits between end users sharing a virtual quantum channel, we derive the maximum coherent information of this virtual quantum channel under noisy classical channel existing between the end users. We also study various communication scenarios using hyper entangled states and quantify the advantages over a single degree of freedom.

I. INTRODUCTION

Superdense coding [1] and teleportation [2] are two important applications in quantum information theory that stem from the power of quantum entanglement. These two protocols can be seen as duals of each other, enabling the transfer of classical bits using quantum states and vice versa. Both protocols make the assumption of noiseless channel existing between nodes sharing the Bell pair. In practice, it may not be possible to create perfect Bell pairs in the laboratory, and classical channels used for classical communication are generally noisy.

Pati and Agrawal [3] considered the use of non-maximally entangled pure bipartite states and evaluated the fidelity of teleportation calling it probabilistic teleportation. Popescu [4] showed that teleportation is possible using mixed bipartite states. It was shown in [5] that not all entangled mixed states can be used for teleportation. To know whether an entangled state can be used for teleportation, Horodecki et al. [6] defined a quantity called the fully entangled fraction. It was shown that as long as this fraction is above a certain threshold, entangled states can be used for teleportation.

In our earlier work [7], under the framework of superdense coding we evaluated the Holevo capacity of a quantum bit flip channel existing between communicating nodes sharing prior entanglement. In this paper we would like to consider the teleportation of qubits as a transfer of qubits over virtual quantum channels when the channel existing between nodes A and B is a binary symmetric channel.

II. TELEPORTATION OVER VIRTUAL QUANTUM CHANNEL

In quantum teleportation [2], two nodes A and B share an entangled pair of qubits called the Bell pair. This Bell pair can be initially prepared and distributed to the two nodes. In the traditional scenario, node A makes joint measurement on its qubit and the qubit to be teleported in the Bell basis and reports the measurement outcome to B via a classical channel. Upon receiving the classical bits, B performs corresponding unitary operations on its qubit to obtain the state of the qubit intended for teleportation by A. However, it is assumed that the channel between nodes A and B is noiseless. In practice, all channels, quantum or classical are noisy. While classical channels induce bit flip errors, quantum channels induce bit flips, phase flips, combination of the two to name a few. In addition, one may not have perfect Bell pair to begin with. We consider a noisy version of the Bell state for communication between nodes A and B. For this, we use the Werner state parameterized by \( \lambda \) for our analysis. We can control \( \lambda \) to go from separable states (\( \lambda = 0 \)) to purely entangled states (\( \lambda = 1 \)). In this section, we would like to consider the teleportation of qubits as a transfer of qubits over virtual quantum channels when the channel existing between nodes A and B is a binary symmetric channel.

A. Binary symmetric channel and quantum teleportation over virtual quantum channel

We explain teleportation briefly below. Suppose that bipartite state namely the Werner state \( \rho_{AB} \) is shared between nodes A and B:

\[
\rho_{AB} = \lambda |\Phi^+\rangle\langle \Phi^+| + \frac{\lambda}{4} \mathbb{1}_4.
\]

Let qubit to be teleported be \( \rho^T \):

\[
\rho^T = \begin{pmatrix} \rho_{00} & \rho_{01} \\ \rho_{10} & \rho_{11} \end{pmatrix}.
\]

The joint state can be expressed as

\[
\rho_{TAB} = \rho^T \otimes \rho_{AB}.
\]

A performs joint measurement on qubit T and qubit A in the Bell basis. Based on the measurement outcome, A conveys the unitary operation that B needs to perform on qubit B to transform its qubit to \( \rho^T \). There are four possible equally likely outcomes of the joint measurement:

\[
|\Phi^\pm\rangle = \frac{|00\rangle \pm |11\rangle}{\sqrt{2}}, \quad |\Psi^\pm\rangle = \frac{|01\rangle \pm |10\rangle}{\sqrt{2}}.
\]
We can associate each of the four outcomes to four unitary operations in terms of Pauli operations that must to be conveyed to B:

\[
|\Phi^+\rangle \leftrightarrow I, \quad |\Psi^+\rangle \leftrightarrow \sigma_1, \quad |\Psi^-\rangle \leftrightarrow i\sigma_2, \quad |\Phi^-\rangle \leftrightarrow \sigma_3.
\]

Depending upon the measurement outcome, the qubit at B attains one of the following states:

\[
|\Phi^+\rangle \leftrightarrow \frac{1}{\sqrt{2}} \begin{pmatrix} (1+\lambda)\rho_{00} + (1-\lambda)\rho_{11} \\ 2\rho_{10} \\ 2\rho_{01} \end{pmatrix}, \quad \rho_{00},
\]

\[
|\Phi^-\rangle \leftrightarrow \frac{1}{\sqrt{2}} \begin{pmatrix} (1+\lambda)\rho_{00} + (1-\lambda)\rho_{11} \\ -2\rho_{10} \\ (1+\lambda)\rho_{01} + (1-\lambda)\rho_{11} \end{pmatrix}, \quad \rho_{01},
\]

\[
|\Psi^+\rangle \leftrightarrow \frac{1}{\sqrt{2}} \begin{pmatrix} (1+\lambda)\rho_{11} + (1-\lambda)\rho_{00} \\ 2\rho_{10} \\ 2\rho_{01} \end{pmatrix}, \quad \rho_{10},
\]

\[
|\Psi^-\rangle \leftrightarrow \frac{1}{\sqrt{2}} \begin{pmatrix} (1+\lambda)\rho_{11} + (1-\lambda)\rho_{00} \\ -2\rho_{10} \\ (1+\lambda)\rho_{01} + (1-\lambda)\rho_{11} \end{pmatrix}, \quad \rho_{11}.
\]

The four unitary operations are encoded into two classical bits that are conveyed over the classical channel. The mapping of unitary operations to classical bits is given below:

\[
I \leftrightarrow 00, \quad \sigma_1 \leftrightarrow 01, \quad \sigma_3 \leftrightarrow 10, \quad i\sigma_2 \leftrightarrow 11.
\]

Any noise in the classical channel affects the transmission and therefore results in teleportation of unintended state. To teleport a quantum state \(\rho\), we model the existential noise in the Werner state and the noisy classical channel together as transmission of \(\rho\) through a virtual quantum channel \(\mathcal{N}\). Any quantum channel is mathematically expressed as a completely positive and trace preserving (CPTP) map that maps density matrices to density matrices [9]:

\[
\mathcal{N} : \mathcal{S}(\mathcal{H}_A) \rightarrow \mathcal{S}(\mathcal{H}_B),
\]

where,

\[
\mathcal{S}(\mathcal{H}) = \{\rho : \rho \geq 0, \text{tr}(\rho) = 1\}.
\]

The classical channel used here is a discrete memoryless channel (DMC) [10] characterized by channel transition probabilities \(\{p(y|x)\}\). For simplicity we consider the binary symmetric channel with cross over probability \(\epsilon\). For transmission of two bits, we can write the channel transition probabilities in the form of a matrix:

\[
P = \begin{pmatrix}
(1-\epsilon)^2 & \epsilon(1-\epsilon) & \epsilon(1-\epsilon) & \epsilon^2 \\
\epsilon(1-\epsilon) & (1-\epsilon)^2 & \epsilon^2 & \epsilon(1-\epsilon) \\
\epsilon(1-\epsilon) & \epsilon^2 & (1-\epsilon)^2 & \epsilon(1-\epsilon) \\
\epsilon^2 & \epsilon(1-\epsilon) & \epsilon(1-\epsilon) & (1-\epsilon)^2
\end{pmatrix},
\]

where \(P = [p_{ij}] = [p(Y = i|X = j)], i, j \in \{0, 1\}^2\). The virtual quantum channel \(\mathcal{N}\) should accommodate all possible types errors that may occur in the transmission of classical bits and reflect in the teleported state:

\[
\mathcal{N}(\rho) = (1-\epsilon)^2 I\rho_{00} I + \epsilon(1-\epsilon)\sigma_1 \varphi_{00} \sigma_1 + \epsilon(1-\epsilon)\sigma_3 \varphi_{00} \sigma_3 + \epsilon^2(\sigma_2) \varphi_{00} (\sigma_2)^d.
\]

**B. Calculation of coherent information of the virtual quantum channel**

We know from classical information theory [10] that the capacity of a classical channel is obtained by maximizing the mutual information between random variables at the communicating nodes over all input distributions. In the quantum scenario, the situation is very interesting. There exist various notions of capacities with and without the use of entanglement defined in [11, 12, 13]. Holevo [14], Schumacher and others [15, 16] gave the expression for classical capacity of a quantum channel that uses quantum states for communication. Smith summarized these in [16]. We are interested in the maximum coherent information of the virtual quantum teleportation channel with prior entanglement shared between the communicating nodes. From the work of Barnum et al. [17, 18], coherent information \(I_c\) plays the same role in quantum capacity as mutual information plays in classical capacity. We maximize the coherent information:

\[
I_c(\mathcal{N}) = \max_\rho S(\mathcal{N}(\rho)) - S_c(\rho, \mathcal{N}) \tag{1}
\]

where \(S\) denotes the Von Neumann entropy of input \(\rho\) and \(S_c(\rho, \mathcal{N})\) is the exchange entropy. Without loss of generality, we can assume that the input \(\rho\) is diagonal in the computational basis. We can then choose the basis states of \(\mathcal{H}_A\) for communication. For instance, we wish to teleport state \(|0\rangle\langle 0|\) or state \(|1\rangle\langle 1|\) chosen with probabilities \(p\) and \(1-p\) respectively. We can write the input state as a mixture:

\[
\rho = p |0\rangle\langle 0| + (1-p) |1\rangle\langle 1| = \begin{pmatrix} p & 0 \\
0 & 1-p \end{pmatrix}.
\]

The uncertainty in the input state \(\rho\) is given by

\[
S(\rho) = h(p).
\]

To characterize the quantum virtual channel completely, we express the action of this map on the basis of space \(S(\mathcal{H}_A)\):

\[
\mathcal{N}(|0\rangle\langle 0|) = \frac{1}{2} \begin{pmatrix} (1+\lambda)(1-\epsilon) + (1-\lambda)\epsilon & 0 \\
0 & (1-\lambda)(1-\epsilon) + (1+\lambda)\epsilon \end{pmatrix},
\]

\[
\mathcal{N}(|1\rangle\langle 1|) = \frac{1}{2} \begin{pmatrix} (1-\lambda)(1-\epsilon) + (1+\lambda)\epsilon & 0 \\
0 & (1+\lambda)(1-\epsilon) + (1-\lambda)\epsilon \end{pmatrix}.
\]

This gives us

\[
\mathcal{N}(\rho) = p\mathcal{N}(|0\rangle\langle 0|) + (1-p)\mathcal{N}(|1\rangle\langle 1|), \quad \mathcal{N}(\rho) = \frac{1}{2} \begin{pmatrix} \zeta_1 & 0 \\
0 & \zeta_2 \end{pmatrix}, \tag{2}
\]

where,

\[
\zeta_1 = p(1-\epsilon)(1+\lambda) + pe(1-\lambda) + (1-p)(1-\lambda)(1-\epsilon) + (1-p)(1+\lambda)\epsilon,
\]

\[
\zeta_2 = pe(1+\lambda) + p(1-\lambda)(1-\epsilon) + (1-p)(1-\lambda)(1-\epsilon) + (1-p)(1+\lambda)\epsilon.
\]

\[
\Rightarrow S(\mathcal{N}(\rho)) = h\left(\frac{\zeta_1 + \zeta_2}{2}\right). \tag{4}
\]

In order to evaluate \(S_c(\rho, \mathcal{N})\), we reiterate the procedure given in [17]. We invoke the Stinespring’s dilation theorem [9] as follows.
**Theorem.** Let \( T : S(\mathcal{H}) \rightarrow S(\mathcal{H}) \) be a completely positive and trace-preserving map between states on a finite-dimensional Hilbert space \( \mathcal{H} \). Then there exists a Hilbert space \( \mathcal{K} \) and a unitary operation \( U \) on \( \mathcal{H} \otimes \mathcal{K} \) such that \( T(\rho) = \text{Tr}_K(U(\rho \otimes |0\rangle\langle 0|)U^\dagger) \) for all \( \rho \in S(\mathcal{H}) \), where \( \text{tr}_K \) denotes the partial trace on the \( \mathcal{K} \)-system. The ancilla space \( \mathcal{K} \) can be chosen such that \( \dim \mathcal{K} \leq \dim^2 \mathcal{H} \). This representation is unique up to unitary equivalence.

It essentially says that propagation of the quantum state \( \rho \) through a quantum channel can be seen as a unitary interaction with an environment \( E \). Suppose \( \{|e_k\}\) forms a basis for the Hilbert space of environment \( E \), then assuming the environment to be initially in the state \( |e_0\rangle \langle e_0| \), we have

\[
\mathcal{N}(\rho) = \text{Tr}_E \left( U \rho U^\dagger \right) = \sum_{k=0}^{\dim \mathcal{K} - 1} \langle e_k | U \rho U^\dagger | e_k \rangle.
\]

There is an equivalent representation of the propagation of the state \( \rho \) through the channel \( \mathcal{N} \) in terms of Kraus operators:

\[
\mathcal{N}(\rho) = \sum_{k=0}^{\dim \mathcal{K} - 1} E_k \rho E_k^\dagger.
\]

The equivalence between the Stinespring’s dilation theorem and Kraus operator representation can be written in terms of basis of \( \mathcal{K} \) and unitary \( U \) [9]:

\[
E_k = \langle e_k | U | e_0 \rangle.
\]

This is the operator sum representation for the effect of the channel on state \( \rho \). \( S_\epsilon(\rho, \mathcal{N}) \) is obtained by evaluating the Von Neumann entropy of the final state of the environment \( W \) after the unitary interaction.

\[
\rho_E = \text{Tr}_A \left( U \rho U^\dagger \right) = W.
\]

In order to maximize (1), we write the operator sum representation of \( \mathcal{N} \). Rearranging equation (2):

\[
\mathcal{N}(\rho) = \frac{(1+\lambda)(1-\epsilon)}{2} \left( \begin{array}{ccc} p & 0 & 0 \\ 0 & 1-p & 0 \\ 0 & 0 & p \end{array} \right) + \frac{(1-\lambda)(1-\epsilon)}{2} \left( \begin{array}{ccc} 1-p & 0 & 0 \\ 0 & p & 0 \\ 0 & 0 & 1-p \end{array} \right) + \frac{(1-\lambda)\epsilon}{2} \left( \begin{array}{ccc} 1-p & 0 & 0 \\ 0 & p & 0 \\ 0 & 0 & 1-p \end{array} \right).
\]

We can express

\[
\mathcal{N}(\rho) = \sum_{k=0}^{3} E_k \rho E_k^\dagger,
\]

where,

\[
E_0 = \sqrt{\frac{(1+\lambda)(1-\epsilon)}{2}} I, \quad E_1 = \sqrt{\frac{(1-\lambda)(1-\epsilon)}{2}} \sigma_1, \\
E_2 = \sqrt{\frac{(1-\lambda)\epsilon}{2}} I, \quad E_3 = \sqrt{\frac{(1+\lambda)\epsilon}{2}} \sigma_1.
\]

Thus, \( \dim \mathcal{K} = 4 \) suffices to represent the action of the channel. The dimension of \( \mathcal{K} \), \( |\mathcal{K}| = 4 \). Therefore,

\[
S_\epsilon(\rho, \mathcal{N}) = S(W) = -\text{Tr}(W \log_2 W),
\]

where,

\[
W_{ij} = \text{Tr}(E_i \rho E_j^\dagger).
\]

This gives us

\[
W = \frac{1}{2} \left( \begin{array}{ccc} (1+\lambda)(1-\epsilon) & 0 & J \\ 0 & (1-\lambda)(1-\epsilon) & 0 \\ J & 0 & (1-\lambda)\epsilon \end{array} \right).
\]

Therefore,

\[
J = \sqrt{(1-\lambda^2)\epsilon(1-\epsilon)}.
\]

We see that the evaluated coherent information is akin to the capacity of a binary symmetric channel for \( \lambda = 1 \). Therefore, using a classical channel, we can teleport qubits from source to destination nodes, that have prior entanglement. For the Werner state, this is true when \( \lambda > \frac{1}{3} \) [19].

**C. Fidelity of noisy teleportation**

We calculate the fidelity of teleportation when pure states are sent through this virtual quantum channel. Fidelity between pure state \( |\varphi\rangle \) and the state received at the end of the channel represented by its density matrix \( \mathcal{N}(|\varphi\rangle\langle \varphi|) \) is given by

\[
F(|\varphi\rangle, \mathcal{N}(|\varphi\rangle\langle \varphi|)) = \langle \varphi | \mathcal{N}(|\varphi\rangle\langle \varphi|) | \varphi \rangle.
\]

If we use the above definition and use it calculate the average fidelity, we get

\[
\mathcal{F} = \rho \left( 0 | \mathcal{N} (|0\rangle\langle 0|) | 0 \rangle \right) + (1 - \rho) \left( 1 | \mathcal{N} (|1\rangle\langle 1|) | 1 \rangle \right),
\]

\[
\rho \left( (1 + \lambda)(1 - \epsilon) + (1 - \lambda)\epsilon \right) \\
+ \frac{1 - \rho}{2} \left( (1 + \lambda)(1 - \epsilon) + (1 - \lambda)\epsilon \right),
\]

\[
= \frac{1}{2} \left( (1 + \lambda)(1 - \epsilon) + (1 - \lambda)\epsilon \right).
\]

We note that if \( \epsilon = 0 \), we get noiseless classical channel and an average fidelity of \( \frac{1 + \lambda}{2} \) supporting a known fact [20]. If an addition, \( \lambda = 1 \), we get the noiseless channel and fully entangled state and hence an average fidelity of 1.

**III. HYPER ENTANGLEMENT AND ITS APPLICATIONS**

Hyper entanglement is relatively new in quantum mechanics and is catching attention of many researchers [21]. [22]. The possibility of a quantum bit to be entangled to two different qubits over different degrees of freedom provides new avenues to exploit for information processing. Various researchers are working in the experimental preparation of hyper entangled states [23]. We consider the nodes A, B and C sharing hyper entangled qubits among them:

\[
|\psi\rangle_{ABC} = |\lambda\rangle_C |\psi^+\rangle^{1}_{AB} |\psi^+\rangle^{2}_{BC} |\varphi\rangle^2_C,
\]
where $|\chi\rangle_C$ is the arbitrary state of C in DOF 1 and $|\varphi\rangle_C$ is the arbitrary state of C in DOF 2. If nodes A and C act as transmitters, then A and C can encode classical information by applying Pauli rotations on their qubits. In other words, A makes use of degree of freedom 1 and C makes use of degree of freedom 2. After encoding both send their qubits to B which performs joint measurement. Joint measurement can be done on compatible observables. This leads to the transmission of four classical bits from source to destination nodes [24]. If we compare it with fully entangled tripartite state, we note that three classical bits can be sent from source nodes to destination node [25]. An additional bit advantage comes from the use of additional degree of freedom for encoding.

We extend this idea to multipartite hyper entangled qubits. Two of the prominent tripartite quantum states are GHZ and W states. These two states are symmetric with respect to permutation. Suppose, GHZ and W states are distributed among nodes A, B and C. Suppose B wants to teleport a qubit to A or C. How well can node B use this geometry for teleportation?

To answer the above question, we need to evaluate the reduced density matrix by tracing out over the third node which is ignored for teleportation. Tracing out node C, we get $\rho^{GHZ}_{AB}$ or $\rho^{W}_{AB}$ depending on whether GHZ or W state is shared between A, B and C. It is a known fact that while $\rho^{GHZ}_{AB}$ is separable, $\rho^{W}_{AB}$ is non-separable and hence entangled. This can be verified using the reduction criteria of Horodecki et al. [26]. In other words, partial trace of fully entangled state $\rho^{GHZ}_{ABC}$ over C’s qubit gives rise to the separable state $\rho^{GHZ}_{AB}$; partial trace of partially entangled state $\rho^{W}_{ABC}$ over C’s qubit gives rise to the partially entangled state $\rho^{W}_{AB}$. The inherent quantum correlations can affect the maximum rate of communication between A and B. The notion of W state is generalized to $n$ qubits [27] and refers to the equal quantum superposition of all possible pure states in which exactly one of the qubits is state $|1\rangle$, while all other ones are in state $|0\rangle$:

$$|\psi^W\rangle = \frac{1}{\sqrt{n}} \left( |000..0\rangle + |010..0\rangle + |0010..0\rangle + |00010..0\rangle + |000010..0\rangle \right), \forall n \geq 3.$$  \hspace{1cm} (11)

If such a quantum state is shared among $n$ nodes and node B wants to teleport a qubit to node A, we need the reduced density matrix $\rho^{W}_{AB}$:

$$\rho^{W}_{AB} = \frac{1}{n} \left( |0\rangle\langle 0| + |1\rangle\langle 1| + |01\rangle\langle 01| + |10\rangle\langle 10| \right).$$

$$= \frac{1}{n} \left( \begin{array}{cccc}
0 & 0 & 1 & 1 \\
0 & 1 & 1 & 0 \\
1 & 1 & 0 & 0 \\
1 & 0 & 0 & 0 \\
\end{array} \right).$$

For the reason that the reduced density matrix of W state is partially entangled, we would like to study it from the perspective of teleportation.

We check if the reduced density matrix is useful for teleportation. This is given by the fully entangled fraction (FEF) [6].

$$\text{FEF} = \max_U \langle \Phi^+ | (U^\dagger \otimes I) \rho^{W}_{AB} (U \otimes I) | \Phi^+ \rangle$$  \hspace{1cm} (12)

where, the maximization is over unitary $U \in \mathbb{C}^{2 \times 2}$. We choose $U$ of the form:

$$U = \begin{pmatrix}
u_{00} & u_{01} \\
u_{10} & u_{11} \end{pmatrix}.$$

Since $U$ is unitary, we have the following conditions:

$$|u_{00}|^2 + |u_{01}|^2 = 1, \quad |u_{00}|^2 + |u_{10}|^2 = 1,$$

$$|u_{01}|^2 + |u_{11}|^2 = 1, \quad |u_{10}|^2 + |u_{11}|^2 = 1,$$

$$\nu_{00}u_{01} + \bar{\nu}_{10}u_{11} = 0, \quad \bar{\nu}_{00}u_{01} + \nu_{10}u_{11} = 0,$$

$$\Rightarrow |u_{01}| = |u_{10}|, \quad |u_{00}| = |u_{11}|.$$  \hspace{1cm} (13)

Substituting (13) in (12), we find that

$$\text{FEF} = \frac{1}{2n} \left\{ \max_{|u_{00}|^2 + |u_{01}|^2 = 1, \quad |u_{00}|^2 + |u_{10}|^2 = 1, \quad |u_{01}|^2 + |u_{11}|^2 = 1, \quad |u_{10}|^2 + |u_{11}|^2 = 1} \right\}\left\{ \begin{array}{c}
(n - 2)|u_{01}|^2 + |u_{10}|^2 \\
+ \nu_{10}u_{01} + \bar{\nu}_{01}u_{10} + |u_{01}|^2 \\
\end{array} \right\},$$

$$= \frac{1}{2n} \left\{ \max_{|u_{00}|^2 + |u_{01}|^2 = 1, \quad |u_{00}|^2 + |u_{10}|^2 = 1, \quad |u_{01}|^2 + |u_{11}|^2 = 1, \quad |u_{10}|^2 + |u_{11}|^2 = 1} \right\}\left\{ \begin{array}{c}
(n - 2)|u_{01}|^2 + |u_{10}|^2 \\
+ \nu_{01}u_{10} + |u_{01}|^2 \\
\end{array} \right\}.$$
Let

\[ u_{01} = re^{i\theta_1}, \quad u_{10} = re^{i\theta_2}. \]

Therefore,

\[ \text{FEF} = \frac{1}{2n} \left\{ \max_{r,\theta_1,\theta_2} (n-2) + r^2 (2 + 2 \cos(\theta_1 - \theta_2) - (n-2)) \right\}. \]

(14)

For maximum value,

\[ \theta_1 - \theta_2 = 2m \pi, m \in \mathbb{Z} \text{ and } r = 1. \]

One such \( U \) that maximizes FEF is

\[ U = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} e^{i\theta}, \]

where, \( \theta \) is arbitrary. This gives

\[ \text{FEF} = \frac{4}{2n} \begin{cases} \frac{1}{2}, & \text{if } n = 3, \\ \frac{1}{2}, & \text{if } n \geq 4. \end{cases} \]

(15)

Horodecki et al. showed that as long as FEF is greater than \( \frac{1}{2} \), it is useful for teleportation [6]. Hence, this reduced state is useful for teleportation only for \( n = 3 \). Therefore, node B in figure [2] can teleport a qubit to any of the nodes and can do so over different degrees of freedom.

IV. CONCLUSION AND FUTURE WORK

In this paper, we looked at teleportation as a transfer of qubits over a virtual quantum channel assisted by noisy classical communication. For this, we considered the Werner state and a binary symmetric channel for classical communication to teleport quantum states. We evaluated the maximum coherent information that can be sent over this virtual quantum channel. We further investigated the use of hyper entangled states for teleportation. We found that W state is useful for teleportation and promises utility of tripartite hyper entangled qubits such as W state for teleportation. It would be interesting to evaluate the coherent information of virtual quantum channels in the setting of multipartite hyper entangled qubits.

ACKNOWLEDGMENT

The authors would like to thank the Space Technology Cell grant ISTC-311 for this work.

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